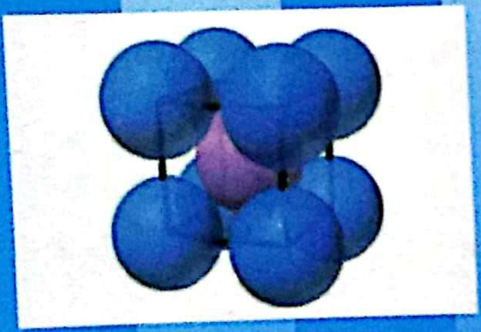
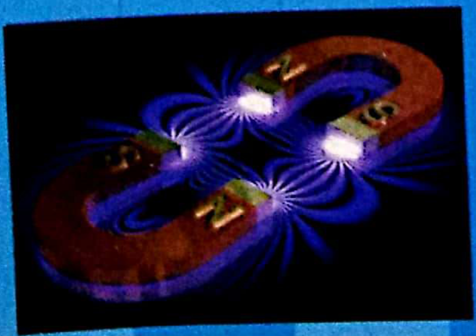
PHYSICS

For Grade **XII**





Khyber Pakhtunkhwa Textbook Board Peshawar

PREFACE

This book is based on national curriculum 2006.

The book include some new approaches of STS (Science Technology and society) connection so that student may aware about the applications of Physics, specially in the field of Medical sciences (MRI, CT Scan, and ECG etc.).

Throughout the book an attempt has been made to present the math symbols in italic for its easy understanding.

In case of any errors we request the readers to point out these errors so that they are eliminated in the next edition. I hope that the teachers students, and experts will keep us informed about the new curriculum and the textbook, so that we may be able to improve the textbook, because there is always room for improvement.

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UNIT 11
ELECTROSTATICS

After studying this chapter the students will be able to

- state Coulomb's law and explain that force between two point charges is reduced in a medium other than free space using Coulomb's law.
- derive the expression $E = 1/4\pi\varepsilon_o\ q/r^2$ for the magnitude of the electric field at a distance 'r' from a point charge 'q'.
- describe the concept of an electric field as an example of a field of force.
- define electric field strength as force per unit positive charge.
- solve problems and analyse information using $E = F/q$.
- solve problems involving the use of the expression $E = 1/4\pi\varepsilon_o\ q/r^2$.
- calculate the magnitude and direction of the electric field at a point due to two charges with the same or opposite signs.
- sketch the electric field lines for two point charges of equal magnitude with same or opposite signs.
- describe the concept of electric dipole.
- define and explain electric flux.
- describe electric flux through a surface enclosing a charge.
- state and explain Gauss's law.
- describe and draw the electric field due to an infinite size conducting plate of positive or negative charge.
- sketch the electric field produced by a hollow spherical charged conductor.
- sketch the electric field between and near the edges of two infinite size oppositely charged parallel plates.
- define electric potential at a point in terms of the work done in bringing unit positive charge from infinity to that point.
- define the unit of potential.
- solve problems by using the expression $V = W/q$.
- describe that the electric field at a point is given by the negative of potential gradient at that point.

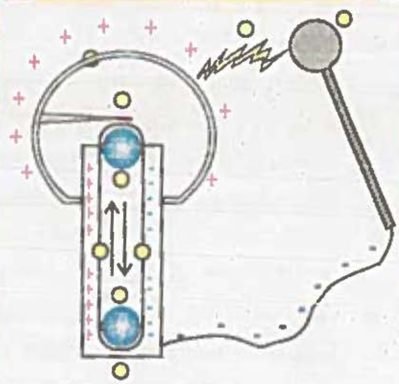
- solve problems by using the expression $E = V/d$.
- derive an expression for electric potential at a point due to a point charge.
- calculate the potential in the field of a point charge using the equation $V = 1/4\pi\varepsilon_o\, q/r$.
- define and become familiar with the use of electron volt.
- define capacitance and the farad and solve problems by using $C=Q/V$.
- describe the functions of capacitors in simple circuits.
- solve problems using formula for capacitors in series and in parallel.
- explain polarization of dielectric of a capacitor.
- demonstrate charging and discharging of a capacitor through a resistance.
- prove that energy stored in a capacitor is $W=1/2\ QV$ and hence $W=1/2\ CV^2$.

Two fundamental processes of electrostatics are introduced: Coulomb's law for force between stationary charges and the principle of superposition for electric field configurations. The electric field at a point in space is defined and used, with Coulomb's law, to derive an expression for the electric field at a distance from a point charge. The concept of work done by an electric field on a charged particle is introduced. For practical purposes, the concept of electric field is translated into concepts of electric potential and electrical potential energy.

The principle of superposition explains the fact that a near-uniform electric field can be produced by two charged parallel conducting plates. The absence of an electric field in hollow conductors is discussed. The presence of strong electric fields in the vicinity of sharp points on charged conductors is identified and applied to corona discharges in relation to photocopiers and laser printers.

Another quantity being discussed that plays an important role in electrical circuits is capacitance and its dependence on the dielectrics.

For your information



A Van de Graaff generator is an electrostatic generator which uses a moving belt to accumulate very high amounts of electrical potential on a hollow metal globe on the top of the stand. A Van de Graaff generator operates by transferring electric charge from a moving belt to a terminal. First invented in 1929, the Van de Graaff generator became a source of high voltage for accelerating subatomic particles to high speeds, making it a useful tool for fundamental physics research.

11.1 PROPERTIES OF CHARGE

In the previous classes we have studied that similar charges repel and opposite charges attract each other with a force, known as force of interaction. There are two different kinds of charges, which are called positive and negative charges.

Electrons have a negative charge and protons have positive charge.

The atom is electrically neutral. In SI units, charge is measured in coulombs (symbol C). The charge carried by an elementary particle is written as e, and its magnitude is

$$1e = 1.6\times10^{-19}C$$

The important characteristic of the charge is that charge is quantized. Quantization of charge means that it exists in discrete packets. Charge q is an integral multiple of minimum elementary charge e, i.e.,

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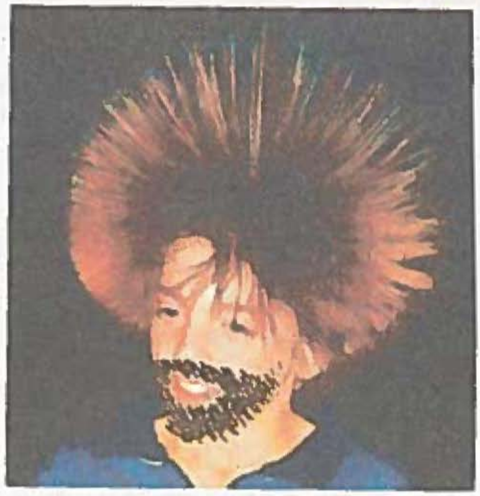


When you charge up your body by touching the Van der Graaf generator your hair stands on end. The hair stands because all hair gains the same electric charge and repel each other. The force of repulsion is so great that it exceeds the weight of each hair strand. Your arms do not lift away from your body though - even they have the same charge as your body. This is because they are too heavy!

$$q=ne \qquad \ldots (11.1)$$

Here we make quantitative analysis of the nature of these forces. We would like to determine the magnitude and direction of such forces.

11.1 COULOMB'S LAW

The quantitative measurement of the force between two electric charges was first madeby Coulomb (1736-1805).

He carried out series of experiments to measure the force between electric charges using an apparatus known as torsion balance.

Coulomb expresses his experimental data in the form of a statement which is known as Coulomb's law.

It states *"The magnitude of the force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them."*

The magnitude of the force $\vec{F}$ between two electric charges q_1 and q_2 separated by distance r can be expressed as

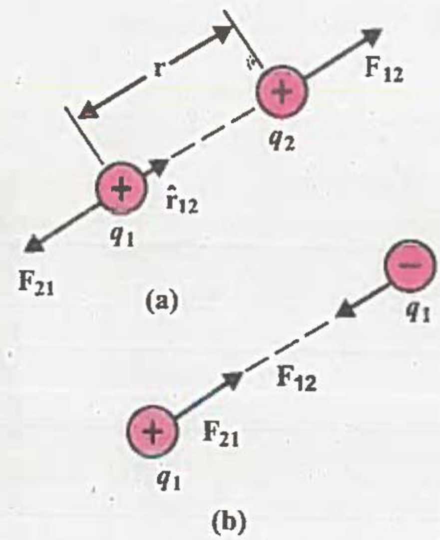


Figure 11.1: Coulomb force between (a) similar charges, and (b) opposite charges.

$$F \propto q_1 q_2 \qquad ...(A)$$

$$F \propto \frac{1}{r^2} \qquad ...(B)$$

combining Eq(A) & Eq(B)

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k\frac{q_1 q_2}{r^2} \qquad ...(11.2)$$

where k is a constant of proportionality and its value depends upon the system of the units used and the medium between the charges. The electric charges q_1 and q_2 are assumed to be point or localized charges, provided the size of the bodies carrying the charges is very small as compared to the distance between them.

In order to show the direction of the force we use unit vector along the line joining the two charges. In fig:11.1(a) $\hat{r}_{12}$ is unit vector, pointing from the charge q_1 towards the charge q_2 which show the force on charge q_2 due to charge q_1 i.e.,

$$\vec{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad ...(11.3)a$$

For like charges the product q_1q_2 will be positive and a force of repulsion between these two charges will be F_{21}. Similarly for unlike charges the product q_1q_2 will be negetive and a force of attraction between these two charges will be F_{12} . Similarly unit vector $\hat{r}_{21}$, pointing from the charge q_2 towards the charge q_1 which show the force on charge q_1 due to charge q_2 is given by

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{21} \qquad ...(11.3)b$$

Since $\hat{r}_{12} = -\hat{r}_{21}$

So from Eq: 11.3(a) and 11.3(b) we can write

$$\vec{F}_{21} = -\vec{F}_{12} \qquad ...(11.4)$$

Where $\vec{F}_{21}$ is the force exerted by the charge q_1 on q_2 and $\vec{F}_{12}$ is the force exerted by the charge q_2 on q_1. Eq 11.4 shows that the two forces are same in magnitude but opposite in direction which is illustrated in fig 11.1(b).

The constant k can be generally expressed in terms of permittivity of the free space ε_o i.e., $k = \dfrac{1}{4\pi\varepsilon_o}$

Experimentally measured value of

$$\varepsilon_o = 8.85\times10^{-12}\,C^2\,N^{-1}\,m^{-2}$$

so $k = \dfrac{1}{4\pi\varepsilon_o} = 9.0\times10^9\ N\,m^2\,C^{-2}$

Coulomb's law in material media

Eq:(11.2) gives the force between two charges when there is air or vacuum between them. But it is experimentaly observed that when an insulator is placed

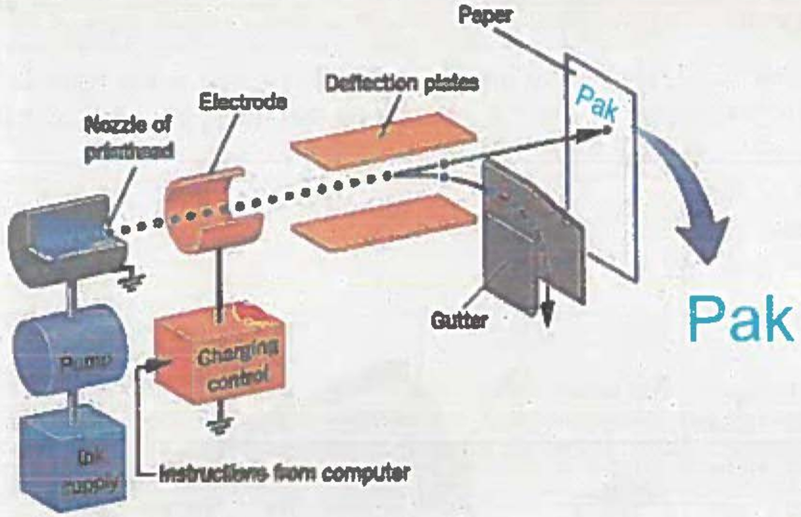


Figure 11.9: charging of ink droplets between two oppositely charged plates.

Whenever ink is to be placed on the paper, the charging control, responding to instructions from the computer, turns off the electric field. The uncharged droplets fly straight through the deflection plates and strike the paper.

Quiz?

What are the basic differences between laser printer and photocopier?
State any two other applications of electrostatic.
State any two hazards of electrostatic.

Example 11.3

A metallic sphere of diameter 30 cm carries a charge of 600 μ C. Find the electric field intensity (a) at a distance of 50cm from the centre of the sphere and (b) at the surface of sphere.

Solution:

The electric field due to a charged sphere has spherical symmetry. Therefore, a charged sphere behaves for external points as if the whole charge is placed at its centre.

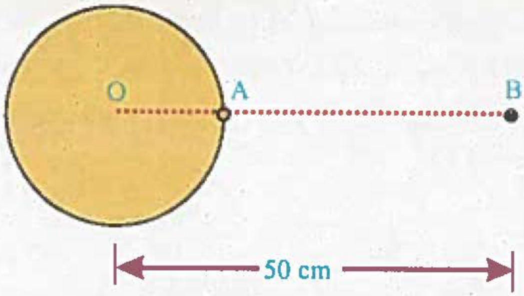


Figure 11.10: field intensity due to a charge sphere

(a) Distance of a charge from the sphere $r = OB = 50\,cm$

$$= 50\times10^{-2}\ m;$$

Charge on sphere $q = 600\mu C = 600\times10^{-6}\ C$

Electric field intensity from the centre of the sphere is $E = k\dfrac{q}{r^2}$

$$= 9\times10^{9}\times\frac{600\times10^{-6}}{(50\times10^{-2})^2}$$

$$= 21.6\times10^{6}\ N/C$$

(b) $r = \dfrac{d}{2} = \dfrac{OA}{2} = \dfrac{30cm}{2} = 0.15m$

Electric field intensity from the surface of a sphere $E = k\dfrac{q}{r^2}$

$$= 9\times10^{9}\times\frac{600\times10^{-6}}{(0.15)^2}$$

$$= 24\times10^{7}\ N/C$$

11.4 Electric Flux

Before we start to define electric flux we start with area A and we take it as a vector quantity. As you may recall in chap two (class 11th) area is geometric representation of a vector product.

In this context A is a vector whose magnitude is equal to the surface area and whose direction is normal to the surface area. The electric flux Φ for uniform field $\vec{E}$ and area $\vec{A}$ is defined as *the number of lines of force that pass through the area placed in the electric field.* i.e.,

$$\Phi = \vec{E}.\vec{A}$$
$$= EA\cos\theta \qquad ...(11.11)$$

Thus flux Φ is the scalar product of the electric field E and plane surface area A. Flux is the flow of field lines through surface area, which is placed in that field. For example Fig 11.11 (a) shows that there are 4 number of lines passing through the surface area of 1m^2 so flux is $4\ \text{Nm}^2\text{C}^{-1}$. The electric flux Φ depends upon the orientation of surface area A with respect to the field lines

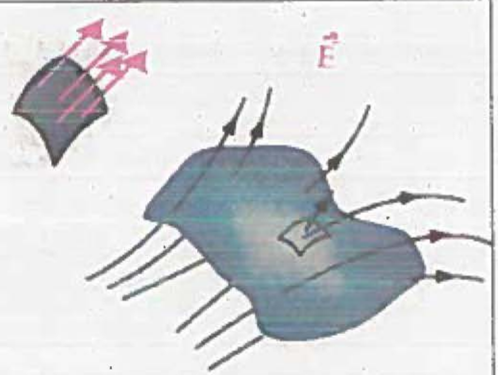


Figure11.11 (a) : electric flux through a unit area and curved surface

Flux at any angle

When the area $\vec{A}$ is tilted such that it is making an angle θ with the electric field lines as shown in Fig 11.11 (b) in this case the electric flux is

$$\Phi = \vec{E}.\vec{A} = EA\cos\theta$$

The number of lines passing through the area will be $E\,(A\cos\theta)$ depending upon angle θ.

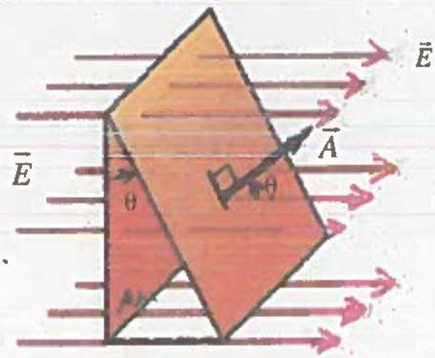


Figure11.11 (b) : electric flux at angle

Maximum Flux:-

If the surface is placed perpendicular to the electric field such that surface area $\vec{A}$ is parallel to electric field $\vec{E}$ (Fig 11.11 b) then maximum electric lines of force will pass

through the surface. Consequently maximum electric flux will pass through the surface.

The electric flux is $\Phi = EA\cos 0 = EA$

Therefore large number of lines passes through the area

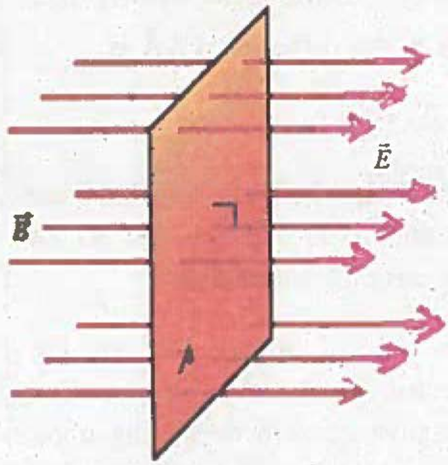


Figure11.11 (c):maximum electric flux

Zero Flux:-

If the surface is placed parallel to the electric field Fig.11.11(d) such that surface area $\vec{A}$ is normal to electric field $\vec{E}$ then no electric lines of force will pass through the surface. Consequently no electric flux will pass through the surface.

Then flux is

$$\Phi = EA\cos 90 = 0$$

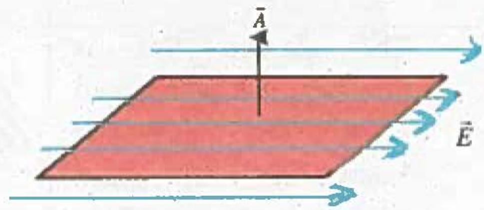


Figure11.11 (d) : electric flux

Electric flux through close surface

If the case is such that either field is non- uniform or the surface is curved, then we divide the surfaces into very small patches of area ΔA called differential area and assume that these are approximately flat and they are so small that electric field E is almost uniform over it. For one of these small patches of area (see Fig.11.12 (a)) electric flux is defined by the relation. (Differential form),

$$\Phi_E = E.\Delta A$$

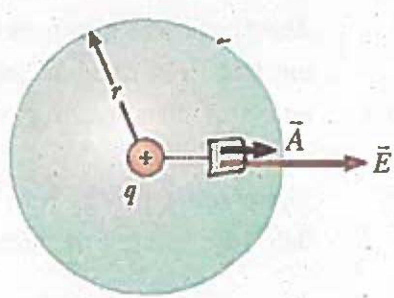


Figure11:12 (a) : electric flux

To calculate total flux through the whole surface we add flux from each patch such that

$$\Phi_E = \Sigma\, \vec{E}\,.\Delta\vec{A}$$

Now let us consider a closed surface of arbitrary shape the positive normal is taken to be "outward" from the volume being enclosed. If ΔA is the magnitude of the differential area and $\hat{n}$ is the unit normal to the surface in positive direction, the vector differential $\Delta \vec{A}$ is

$$\Delta \vec{A} = \Delta A \, \hat{n}$$

The fields lines in this case are parallel to surface area and do not intersect the surface.

In case of closed surface the electric flux may be positive negative or zero depending upon the number of lines entering or leaving the surface:

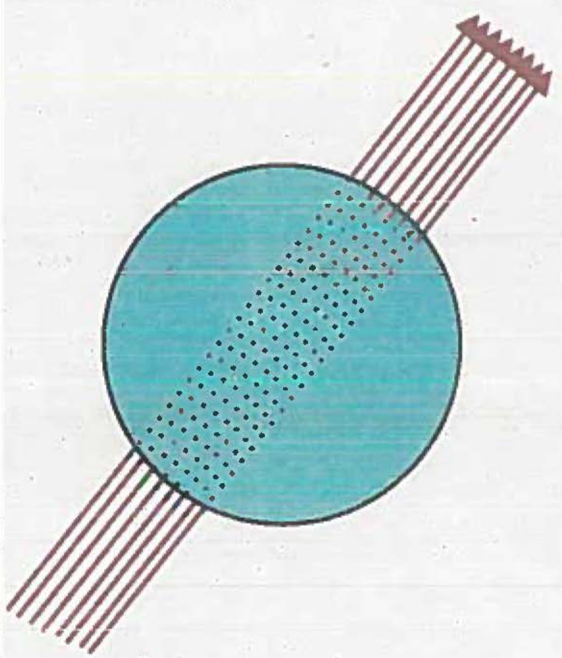


Figure 11.12b: positive, negative and zero flux through a close surface

(i) The electric flux is positive if net numbers of electric field lines are leaving the surface, that is, there is source of field lines in the closed surface.

(ii) The electric flux is negative if net numbers of electric field lines are entering the closed surface or more field lines are entering than leaving the surface; there is a sink of field lines in the closed surface.

(iii) The electric flux is zero if numbers of field lines entering are equal to field line leaving the surface or no field line intercepting the surface.

11.5 Gauss's law

The electric field of a given charge distribution can be calculated using Coulomb's law. The examples discussed before showed however, that the actual calculations can become quite complicated.

An alternative method to calculate the electric field of a given charge distribution relies on a theorem called Gauss' law. Gauss' law provides a relationship between the net electrical flux Φ through a closed surface and the net charge q enclosed by that surface.

Gauss' law states that *the net electric flux through a closed surface is equal to the total charge q enclosed by the surface divided by the permittivity of free space* ε_o.

In order to derive the expression for Gauss's law consider a spherical closed surface of radius r having a point charge q at its center as shown in fig.11.13.

Gauss was a German mathematician and scientist who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geophysics, electrostatics, astronomy and optics.

To calculate the electric flux through the whole surface it is divided into n-number of small pieces having area $\Delta A_1, \Delta A_2, \Delta A_3, \Delta A_4 \ldots\ldots \Delta A_n$. The intensity of electric field is same at every point as they are at equidistant from the charge.

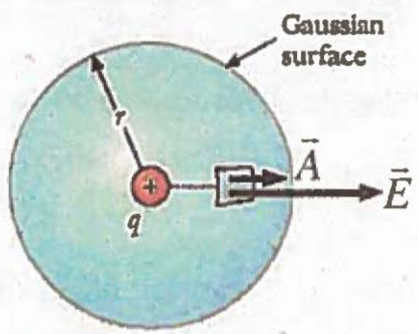


Figure 11.13 : flux through a close surface

The electric flux through the small elements ΔA_1 is

$$\Phi_1 = \vec{E}_1.\Delta\vec{A}_1 = E_1\,\Delta A_1\cos 0 = E_1\Delta A_1$$

The electric flux through the other small element ΔA_2 is

$$\Phi_2 = \vec{E}_2.\Delta\vec{A}_2 = E_2\Delta A_2\cos 0 = E_2\Delta A_2$$

Similarly the electric flux through ΔA_n is

$$\Phi_n = \vec{E}_n.\Delta\vec{A}_n = E_n\Delta A_n\cos 0 = E_n\Delta A_n$$

The total flux through the entire surface is

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \ldots\ldots\ldots + \Phi_n$$

$$\Phi = \Sigma_{surface}\, E\Delta A \qquad \text{...(11.12)}$$

But as electric field intensity E is constant over the sphere

Therefore

$$\Phi = E\,\Sigma_{surface}\,\Delta A$$

Since $$E = \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}$$

Thus the above equation can be written as

$$\Phi_E = \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}\Sigma_{surface}\Delta A$$

$$= \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}\text{ (total area enclosed by spherical surface)}$$

$$= \frac{q}{4\pi\varepsilon_o} \frac{1}{r^2}\left(4\pi r^2\right)$$

$$= \frac{q}{\varepsilon_o} \qquad \ldots(11.13)$$

The above equation shows that electric flux does not depend upon the shape or geometry of a closed surface. But it depends upon the medium and the charge enclosed by that surface.

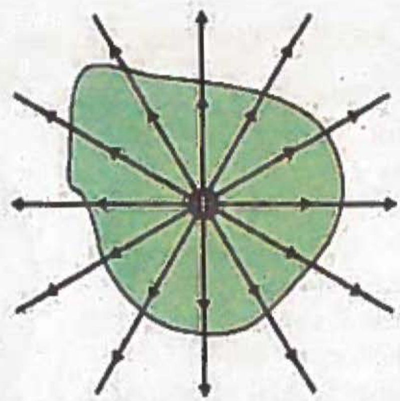


Fig 11.14 : flux through an irregular close surface

Let us consider an irregular closed surface S, enclosing a point charges $q_1, q_2, q_3 \ldots\ldots q_n$ as shown in fig 11.15.

Then the total electric flux through that closed surface is

$$\Phi_E = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \frac{q_3}{\varepsilon_o} \ldots\ldots\ldots \frac{q_n}{\varepsilon_o}$$

$$\Phi_E = \frac{1}{\varepsilon_o}(q_1 + q_2 + q_3 + \ldots\ldots\ldots q_n)$$

$$\Phi_E = \frac{1}{\varepsilon_o}\sum_{i=1}^{n} q_i$$

$$= \frac{1}{\varepsilon_o} \text{ (total charge enclosed in surface)}$$

$$= \frac{Q}{\varepsilon_o}$$

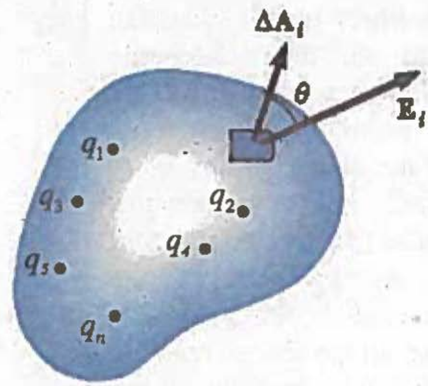


Fig: 11.15: number of charges in a closed surface

Thus Gauss's law shows that *the electric flux through any closed surface is $1/\varepsilon_o$ times the total charge enclosed in it.*

11.5.1 Applications of Gauss's law

Gauss's law provides a convenient method to calculate E in the case of sufficiently symmetrical charge distribution. Some of examples are now presented.

11.5.1.1 Location of Excess Charge on a Conductor

We know that the electric field E = 0 at all points due to electrostatic equilibrium in a conductor. We can make an imaginary Gaussian surface S, in the interior of a conductor, as shown in fig:. Because E = 0 everywhere in this surface, the net charge inside the surface has to be zero. Since E = 0 everywhere on the Gaussian surface S, the net charge inside that point is zero. That also means that there cannot be a net charge at any point within the conductor, because that tiny point could be put anywhere in the conductor. If that's so, that means all the charge must be on the outer surface of the conductor, as shown in fig 11.16.

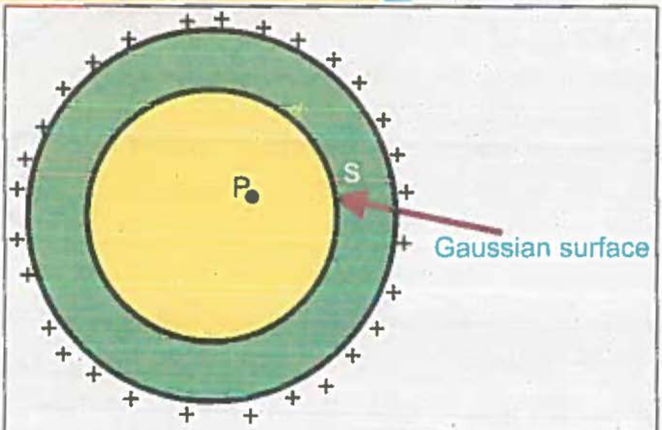


Fig: 11.16: Gaussian surface inside a conductor

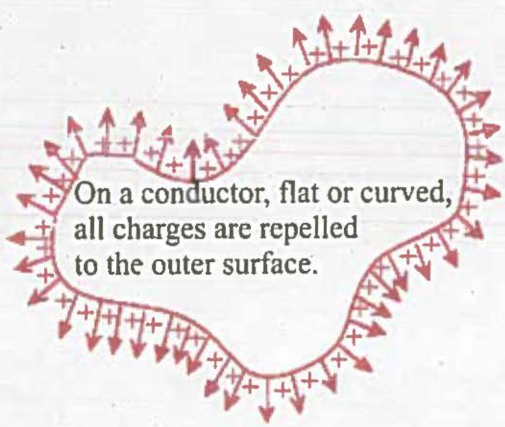


Fig: 11.17: Excess of charge on the surface of a conductor

Now let's consider a hollow conductor as shown in the illustration 11.18.

Since the conductor is hollow so there is no net charge at any point within the conductor. The cavity is surrounded by a Gaussian surface S which encloses no net charge and so there is no charge on the internal cavity. Again all the charge is deposited on the outer surface of the conductor.

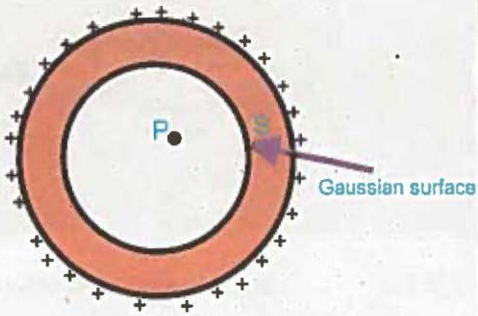


Figure 11.18 : an excess of charge on the outer surface of a hollow conductor

In the third case as shown in the fig: 11.19. Let's put a charge q inside the hollow conductor. We insulate it so no charges can jump from one surface to another. Now if we use Gauss's Law with Gaussian surface S again, the net charge of what it encloses has to be zero because there was no charge transfer between the charge and the conductor. Initially the conductor was uncharged but when charge q is inserted, then there will be negative charge on the inside cavity in order to maintain its neutral status.

So the other surface must have a charge equal but opposite the charge of the internal cavity, or the outer surface's charge is equal to that of charge q.

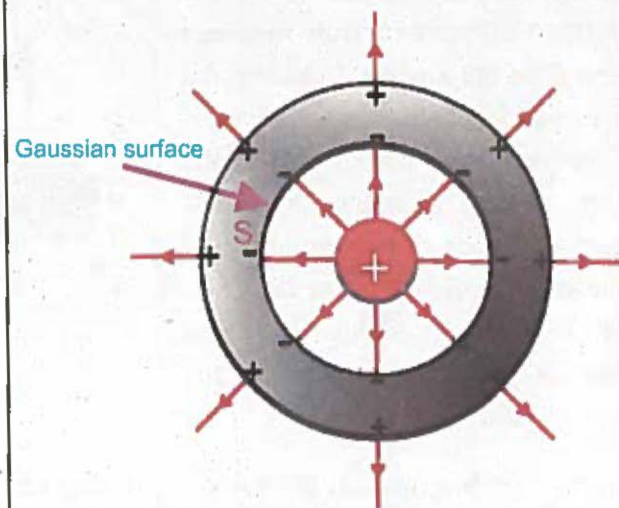


Figure 11.19 : charge q inside the hollow conductor having a Gaussian surface S

For your information

Figure 11.20 : Airlines Airbus A380 flew through a storm when 500 peoples were on the board but none of them were injured. As there is no electric field, no potential difference inside a metal shell, so one of the safest way to be inside a metal shell during thunderstorm.

11.5.1.3 Electric field Intensity Due To an Infinite Sheet of Charge

Consider an infinite plane sheet of charge with surface charge density σ. Let P be the point at a distance *r* from the sheet and *E* be the electric field at point P due to positive sheet of charges. Consider Gaussian surface in the form of a cylinder of cross-sectional area *A* perpendicular to the sheet of charge. The direction of *E* is perpendicular to face containing P and parallel to the curved surface.

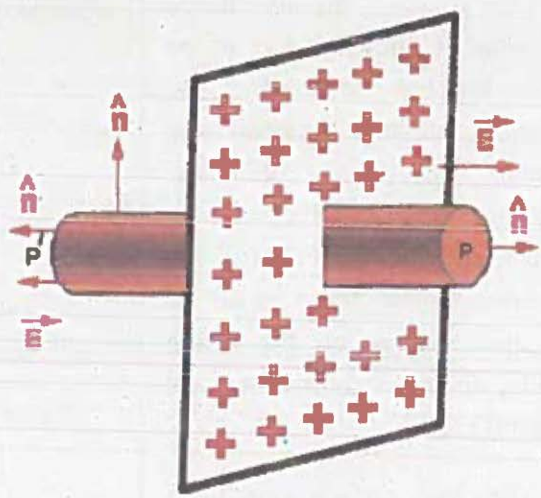


Figure 11.21 : Gaussian surface in the form of a cylinder perpendicular to the sheet of charge

Hence the flux through the curved surface is zero while it is 2 *EA* through the two end faces of the closed surface, where *A* is the area of cross section, as shown in Fig11.21. The charge enclosed by the surface will be σ *A*. Since we know by Gauss's law

$$\text{Total electric flux} = \frac{1}{\varepsilon_0} \times (\text{Charge enclosed by the closed surface})$$

$$\Phi=\frac{1}{\varepsilon_0}\times(Q)$$

As surface charge density $\sigma=Q/A$ or,

$$\sigma A=Q \qquad ...(11.14)$$

$$\Phi_E=\frac{\sigma A}{\varepsilon_0}$$

The electric flux through the end faces is

$$\vec{E}.\Delta\vec{A}+\vec{E}.\Delta\vec{A}=\frac{\sigma A}{\varepsilon_o}$$

Hence, $2EA=\dfrac{\sigma A}{\varepsilon_0}$

or $E=\dfrac{\sigma}{2\varepsilon_0}$ ...(11.15)

and $\vec{E}=\dfrac{\sigma}{2\varepsilon_0}\hat{r}$ ...(11.16)

Where $\hat{r}$ is the unit vector normal to the sheet and directed away from it.

11.5.1.3 Electric field intensity between two oppositely charged parallel plates.

Consider two oppositely charged parallel metal plates. The charge densities of these metal plates are $+\sigma$ and $-\sigma$. These plates are assumed to be of infinite length in order to avoid fringing field at end. Under these conditions, the field intensity will be uniform and normal to the plates. In order to find the electric field intensity E, at point P between the plates consider a Gaussian surface in the form of a box as shown in fig (11.22). Let A be the cross sectional area of the box but as the box sides are parallel to electric field E so the flux through sides will be

zero, the electric field intensity E is normal to the lower surface of Gaussian box. Therefore by Gauss's Law electric flux is

$$\Sigma \vec{E}.\Delta\vec{A} = \frac{Q}{\varepsilon_o}$$

$$\vec{E}.\Delta\vec{A} + \vec{E}.\Delta\vec{A} + \vec{E}.\Delta\vec{A} = \frac{Q}{\varepsilon_o}$$

$$\vec{E}\,\vec{A} + 0 + 0 = \frac{Q}{\varepsilon_o}$$

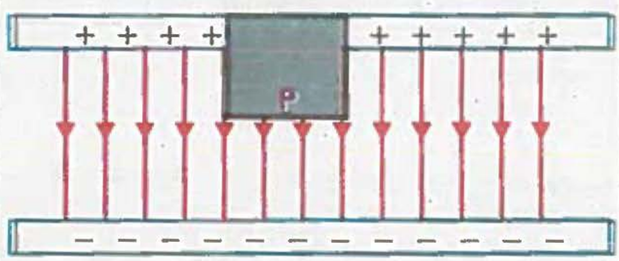


Figure 11.22: Gaussian surface in the form of a box between the plates

As the charge per unit area is σ and the charge enclosed in the box is σA. So

$$EA = \frac{\sigma A}{\varepsilon_o} \qquad \therefore Q = \sigma A$$

$$E = \frac{\sigma}{\varepsilon_o} \qquad \ldots(11.17)$$

and $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{r}$

This gives the electric field between oppositely charged parallel plates. The magnitude of the field is independent of the position between plates.

11.6 ELECTRIC POTENTIAL

Let us consider a positive charge $+q$ placed in oppositely charged parallel plates. A charge experienced a force in an electric field. If the charge is allowed to move freely in an electric field it will move from plate A to plate B and acquire kinetic energy as shown in Fig:11.23(a). On the other hand external force is required to move the charge against electric field. In order to move the charge q_o, with uniform velocity from B to A an external force F must be applied which is equal and opposite to q_oE as shown in Fig 11.23(b) .This force will maintain the equilibrium by preventing the charge q_o from acceleration while, moving from A to B.

Let W_{BA} be the work done by the force in carrying a positive charge q_o from point B to point A without disturbing the equilibrium state of the charge. The change in potential energy ΔU of charge q_o is defined to be equal to the work done by the force in carrying the charge q_o from one point to the other against the electrical field, i.e.,

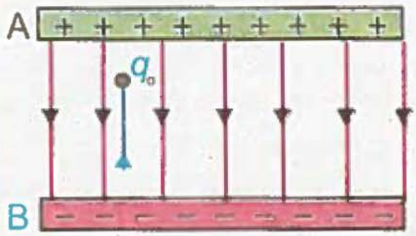


Figure 11.23 (a) : potential difference between point A & B in electric field.

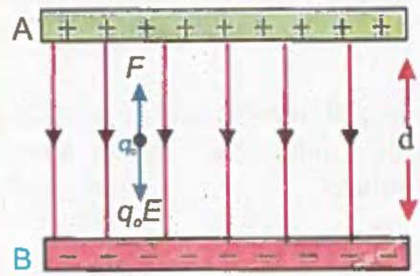


Figure 11.23 (b) : potential difference between point A & B in electric field.

$$\Delta U = W_{BA}$$

$$U_A - U_B = W_{BA} \qquad \text{... (11.18)}$$

where U_A and U_B represent the potential energies at point A and B respectively.
If this charge is released from point A, it will move from A to B and will gain an equivalent amount of K.E. Potential difference between two points is the work done in moving a unit positive charge from one point to another keeping the charge in electrostatic equilibrium. It implies that

$$\frac{\Delta U}{q_o} = \frac{W_{BA}}{q_o} \qquad \text{... (11.19)}$$

$$\frac{\Delta U}{q_o} = \frac{U_A}{q_o} - \frac{U_B}{q_o} = V_A - V_B = \Delta V \qquad \text{...(1)}$$

where V_A and V_B are the electric potentials at point A and B, respectively. The defining equations give.

$$U_A = q_o V_A, \; U_B = q_o V_B$$

Thus

$$\Delta U = q_o \Delta V = W_{BA} \qquad \text{...(11.20)}$$

Where ΔV is potential difference between A and B. Potential difference is the joule per coulomb which is termed as volt.

$$1\text{ volt} = \frac{1\text{Joule}}{1\text{Coulomb}}$$

One volt is the potential difference between two points in an electric field if one joule of work is done in moving one coulomb of charge from the one point to the other. Other multiples and sub-multiples of volt are

1 milli volt (mV) = 10^{-3} V, 1 micro volt (μV) = 10^{-6} V
1 kilo volt (kV) = 10^{3} V, 1 mega volt (MV) = 10^{6} V
1 giga volt (GV) = 10^{9} V

If an amount of work W is required to move a charge Q from one point to another, then the potential difference between the two points is given by,

$$V = \frac{W}{Q}$$

The electric potential at any point in an electric field is equal to work done in bringing a unit positive charge from infinity to that point keeping it in equilibrium.

Example 11.4

What is the electric potential energy of a 7 n C charge that is 2 cm from a 20 n C charge?

Solution:

We will use the equation: $U = k\frac{q_1 q_2}{r}$

Putting values:

$$U = 9.0 \times 10^{9}\ \frac{(7\times10^{-9})(20\times10^{-9})}{0.02\text{m}}$$

$$U = 6.3\times10^{-5}\text{J}$$

Example 11.5

What is the potential difference between two points in an electric field if it takes 600 J of energy to move a charge of 2 C between these two points?

Solution

We will use the equation: $V = \frac{W}{Q}$

$$= \frac{600}{2}$$
$$= 300\ \text{V}$$

Electric Potential Energy and Potential due to a point charge

Like earth gravitational field every charge has electric field which theoretically expands up to infinity. Consider an isolated charge +Q fixed in space as shown in fig11.24. If a test charge q is placed at infinity. The force on it due to charge +Q is zero. As the test charge is at infinity which is moved towards +Q as force of repulsion acts on it. So work is required to be done to bring it to point A. Hence when the test charge is at point A it has some amount of electric potential energy. The closer the test charge to the charge +Q the higher will be the electric potential energy. Electric field does not remain constant but varies as the square of the distance from the charge. The equation for electric field intensity is

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\hat{r} \qquad (1)$$

In order to keep E constant the test charge is moved through infinitesimally small displacements Δr.

The work done is

$$\Delta W = -q\,\vec{E}.\Delta\vec{r} = -\,q\,E\Delta r\cos 180^o$$
$$= q\,E\Delta r \qquad \text{... (11.21)}$$

The negative sign in the above equation shows that E and Δr, are in opposite direction. Now putting the value of E from Eq (1) in Eq (11.21) we have

$$\Delta W = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r^2}\Delta r$$

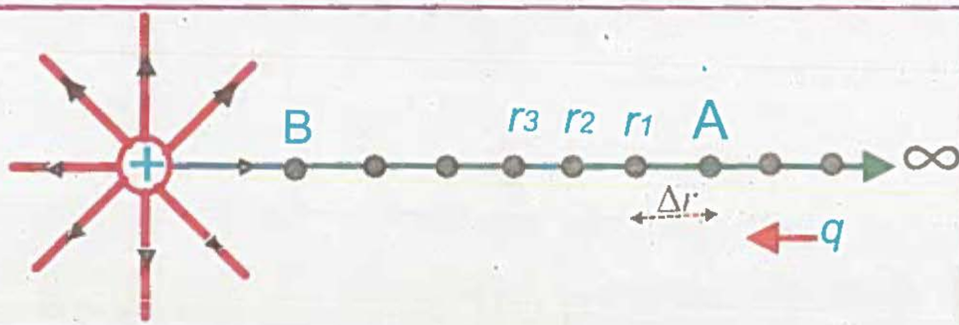


Figure 11.24 : work done on a test charge *q* in moving it towards source charge +*Q*

Now let the test charge is at a large distance r_A from charge Q. We divide the distance between r_A and r_B into infinitesimally small displacement so that the field intensity over each displacement remains constant. At the beginning of the first displacement E varies as $1/r_A^2$ and at the end E varies as $1/r_1^2$ since the average $1/r^2$ over each small displacement is

$$\frac{1}{<r^2>} = \frac{1}{<rr>}$$

Therefore one r is replaced by r_A and another by r_1.

$$\frac{1}{<r^2>} = \frac{1}{r_A r_1}$$

To calculate work done in this small displacement we put $1/r^2 = 1/r_A r_1$ and $\Delta r = r_A - r_1$

We get

$$\Delta W_{r_1 \to r_A} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{r_A - r_1}{r_A r_1}\right)$$

By division

$$\therefore\left(\frac{r_A}{r_A r_1}-\frac{r_1}{r_A r_1}\right)=\left(\frac{1}{r_1}-\frac{1}{r_A}\right)$$

$$\Delta W_{r_A\to r_1}=\frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1}-\frac{1}{r_A}\right) \qquad \text{... (11.22)}$$

Similarly for second small displacement $\Delta r = r_1 - r_2$ Work done is

$$\Delta W_{r_1\to r_2}=\frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

And

$$\Delta W_{r_n\to r_B}=\frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_B}-\frac{1}{r_n}\right) \qquad \text{... (11.23)}$$

The total work done in moving a charge q from r_A to r_B can be calculated by taking its sum.

$$\Delta W_{r_A\to r_B}=\frac{Qq}{4\pi\varepsilon_o}\left(-\frac{1}{r_A}+\frac{1}{r_1}-\frac{1}{r_1}+\frac{1}{r_2}-\frac{1}{r_2}+\dots\dots\dots+\frac{1}{r_B}\right)$$

$$\Delta W_{r_A\to r_B}=\frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_B}-\frac{1}{r_A}\right) \qquad \text{... (11.24)}$$

The work done to move a test charge q from infnity to a distance r from Q is

$$W_{r_A\to r_B}=\frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_B}-\frac{1}{\infty}\right)$$

$$=\frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_B}\right) \qquad \text{... (11.25)}$$

The electric potential energy at distance r from Q is

$$U = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right) \qquad \ldots (11.26)$$

The electric potential at distance r from Q is

$$V = \frac{W}{q} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} \qquad \ldots (11.27)$$

Example 11.6

A point charge of 3µC is placed at point O between M and N, 3cm apart. Point M is 2cm from the charge and N is 1cm from the charge. What is the potential difference $V_M - V_N$?

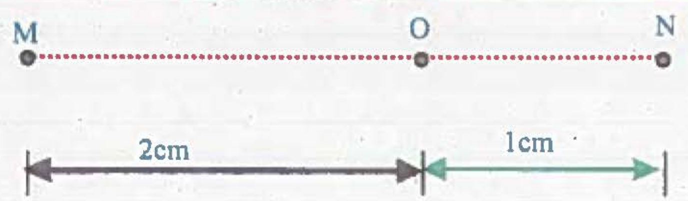


Figure 11. 25: potential due to a point charge

Solution

Potential at M due to the charge is $V_M = k\frac{q}{r}$

$$V_M = 9\times10^9\times\frac{3\times10^{-6}}{2\times10^{-2}} = 13.5\times10^5 \text{ V}$$

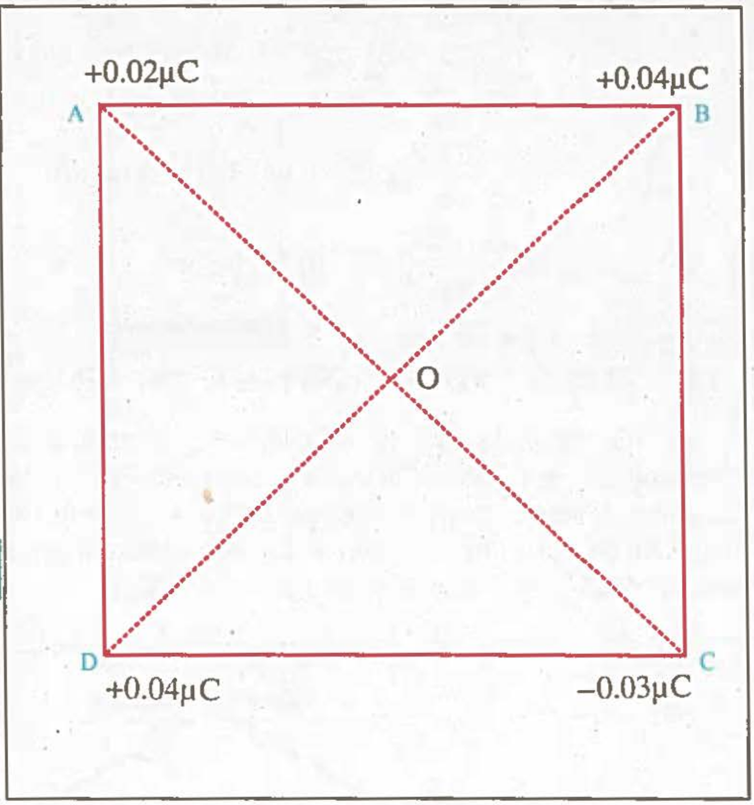
Potential at N due to the charge is

$$V_N = 9\times10^9\times\frac{3\times10^{-6}}{1\times10^{-2}} = 27\times10^5 \text{ V}$$

$$\therefore \quad V_M - V_N = 13.5\times10^5 - 27\times10^5 = -13.5\times10^5 \text{ V}$$

Example 11.7

Four point charges of $+0.02\mu C$, $+0.04\mu C$, $-0.03\mu C$ and $+0.04\mu C$ are placed at the corner A, B, C and D of a square ABCD respectively. Find the potential at the centre of the square if each side of the square is 1.5m apart.



Solution:

Fig: shows the square ABCD with charges placed at its corners. The diagonals of the square intersect at point O. Clearly, point O is the centre of the square. The distance of each charge from point O is

$$=\frac{1}{2}\sqrt{(1.5)^2+(1.5)^2}=1.0606\text{m}$$

The potential at point O due to all charges is equal to the algebraic sum of potentials due to each charge.

$\therefore$ Potential at O due to all charges

$$V=k\left[\frac{q_A}{r_A}+\frac{q_B}{r_B}+\frac{q_C}{r_C}+\frac{q_D}{r_D}\right]$$

Putting values we get

$$=9\times10^{9}\left[\frac{0.02\times10^{-6}}{1.0606}+\frac{0.04\times10^{-6}}{1.0606}+\frac{-0.03\times10^{-6}}{1.0606}+\frac{0.04\times10^{-6}}{1.0606}\right]$$

$$=\frac{9\times10^{9}}{1.0606}\left[(0.02+0.04-0.03+0.04)10^{-6}\right]$$

$$=\frac{9\times10^{9}}{1.0606}\times0.07\times10^{-6}=593.9V$$

11.8 FIELD AND POTENTIAL GRADIENT

Electric fields is very difficult to represent in a diagram. Both strength and direction can be indicated at every point in the field. As an alternative to field-line diagrams, 'contour maps' of the electric field can be drawn using equipotential lines. An equipotential line connects points in space where the potential of an electric field is the same as shown in fig (11.26).

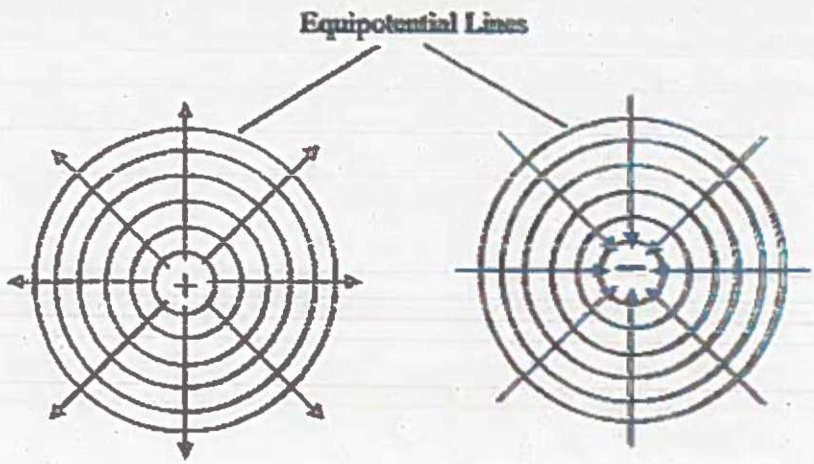


Figure 11.26(a) : equipotential lines around positive and negetive charges

For a point charge $V=\frac{q_o}{4\pi\varepsilon_o r}$

so all points at the same distance r from the charge will have the same potential — the equipotential lines form circles centred on the charge.

If there are two or more charges present, then the potential at any point is the sum of the potentials due to each charge. Potential is a scalar quantitiy. However; potential can be either positive or negative, depending upon the sign of the charge.

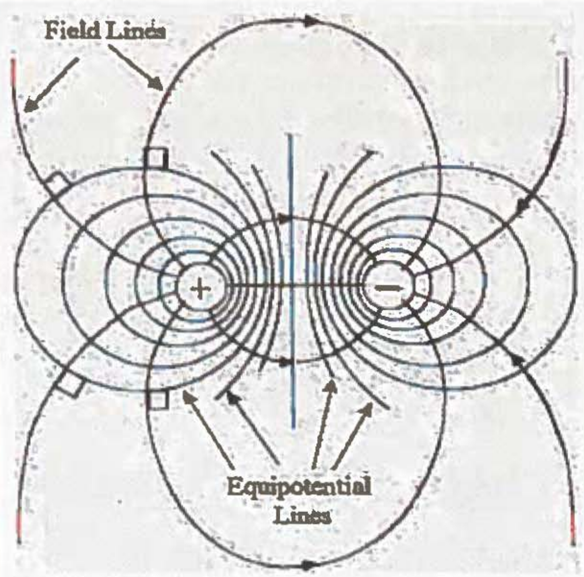


Figure 11.26(b) : equipotential lines due to two charges

The same basic idea is true on an equipotential map the closer the lines are together; the stronger the field is at that point if the equipotential lines are close together, the electric potential energy must be changing by large amounts in small distances, and there must be a large force acting. The exact relationship can easily be derived for the field of a point charge if a test charge q_o is moved a small distance Δr from point A to B then

work done on test charge = force acting × distance moved

$$W = F\,\Delta r$$

In an electric field, the force acting is equal to the charge times the field strength

$$F = q_o\,E$$

$$\text{work done} = W = E\,q_o\,\Delta r$$

Now the work done on the charge is equal to the decrease in electric Potential energy (remember that the potential energy is the charge times potential)

$$\therefore W = -\,q_o\,\Delta V$$

The negetive sign is applied because the work done on q_o is against field force $q_o E$

, so

$$q_o\,\Delta V = -E\,q_o\,\Delta r$$

$$E = -\frac{\Delta V}{\Delta r} \qquad \text{... (11.28)}$$

The strength of the field is equal to the potential gradient.
The rate of change of electric potential ΔV with respect to displacement Δr is known as potential gradient.
The negative sign indicates that the direction of the field is opposite to the direction in which the potential is increasing. This relationship between field strength and potential gradient is analogus to gravitational fields.

11.9 THE ELECTRON VOLT (eV)

One elecrton volt is the the amount of energy acquired or lost by an electron when it is displaced across two points between which potential difference is one volt.
By definition

$$\Delta(KE) = 1\ \text{eV, when } q = e = 1.602 \times 10^{-19}\ \text{C and } \Delta V = 1\text{V}.$$

thus

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{C} \times 1\ \text{V}$$
$$= 1.602 \times 10^{-19}\ \text{J}$$

The electron volt (eV) is just another unit of energy like the joule.
This is the smaller unit of energy. Its bigger units are multiples of 1 eV that are in frequent use are given below.
I Million electron volt = I Mev = 10^6 eV
I Giga electron volt = I GeV = 10^9 eV

Example.11:8

A particle carrying a charge ($3e$) falls through a potential difference of 5V. Calculate in joules the energy acquired by the particle.

Solution:

The energy acquired by the charged particle is
$\Delta(KE) = q\,\Delta V = (3e)\,(5\text{ V}) = 15\text{eV}$
$= 15 \times 1.602 \times 10^{-19}\text{ J} = 2.4 \times 10^{-18}\text{ J}$

11.10 Capacitor

A device which is used for storing electric charges is called capacitor. It consists of two parallel metal plates, separated by small distance. The medium between the two plates is air or a sheet of some insulator. This medium is known as dielectric.

When a charge is transferred to one of the plate say (A) due to electrostatic induction it would induce charge Q on the inner surface of the other plate B. The capacitor is commonly charged by connecting its plates for a while to the opposite terminals of a battery. In this way some electrons are transferred through the battery from the positive plate to the negative plate. Charge $+Q$ and $-Q$ appear on the plates. Mutual attraction between the charges keeps them bound on the inner surface of two plates and thus the charge remains stored in the capacitor even after removal of the battery.

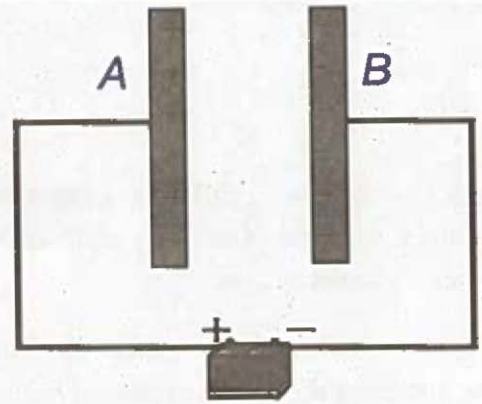


Figure 11.27 : parallel plates of a capacitor

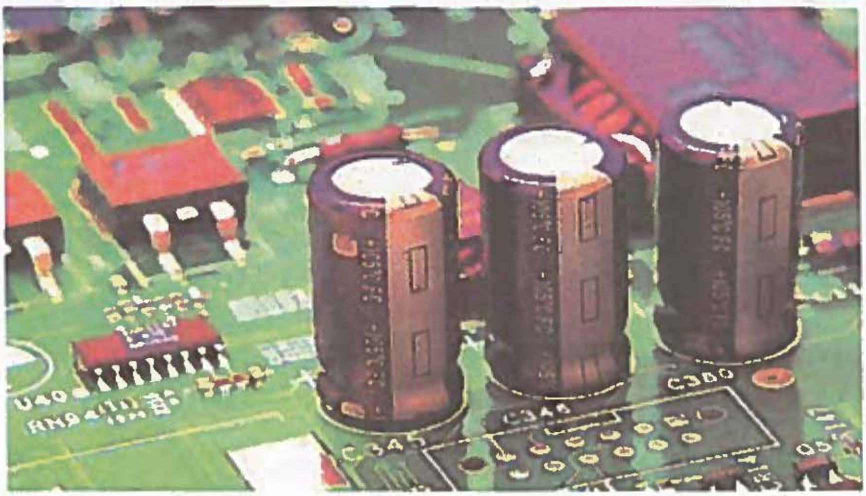


Fig:11.28 Capacitors use in different electronic devices.

11.10.1 Capacitance of a capacitor and its unit

The capability of a capacitor to store charges is called it capacitance. When a charge Q is transferred on one of the plates of a capacitor, the potential difference V between the plates also increases. In other words, the charge 'Q' on the plate of

a capacitor is directly proportional to the electric potential difference V between them i.e.

$$Q = CV \qquad ...(11.29)$$

OR $$\Rightarrow C = \frac{Q}{V}$$

Where C is a constant called the capacitance of capacitor. The value of C depends upon the area of the plates, the distance between the plates and the medium (dielectric) between them.

The capacitance is thus defined as *the ratio of magnitude of charge on either plate to the potential difference produced between the plates.*

Substituting V with 1 volt in Eq. (11.29) it reduce to Q = C. This implies that if the potential difference between the plates of capacitor is 1 volt then the amount of charge stored on its plates is equal to its capacity. The SI unit of capacitance is called farad. Farad (F) is defined as "*the capacity of that capacitor which stores a charge of 1 coulomb having the potential difference of 1 volt between the plates*".

Convenient sub-multiples of farad are:

$$1\mu F(\text{micro Farad}) = 10^{-6}\,F \;\&\; 1\mu\mu F(\text{Pico Farad}) = 10^{-12}\,F$$

11.10.2 CAPACITANCE OF A PARALLEL PLATE CAPACITOR

Let us consider a parallel plate capacitor connected to a voltage source V. The source, charges the capacitor plates till the potential difference across the plates builds to V. Let the charges on the plates are $+Q$ and $-Q$ when the potential difference is V. If the positive plate is at potential V_1 and negative plate is at potential V_2, then the electric field strength between the plates is

$$E = \frac{-\Delta V}{\Delta r} = \frac{-(V_2 - V_1)}{d} = \frac{V_1 - V_2}{d} = \frac{V}{d} \qquad (1)$$

Where V_1- V_2 = V, the P.D. between the plates, and d is the separation between the plates. The strength of the electric field also depends on the number of charges on the plates. The charge density is the total charge per area of the plate. $\sigma = \frac{Q}{A}$

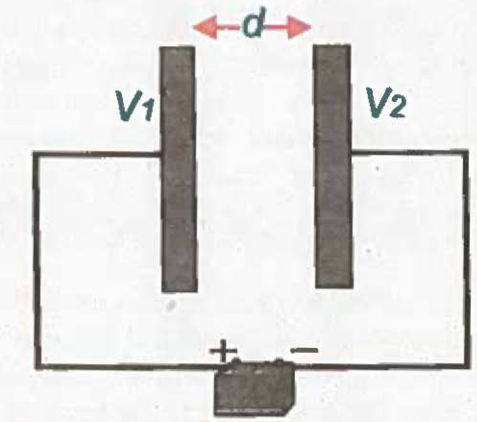


Figure 11.29 : plates of a capacitor separated by a distance *d*

By using Gauss's law the electric field intensity E between the plates is

$$E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{\varepsilon_o A} \qquad (2)$$

From Eq (1) & (2) we have

$$E = \frac{V}{d} = \frac{Q}{\varepsilon_o A}$$

Or

$$Q = \frac{\varepsilon_o A V}{d} \qquad \text{... (11.30)}$$

By using $C_{vac} = \frac{Q}{V}$ Eq (11.30) becomes

$$C_{vac} = \frac{Q}{V} = \frac{\varepsilon_o A}{d} \qquad \text{... (11.31)}$$

When a dielectric is inserted between the plates of a capacitor, then it is seen that the charge storing capacity of a capacitor is enhanced by the dielectric which permits it to store ε_r times more charge for the same potential difference. ε_r is a dimensionless quantity which is always greater than unity for dielectric and is independent of the size and shape of the dielectrics. It is called dielectric constant or relative permittivity.

In case of a parallel plate capacitor completely filled with a dielectric the capacitance is

$$C_{med} = \frac{\varepsilon_o \varepsilon_r A}{d} \quad \text{... (11.32)}$$

Eq11.32: shows the dependence of capacitance upon the area of the plates, the separation between the plates and the medium between them. The larger the plates, the closer they are and higher the dielectric constant of the separating medium the greater will be the capacitance of the capacitor.

From Eqs(11.31) and (11.32) we have

$$\varepsilon_r = \frac{C_{med}}{C_{vac}} \quad \text{... (11.33)}$$

This relation provides us with the definition of relative permittivity or dielectric constant or specific inductive capacity:

The ratio of the capacitance of a capacitor with a given material filling the space between the conductors to the capacitance of the same capacitor when the space is evacuated is the relative permittivity ε_r of the material.

11.10.3 Combinations of Capacitors

We know that the capacitors can be connected either in series or in parallel. We want to find out an equivalent capacitor that has the same capacitance as that of the combination of capacitors.

Series Combination of capacitors

When the capacitors are connected plate to plate i.e. the right plate of one capacitor is connected to the left plate of the next capacitor so on as shown in fig:11.30 (a) then it is called series combination. A battery of voltage V is connected between points A and B. Then it supplies $+Q$ charge to the left plates of the capacitor C and $-Q$ charge is induced on its right plates. As a result of this charging each capacitor gets an equal amount of charge Q on each of its plates. The potential difference V must be equal to the sum of potential difference, V_1, V_2 & V_3 across the capacitors i.e. $V = V_1 + V_2 + V_3$ (i)

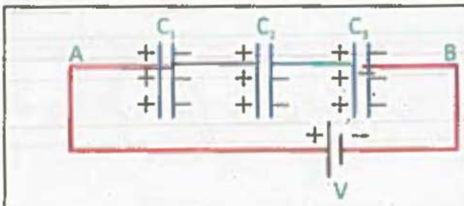


Figure 11.30(a) : series combination of capacitors

We know that: $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, & $V_3 = \frac{Q}{C_3}$ Substituting these expressions into the above Equation(i),

we have $V=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}$ or $V=Q(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3})$

Let C_e be the capacitance of an equivalent capacitor, which would hold the same charge when the potential difference V is applied. That is $V=\frac{Q}{C_e}$

Therefore $\frac{Q}{C_e}=Q(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3})$

or $\frac{1}{C_e}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$ ... (11.34)

Thus the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.

Parallel Combination of capacitors

When two or more capacitors are connected between the same two points in a circuit, as shown in fig:11.30(b), then it is called parallel combination of capacitors. Three capacitors C_1, C_2, & C_3 are connected in parallel between two points A and B. The potential difference between the plates of each capacitor is the same and is equal to the applied potential difference V i.e $V=V_1=V_2=V_3$.

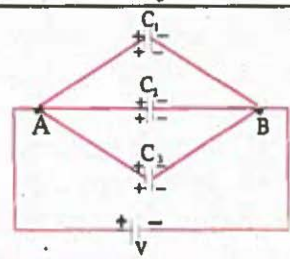


Figure 11.30 (b): parallel combination of capacitors

When Q charge is supplied to the capacitors C_1, C_2, and C_3, they acquire different amount of charges Q_1, Q_2, and Q_3 respectively depending upon their capacitances. Let C_e be the capacitance of an equivalent capacitor, which would hold the same amount of charge as all the three capacitors C_1, C_2, and C_3 hold under the same potential difference.

$\therefore Q=Q_1+Q_2+Q_3$ But $Q_1=C_1V, Q_2=C_2V, Q_3=C_3V$ & $Q=C_eV$

so $C_eV=C_1V+C_2V+C_3V=(C_1+C_2+C_3)V$

$C_e=C_1+C_2+C_3$ (11.35)

Thus the equivalent capacitance of a parallel combination is always larger than any individual capacitance in the combination.

11.11 ELECTRIC POLARIZATION

When insulating material with relative permittivity (or dielectric constant) ε_r is inserted into an initially charged parrallel plates of a capacitor. Then negative charges appear on the left face and positive charges on the right face of

the dielectric as shown in fig11.31. The phenomenon is known as electric polarization and dielectric is said to be polarized under such condition. The charges on the dielectric faces are called induced charges; they are induced by the external field and appear on the dielectric faces only. The electric field from the free charges is left to right whereas the electric field due to induced charges is right to left.

Molecules in the dielectric material have their positive and negative charges separated slightly, causing the molecules to be oriented slightly in the electric field of the charged capacitor. As electric field due to induced charges is opposite to the external electric field so it reduces the intensity of external field due to oppositely charged plates of the capacitor.

We know that dielectric materials are made up of the two types of molecules ;polar and non polar molecules. A polar molecule behaves like a permanent dipole; although electrically neutral as a whole. For example in NaCl the end with Sodium ion is positive while the end with Chlorine ion is negative.

On the other hand, a non-polar molecule like oil has no electric dipole moment in the absence of an external field. The centre of positive and negative charges coincide in the absence of an extenal electric field. When a non polar dielectric material is placed in an external field, it gets polarized, that is, it displaces the electrons to the opposite of electric field E as shown in fig. Under the action of external field the centres of negative and positive charges shift and form dipoles (Two opposite point charge separated by a finite distance d constitutes a dipole).

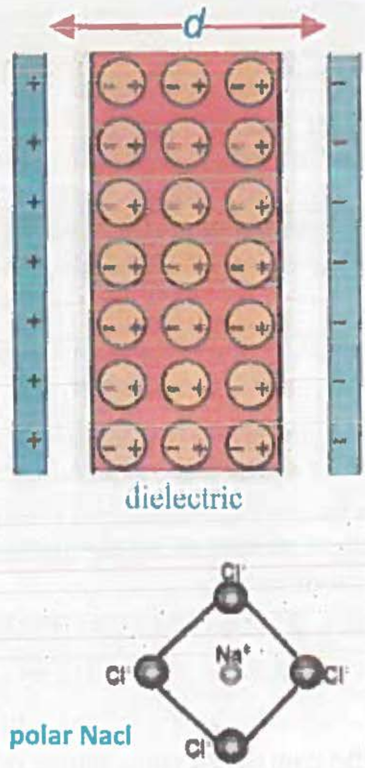


Figure 11.31: Polarization of a dielectric in an electric field gives rise to thin layers of bound charges on the surfaces, creating positive and negative surface charge densities.

The system in which two charges of equal magnitude but of opposite sign separated by the distance d, are present is termed as a dipole.

Electric dipole moment is represented by *P*, which is equal to the product of the charge *q* present in the dipole and the distance *d* between the two charges of the dipole.

$$P = |qd| \qquad \ldots(11.36)$$

Where *P* is a vector quantity.

Example 11.9

Fig:(11.32). shows a different combination of capacitors, if the total charge is 600μC. Then determine the values of V_1, and C_2.

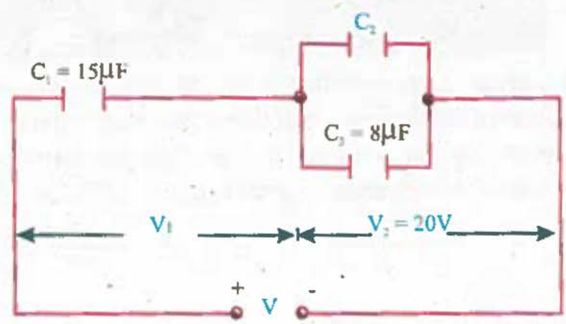


Figure 11.32: capacitors C_1, C_2, & C_3 connected to potential V

Solution:

$$\text{P.D. across capacitor } C_1: V_1 = \frac{Q}{C_1} = \frac{600\times10^{-6}}{15\times10^{-6}} = 40\text{V}$$

$$\text{Total p.d.: } V = V_1 + V_2 = 40 + 20 = 60\text{V}$$

$$\text{Charge on capacitor } C_3 \text{ is}: Q_3 = C_3 \times V_2$$

$$= (8\times10^{-6})\times 20$$

$$= 160\times10^{-6}C = 160\mu\text{C}$$

$$\therefore \text{Charge on capacitor } C_2 \text{ is}: Q_2 = 600 - 160 = 440\,\mu\text{C}$$

$$\therefore \text{Capacitacnce of capacitor } C_2 = \frac{440\times10^{-6}}{20}$$

$$= 22\times10^{-6}\,\text{F} = 22\,\mu\text{F}$$

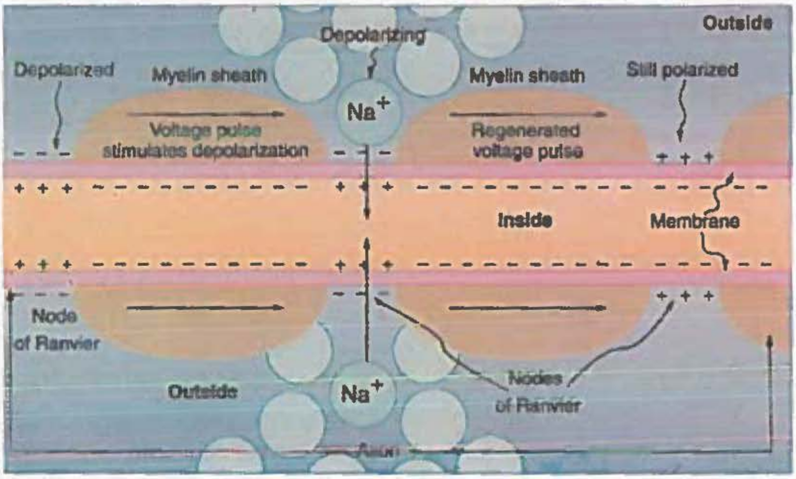


Figure 11.33: an axon membrane acts as a capacitor. Axon of a resting human nerve cell has a potential difference of 65mV. Current enters the axon (or dendrite) through ion channels (e.g. Na+ channels) in a region of membrane, depolarizing that region. The intracellular + charge is attracted to adjacent negatively charged regions of membrane.

Example 11.10

A 6μF capacitor is charged to a P.D. of 200V and then connected in parallel with an un-charged 3μF capacitor. Calculate the P.D. across the parallel plate capacitors.

Solution:

Capacitance of charged capacitor = $C_1 = 6\mu F$

Capacitance of un-charged capacitor = $C_2 = 3\mu F$

Charge on capacitor, C_1 is: $Q = C_1 V = (6\times10^{-6}) \times 200 = 0.0012C$

The equivalent capacitance of parallel combination of capacitors is

$C_{eq} = C_1 + C_2 = 6 + 3 = 9\mu F.$

The charge 0.0012 C is distributed between the two capacitors to have a common P.D.

$$\therefore \text{P.D. across Parallel plate capacitors}: V = \frac{Q}{C_{eq}} = \frac{0.0012}{9\times10^{-6}} = 133.3\text{V}$$

11.12 ENERGY STORED IN A CAPACITOR

Let us suppose that initially the capacitor is uncharged when voltage is zero. When it is connected to source of potential difference V it is charged. Initially, when the capacitor is uncharged, the potential difference between the plates is zero. Finally when charge $+Q$ and $(-Q)$ are deposited on the plates, the potential difference between the plates becomes V. The average voltage on the capacitor during the charging process is $V/2$. Thus the energy stored in a capacitor, is

Figure 11.34: Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged.

$$\text{Energy} = U = \frac{QV}{2} \qquad \text{... (11.37)}$$

Where Q is the charge on a capacitor with a voltage V applied. Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$ and so the expression for U can be written in three equivalent expressions as:

$$U = \frac{QV}{2} = \frac{CV^2}{2} \qquad \text{... (11.38)}$$

$$= \frac{Q^2}{2C}$$

It is also possible to regard the energy as being stored in the electric field between the plates rather than the potential energy of the charges on the plates.
Such a view point is useful when the Electric field between the plates is considered rather than the charges on the plates causing the field is to be considered .

We know that

$$V = E\,d$$

and

$$C = \frac{A\varepsilon_r\varepsilon_o}{d}$$

Substituting these values in Eq.(11.38). We get

$$U = \frac{1}{2}\times\frac{A\varepsilon_r\varepsilon_o}{d}\times(Ed)^2$$

$$U = \frac{1}{2}\varepsilon_r\,\varepsilon_o\,E^2\times(Ad) \qquad ...(11.39)$$

The product (Ad) is volume between the plates. Let u denote the energy density that is, the energy contained in a unit volume of the field. Then

$$u = \frac{\text{Energy}}{\text{Volume}} = \frac{U}{Ad}$$

$$= \frac{1}{2}\varepsilon_r\varepsilon_o E^2 \qquad ...(11.40)$$

11.13 CHARGING AND DISCHARGING A CAPACITOR

Electronic flashguns for cameras have to be left for a short period of time between flashes.

There is a capacitor inside that stores energy,it needs to be charged up again by the battery in the flashgun. The time taken for this depends on the rate at which the charge is flowing, which in turn is determined by the resistance of the circuit.It can take a couple of seconds before the capacitor is fully charged.

Fig.(11.35)shows a resistor capacitor circuit called *RC* circuit.

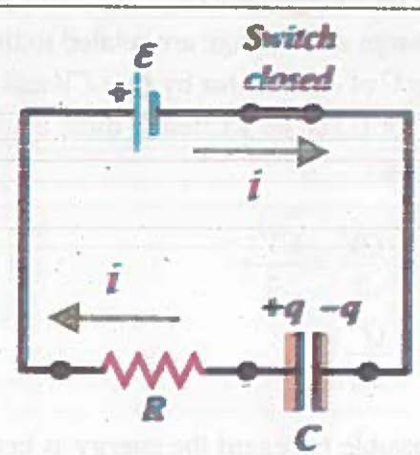


Fig: 11.35 : RC circuit

When the switch S is closed to connect the upper circuit, a battery of voltage V, starts charging the capacitor through the resistor *R*.

The charge builds up gradually on the plates to the maximum value of q_0. Suppose at $t = 0$ charge on a capacitor is zero .i.e $q = 0$.
It can be shown that after time t, as charge builds up on the plates, it repels more charge than is arriving, and the current drops as the charge on the plates increases. Charging will stop when the P.D. between the capacitor plates is equal to the e.m.f. of the battery.

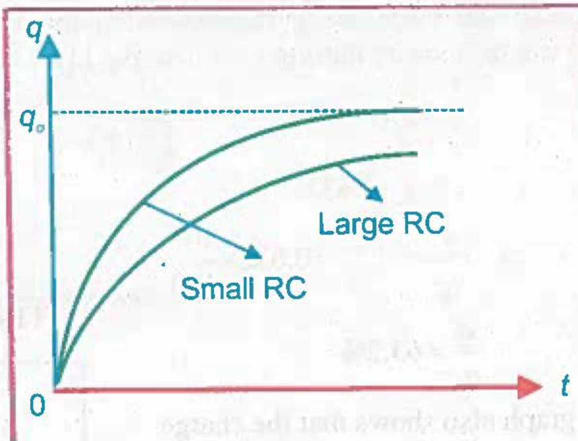


Fig: 11.36 : Growth of charge on the capacitor

maximum charge on capacitor = capacitance × e.m.f. of battery

Experiments shows that the charging process of a capacitor exhibits the exponential behavior therefore we can write its Eq: as

$$q = q_o(1 - e^{-t/RC}) \quad \text{.... (11.41)}$$

where e is a constant. Its value is 2.7182 Fig. (11.35) shows a graph between time t and charge q. According to this graph, $q = 0$ at $t = 0$ and increases gradually to its maximum value q_o.

Time constant

The time taken to charge a capacitor in a given circuit is determined by the time constant of the circuit. The bigger the capacitance, the longer it takes to charge the capacitor. The larger the resistance, the smaller the current, which also increases the charging time. The factor RC is called 'time constant. The time constant is the duration of time for the capacitor in which 63.2 % of

For your information

In principle, a capacitor can never charge up fully, because the rate of charging decreases as the charge increases. In practice, after a finite time the charging current becomes too small to measure, and the capacitor is effectively fully charged.

its maximum value charge is deposited on the plates.
This can be seen by putting $t = RC$ in Eq(11.41)

$$q = q_o(1-e^{-1}) = q_o\left(1-\frac{1}{2.718}\right)$$

$$q = q_o(0.632)$$

$$\Rightarrow \frac{q}{q_o} = 0.632 = 0.632\times\frac{100}{100}$$

$$\frac{q}{q_o} = 63.2\% \quad \text{... (11.42)}$$

The graph also shows that the charge reaches its maximum value sooner when the time constant is small.

Fig.11.37 : illustrates the discharging of a charged capacitor through a resistor. When the switch S is closed. the charge $+q$ on the right plate can now flow clockwise through the resistance and neutralize the charge $-q$ on the left plate. Assuming the fully charged capacitor begins discharging at time $t = 0$, it can be shown that charge left on either plate at time t is

$$q = q_o e^{-t/RC} \quad \text{... (11.43)}$$

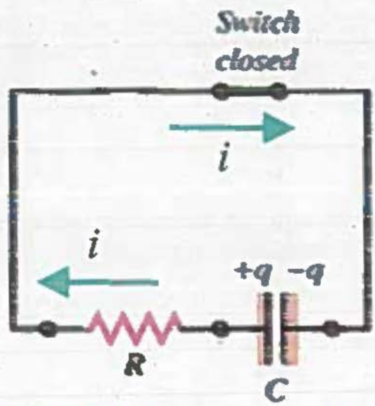


Fig: 11.37: Discharging a capacitor

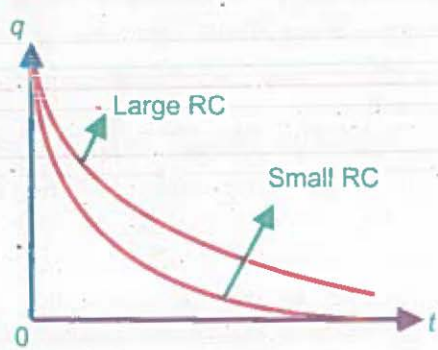


Fig: 11.38: decay of charge on capacitor

The corresponding graph in fig. 11.38 shows that discharging begins at $t = 0$ when $q = q_o$, and decreases gradually.

Smaller values of time constant RC lead to a more rapid discharge. When $t = RC$, the magnitude of charge remaining on each plate is,

$$q = q_o(0.367)$$

$$\Rightarrow \frac{q}{q_o} = 0.367$$

Thus,

$$\frac{q}{q_o} = 36.7\% \qquad ...(11.44)$$

The charging and discharging of a capacitor has many applications.

Capacitor discharge ignition (CDI) is a type of automotive electronic ignition system which is widely used in motorcycles, lawn mowers, chain saws, small engines, turbine powered aircraft, and some cars. It was originally developed to overcome the long charging times associated with high inductance coils used in inductive ignition systems, making the ignition system more suitable for high engine speeds (for small engines, racing engines and rotary piston engines). It can enhance the capability of power supply and make the spark much stronger.

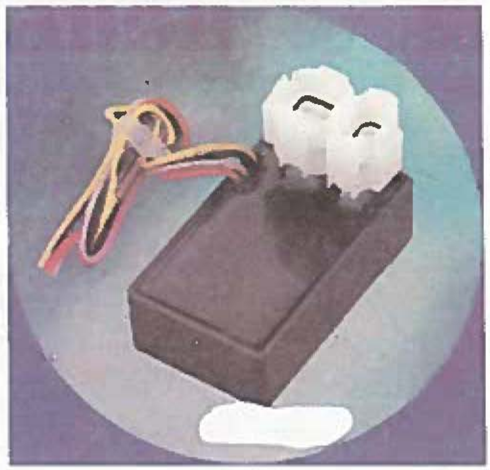


Figure 11.39 : Electronic Ignition Cdl

Key points

- According to Coulomb law the electric force between two point charges is directly proportional to the product of magnitudes of the charges and inversely proportional to the square of the distance between them.
- An electric field is a region around a charge in which an electric test charge would experience an electric force. The existence of electric field can be proved by bringing a test charge q_o into its field.
- The applications of electrostatics are photocopier and inkject printer.
- For electric flux area is considered as vector quantity.
- The electric flux Φ is defined as the number of lines of force that pass through the area placed in the electric field. $\Phi = E.A = EA\cos\theta$
- When area A is normal to the electric field E then electric flux is maximum.
- The electric flux through any closed surface is $1/\varepsilon_o$ times the total charge enclosed in it.
- A device which is used for storing electric charges is called capacitor. The SI unit of capacitance is called farad.
- The electron volt (eV) is another unit of energy and is related to joule as $1\ eV = 1.602 \times 10^{-19}\ J$
- The dielectric materials are made up of the two types of mole les ;polar and non polar molecules.
- The system in which two charges of equal magnitude but of opposite sign separated by the distance d are present is termed as a dipole.
- the energy stored between the plates of a capactor is in the form of electric field. $U = \frac{1}{2}\varepsilon_r \varepsilon_o E^2 \times (Ad)$
- The charging process of a capacitor exhibits the exponential behavior so we can write its Eq: as $q = q_o(1 - e^{-t/RC})$
- The time constant is the duration of time for the capacitor in which 63.2 % of its maximum value charge is deposited on the plates.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. A charge Q is divided into two parts q and $Q-q$ and separated by a distance R. The force of repulsion between them will be maximum when:
   a. $q = Q/4$
   b. $q = Q/2$
   c. $q = Q$
   d. None of these

2. Some charge is being given to a conductor. Then its potential
   a. Is maximum at surface
   b. Is maximum at centre
   c. Is remain same throughout the conductor
   d. Is maximum somewhere between surface and centre

3. Electric potential of earth is taken to be zero because the earth is good:
   a. Semiconductor
   b. Conductor
   c. Insulator
   d. Dielectric

4. A proton is about 1840 time heavier than an electron. When it is accelerated by a potential difference if 1 kV, its kinetic energy will be:
   a. 1840 keV
   b. 1/1840 keV
   c. 1 keV
   d. 920 keV

5. A capacitor is charged with a battery and then it is disconnected. A slab of dielectric is now inserted between the plates, then
   a. The charge in the plates reduces and potential difference increase
   b. Potential difference between the plates increase, stored energy decreases and charge remains the same

c. Potential difference between the plates decreases, stored energy decreases and charge remains unchanged
d. None of the above

6. A one microfarad capacitor of a TV is subjected to 4000 V potential difference. The energy stored in capacitor is
a. 8 j
b. 16 j
c. 4 x 10^{-3} j
d. 2 x 10^{-3} j

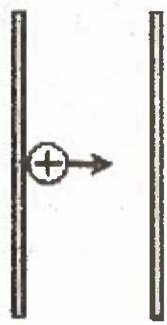
7. In the figure below, the charge on 3 μF capacitor is

a. 5 μC
b. 10 μC
c. 3 μC
d. 6 μC

8. The electric potential between two points A and B is ΔV. The work done W by the field in moving a charge q from A to B is
a. $W = -q\ \Delta V$
b. $W = q\ \Delta V$
c. $W = -\ \Delta V/q$
d. $W = \ \Delta V/q$

9. The electric flux through the surface of a sphere due to a charge q placed at its centre depends upon
a. the radius of the sphere
b. the quantity of charge outside the sphere
c. the surface area of the sphere
d. the quantity of charge inside the sphere

10. Two parallel, metal plates are a distance 8.00 m apart. The electric field between the plates is uniform, directed toward the right, and has a magnitude of 4.00 N/C. If an ion of charge+2e is released at rest at the left-hand plate, what is its kinetic energy when it reaches the right-hand plate?
a. 4 eV.
b. 64 eV.
c. 32 eV.
d. 16 eV



Comprehensive questions

1. State and explain coulombs law. Do include the case when the charges are placed in dielectrics. Discuss how the unit of charge coulomb is defined?
2. Explain the concept of electric field and hence define electric field intensity. Discuss the direction as well as the unit of $\vec{E}$
3. Explain the concept of electric flux. Using mathematical expressions of electric flux to show that how electric flux is maximum and minimum.

4. State and prove the Gauss law for electrostatics. Also discuss its applications with daily life example.
5. Explain the concept of electric potential. Derive an expression for electric potential at a field point due to a source charge.
6. Describe the construction of capacitor and derive an expression for the energy stored in a capacitor.
7. Describe the concept of equipotential surfaces and derive an expression for electric field as a negative of potential gradient.
8. Explain the phenomenon of electric polarization. Discuss how the phenomenon of polarization account for the increase in capacitance of a capacitor when instead of air, dielectric is inserted between its plates?
9. Derive an expression for the capacitance of a parallel plate capacitor when a dielectric is inserted between the plates of a capacitor.
10. Describe the process of charging and discharging a capacitor. Give the diagram and mathematical expressions for the growth and decay of charge on the capacitor.

Conceptual questions

1. The electric potential is constant through a given region of space. Is the electric field zero or non-zero in this region? Explain.
2. If a point charge q of mass m is released in a non-uniform electric field with field lines pointing in the same direction, will it make a rectilinear motion?
3. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?
4. Voltages are always measured between two points. Why?
5. How are units of volts and electron volts related? How do they differ?
6. In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

7. Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.
8. What is an equipotential line and equipotential surface?
9. Can different equipotential lines cross each other ? Explain.
10. Water has a large dielectric constant, but it is rarely used in capacitors. Explain why?
11. A capacitor is connected in series with a resistor and charged. Explain why the potential difference across the resistor decreases with time during the charging.
12. Sketch the graphs of potential difference against time for (a) a discharging capacitor (b) a charging capacitor.
13. Compare the formula for capacitors in series and parallel with those for resistors in series and parallel. Explain why the pattern is different.
14. Explain why capacitors are of little use for storage of energy for normal domestic purposes of lighting heating and so on.

Numerical problems

1. What is the magnitude of the force of attraction between an iron nucleus bearing charge $q = 26e$ and its innermost electron, if the distance between them is 1×10^{-12} m. (6×10^{-3} N)
2. Charges 2 μC, -3 μC, and 4 μC are placed in air at the vertices of an equilateral triangle of sides 10 cm. what is the magnitude of resultant force acting on 4 μC charge?

(15.7N)

3. A charge q is placed at the centre of the line joining the two charges, each of magnitude Q. Prove that the system of three charges will be in equilibrium if $q = -Q / 4$.

4. Two equal and opposite charges of magnitude 2×10^{-7} C are placed 15cm apart. What is the magnitude and direction of electric intensity (E) at a point mid-way between the charges? What force would act on a proton (charge = $+1.6\times10^{-19}$ C) placed there?

   ($.64\times10^{6}$ N/C along AB, 1.024×10^{-13} N along AB)

5. Two positive point charges of 15×10^{-10} C and 13×10^{-10}C are placed 12cm apart. Find the work done in bringing the two charges 4 cm closer.

   (7.31×10^{-8} J)

6. A hollow sphere is charged to 14µC. Find the potential (a) at its surface (b) inside the sphere (c) at a distance of 0.2m from the surface. The radius of the sphere is 0.3m.

   (42×10^{4} V, 42×10^{4} V, 25.2×10^{4} V)

7. If 280 J of work is done in carrying a charge of 2C from a place where the potential is -12V to another place where potential is V, calculate the value of V. (128 V)

8. Calculate the electric potential at the surface of a silver nucleus having radius 3.4×10^{-14} m. The atomic number of silver is 47 and charge on a proton = 1.6×10^{-19}C. (1.99×10^{6} V)

9. The electric field at a point due to a point charge is 26 N/C and the electric potential at that point is 13 J/C. Calculate the distance of the point from the charge and magnitude of charge.

   (0.5 m, 0.722×10^{-9} C)

10. Two point charges of 8µC and -4µC are separated by a distance of 10cm in air. At what point on the line joining the two charges is the electric potential zero?

    (6.6 cm from 8 µC and, 3.3 cm from −4µ F charge)

11. An electron with an initial speed of $29\times10^5 ms^{-1}$ is fired in the same direction as a uniform electric field with a magnitude of 80 NC^{-1}. How far does the electron travel before being brought to rest momentarily and turned back?

(.299m)

12. Two capacitors of capacitance 4 μF and 8 μF are first connected (a) in series and then (b) in parallel. In each case external source of voltage is 200 V. Calculate in each case the total capacitance, the potential drop across each capacitor, and the charge on each capacitor.

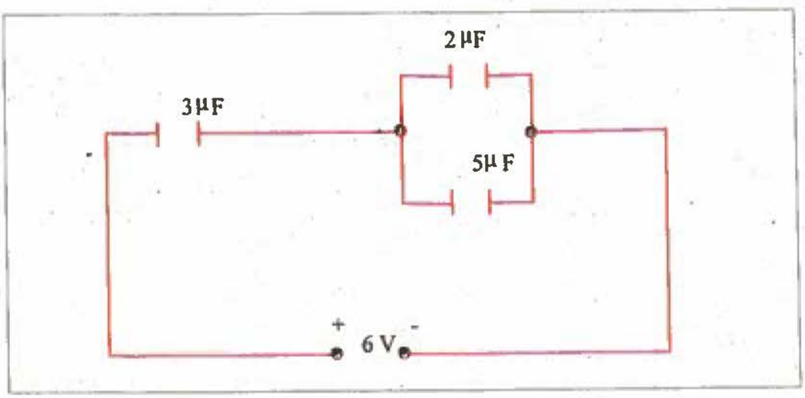
(2.66μF, 5.33×10^{-4}C, 133.2V, 66.6V, 12μF, 200V, .08μC, .16μC)

13. Three capacitors of capacitance 4μF, 6μF and 8μF respectively are connected in series to a 250V d.c. supply. Find (i) the total capacitance (ii) charge on each capacitor and (iii) P.D. across each capacitor.

(1.84 μ F, 460×10^{-6} C, 115 V, 76.6V and 57.7 V)

14. If C_1 = 14μF, C_2 = 20 μF, C_3 = 12μF and the insulated plate of C_1 be at potential of 100V, one plate of C_3 being earthed, what is the potential difference between the plates of C_2, three capacitors being connected in series? (24.4 μV)

15. Find the charge on 5μF capacitor in the circuit shown in Fig. .



(9 μ C)

16. Two parallel plate capacitors A and B having capacitance of 2 μF and 6 μF are charged separately to the same potential of 120V. Now positive plate of A is connected to the negative plate of B and the negative plate of A is connected to the positive of B. Find the final charge on each capacitor.

(120 μ C, 360 μ C)

17. A 6μF capacitor is charged to a P.D. of 120V and then connected to an un-charged 4 μF capacitor. Calculate the P.D. across the capacitors.

(72 V)

18. Two capacitor of capacitance 8μF and 10 μF respectively are connected in series across a P.D. of 180V. The capacitors are disconnected from the supply and are reconnected in parallel with each other. Calculate the new P.D. and charge on each capacitor.

(88.8 V, 710 μ C, 888 μ C)

UNIT 12

.....Current Electricity.....

After studying this chapter the students will be able to

- describe the concept of steady current.
- state Ohm's law.
- define resistivity and explain its dependence upon temperature.
- define conductance and conductivity of conductor.
- state the characteristics of a thermistor and its use to measure low temperatures.
- distinguish between e.m.f and p.d. using the energy considerations.
- explain the internal resistance of sources and its consequences for external circuits.
- describe some sources of e.m.f.
- describe the conditions for maximum power transfer.
- describe thermocouple and its function.
- explain variation of thermoelectric e.m.f. with temperature.
- apply Kirchhoff's first law as conservation of charge to solve problem.
- apply Kirchhoff's second law as conservation of energy to solve problem.
- describe the working of rheostat in the potential divider circuit.
- describe what is a Wheatstone bridge and how it is used to find unknown resistance.
- describe the function of potentiometer to measure and compare potentials without drawing any current from the circuit.

Current electricity is the study of charges in motion. Simple electrical circuits can be solved by applying Ohm's law. For other circuits, Kirchhoff rules are applied.

We all enjoy the comforts and benefits of using electricity. Electricity has characteristics that have made it uniquely appropriate for powering an emerging technological society. Electricity is also relatively easy to distribute.

Electricity authorities use high-voltage transmission lines and transformers to distribute electricity to homes and industries in a town. Voltages can be as high as 5×10^5 volts from power stations but by the time this reaches homes, the electricity has been transformed to 220 volts in Pakistan. While it is relatively economical to generate electric power at a steady rate, there are both financial and environmental issues that should be considered when assessing the long-term impact of supplying commercial and household power.

For Your Information

The electric eel is an electric fish which is capable of generating powerful electric shocks of up to 600 volts. It uses electric shock for hunting and self-defense.

In making transition from electrostatics to moving charges, we have to study phenomena like magnetic field produced due to the motion of charges, resistance offered but the materials to the moving charges through them and lastly the dissipation of energy when the charges move through the conductors. In this chapter we shall discuss briefly the various features associated with the moving charges.

12.1 STEADY CURRENT

The continuous flow of free electrons is called steady current. The flow of electronic current can be easily explained by referring Fig 12.1 The conductor has a large number of free electrons when electric field or voltage is applied, then

free electrons, being negatively charged, will start moving towards the positives terminal around the circuit.

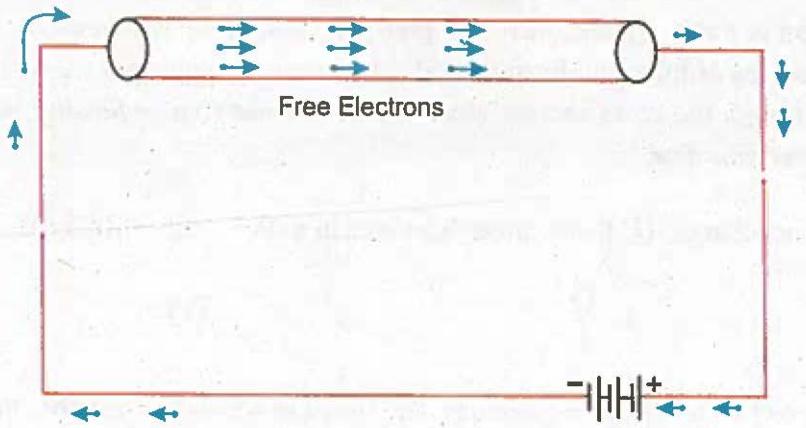


Fig: 12.1: Flow of electrons in a conductor.

This directed flow of electrons is called electric current.

Following are the main points:

1. Current is due to flow of electrons and electrons are the constituents of matter. Therefore, electric current is matter (i.e. free electrons) in motion.
2. The actual direction of current is from negative terminal to the positive terminal throughout that part of the circuit external to the cell. However, prior to electron theory, it was assumed that current flows from positive terminal to the negative terminal of the cell via the circuit. This convention is so firmly established that it is still in use. This assumed direction is now called conventional current.
3. Those substances which have larger number of free electrons will permit current flow easily. Such substances are called conductors, for example Copper, Zinc, Silver, Aluminum etc.

In different current carrying materials the charges on the moving particles may be positive or negative. In metals the moving charges are always electrons (negative charges). While in gases the moving charges are negative and positive ions. In semiconductor material such as germanium or silicon conduction is partly by electrons and partly by motion of vacancies, or holes. These are sites of missing electrons and act like positive charges we define the current through the cross-section area A to be *the net charge flowing through the area per unit time.*

Thus if a net charge 'Q' flows through an area in a time t, the current I is

$$I = \frac{Q}{t} \qquad (12.1)$$

The charge Q is measured in coulombs and time t in seconds. Therefore the unit of electric current will be coulombs/sec or ampere (A). *One ampere is the current when one coulomb of charge flows across any cross section of a conductor in one second.* The submultiples of ampere are:

1 milliampere $= 1\ \text{mA} = 1 \times 10^{-3}\ \text{A}$

1 microampere $= 1\ \mu\text{A} = 1 \times 10^{-6}\ \text{A}$

Do you know?

Current is a scalar quantity as it does not follow vector law of addition.

12.2 DRIFT VELOCITY IN CONDUCTOR

Every metal has a large number of free electrons which wander randomly within the body of the conductor. The average speed of free electrons is sufficiently high ($\approx 10^5\ \text{ms}^{-1}$). During random motion, the free electrons collide with atoms of conductor again and again and after each collision their direction of motion changes. Due to random motion of all free electrons there is no net flow of charges in any particular direction.

Consequently no current is established in the conductor.

When the potential difference is applied across the ends of a conductor, an electric field $\bar{E}$ is set up at every point within the wire. The electrons experience a force in a direction opposite to electric field, $\bar{E}$. As a result of this force and the continuous collision with the atoms, the electrons acquire a net drift velocity.

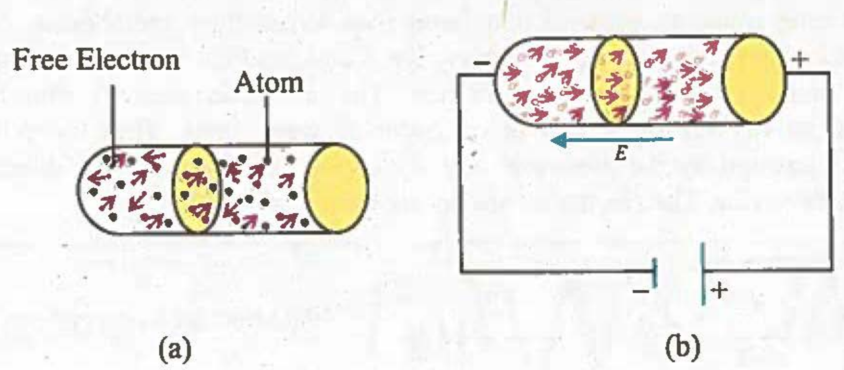


Figure 12.2 : drift of free electrons

The average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called drift velocity ($\bar{V}_d$).

The drift velocity of free electrons is of the order of 10^{-5} m/s.

Do you know?

Now if electrons drift so slowly, how room light turns on quickly when switch is closed? The answer is that propagation of electric field takes place with the speed of light.

Electroencephalogram (EEG)

Electroencephalography, or EEG, is a neurological test that uses an electronic monitoring device to measure and record electrical activity in the brain.

Special sensors (electrodes) are attached to your head and hooked by wires to a computer. EEG measures voltage fluctuations resulting from ionic current flows within the neurons of the brain. The brain's electrical charge is maintained by billions of neurons. Neurons are electrically charged ((or "polarized ") by membrane transport proteins that pump ions across their membranes. Brain electrical current consists mostly of Na+, K+, Ca++, and Cl- ions that are pumped through channels in neuron membranes. The computer records your brain's electrical activity on the screen or on paper as wavy lines. Thus the potential difference created by the electrical activity of the brain is used for diagnosing abnormal behavior. The electrodes are not harmful to your body.

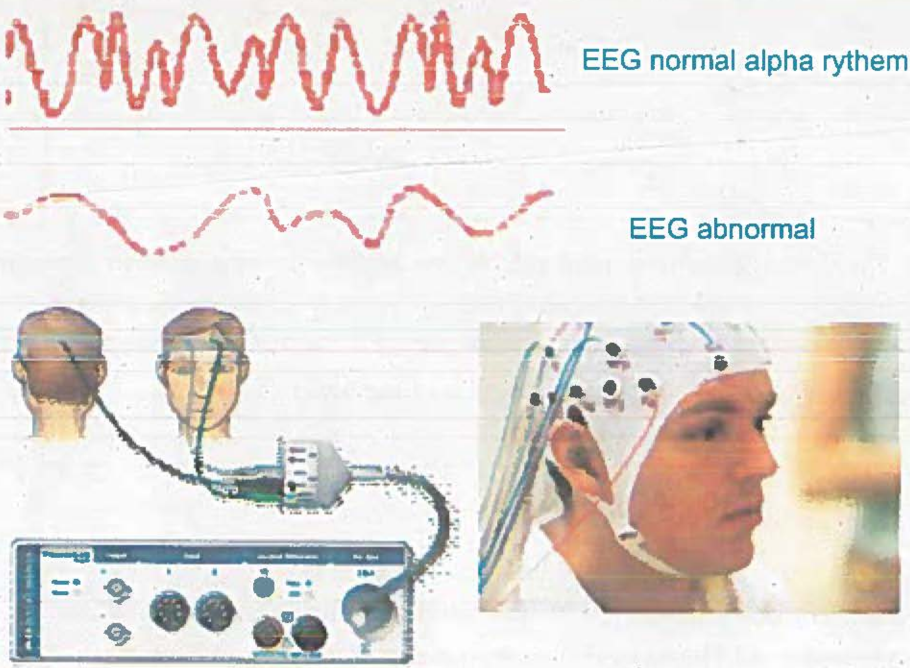


Figure 12.2(c): An electroencephalograph, or EEG, detects and measures electrical signals.

Example 12.1

Each second 10^{18} electrons flow from right to left across a cross-section of a wire attached to the two terminals of a battery.(a) Calculate the current in the wire, (b) in which direction the current is flowing.

Solution:

Electric current $I = \frac{Q}{t} = \frac{ne}{t}$

Here $n = 10^{18}$;

Charge on single electron: $e = 1.6 \times 10^{-19}$ C,

$t = 1$ s

$$I = \frac{10^{18} \times 1.6 \times 10^{-19}}{1} = 1.6 \times 10^{-1}\,\text{A} = 160\,\text{mA}$$

The current is flowing from left to right i.e. in opposite direction of electron flow.

12.3 OHM'S LAW

Many materials contain some 'free' electrons which can move in response to an applied electric field. By attaching a pair of metal wires and applying a voltage between them we can move these charge carriers through the material. Ohm's law is the most important, basic law of electricity. The relationship between the three fundamental electrical quantities: voltage (V), the current (I) and resistance (R) in a D.C. circuit was first discovered by German scientist George Simon Ohm in (1826). When a voltage is applied to a circuit containing only resistive elements, current flows according to Ohm's Law, which is $I = V / R$. If a voltage V is applied across a conductor and current (I) flows through it then according to ohm law

The magnitude of the current in metals is proportional to the applied voltage as long as the temperature of the conductor is kept constant.

Mathematically $\quad I \propto V \text{ or } I = \frac{V}{R}$

or $\quad V = IR \qquad (12.2)$

Where R is constant, which is known as the resistance of the conducting material. Resistance depends upon the nature, dimension and physical state of the conductor. For a conductor that obeys Ohm's law, a graph of current I as a function of voltage 'V' is a straight line passing through the origin Fig. (12.3(a).

The slope of the line is $\tan\theta = \frac{I}{V} = \frac{1}{R}$

As R is constant the slope is constant for ohmic conductors. The Ohm's law is not valid for all conducting material. Those materials for which the slope of I verses V graph is not constant are called non-Ohmic materials. For examples Fig 12.3 (b, c & d) illustrates the non-Ohmic characteristics of the filament of an electric bulb, a thermistor and of a semiconductor diode respectively. The I-V graph for a filament bulb shows that graph bends over as V and I increases, indicating that a given change of V causes a correspondingly smaller change in I at larger values of V. Thus slope decreases with the increase of voltage.

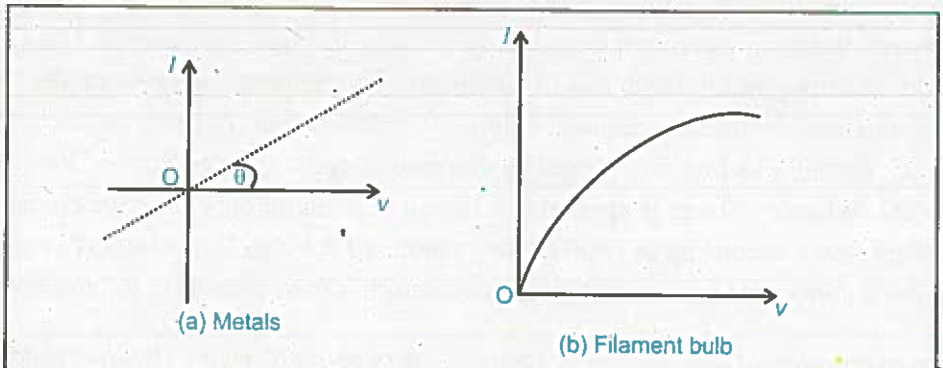
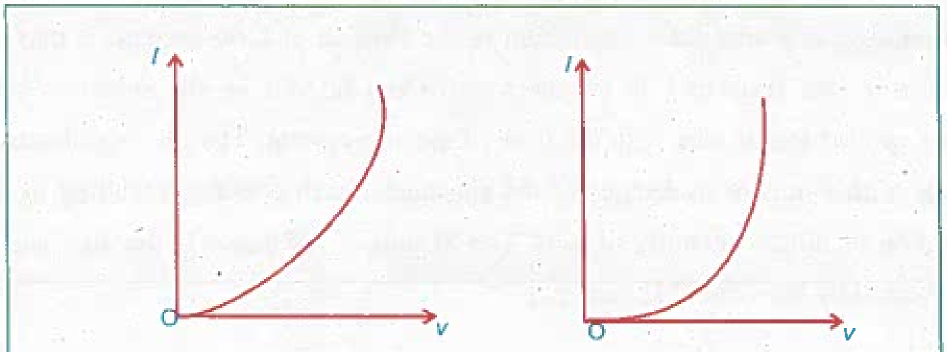


Figure 12.3: V-I graph



(c) thermistor (d) Semiconductor Diode

Figure 12.3: V-I graph

The I-V graph for thermistor bends upward shows that resistancc decreases sharply as their temperature rises. The I-V graph of a semiconductor diode shows that it is also non-linear graph. The current passes when the voltage is applied in one direction but it is almost zero when its acts in the opposite direction.

Example: 12.2

The high voltage in a TV receiver is 17 kV. The maximum allowable current is 150μA. What is the least permissible value of load resistance?

Solution

$$V = 17KV = 17\times10^{3}\ \text{V}$$

$$I = 150\mu\text{A} = 150\times10^{-6}\ \text{A}$$

$$R = \frac{V}{I} = \frac{17\times10^{3}}{150\times10^{-6}} = 113.3\times10^{6}\Omega$$

12.4 ELECTRICAL RESISTANCE

Resistance is the opposition offered by the substance to the flow of free electrons. This opposition occurs because atoms and molecules of the substance obstruct the

flow of these electrons. Certain substances (metals such as silver, copper, aluminium) offer very little opposition to the flow of electric current. It may be noted here that resistance is the electric friction offered by the substance and causes production of heat with the flow of electric current. The moving electrons collide with atoms or molecules of the substance, each collision resulting in the liberation of minute quantity of heat. The SI unit of resistance is the ohm and is represented by the symbol Ω (omega).

It is defined as, *the resistance of wire is to be 1 ohm if a potential difference of one volt across its ends causes 1 ampere current to flow through it.*

$$1\Omega = \frac{1\text{V}}{1\text{A}}$$

12.4.1 FACTORS UPON WHICH RESISTANCE DEPENDS

When electrons flow through a wire they experience resistance and lose energy, as the electrons flow on longer path they lose more energy. It is observed experimentally that the total resistance of a wire

1) Is directly proportional to its length i.e.

$$R \propto L$$

2) Is inversely proportional to its area of cross-section i.e.

$$R \propto \frac{1}{A}$$

Therefore thicker wires have less resistance per meter and will cause less energy to be lost as heat.

3) Depends upon the nature of material

4) Is directly proportional to the temperature.

From the first three points (keeping the temperature constant), the resistance of a conductor is,

$$R \propto \frac{L}{A}$$

or $$R = \rho \frac{L}{A} \qquad (12.3)$$

Where ρ (Greek LETTER 'Rho') is a constant and is known as resistivity or specific resistance of the material.

12.5 SPECIFIC RESISTANCE OR RESISTIVITY

We have seen above that

$$R = \rho \frac{L}{A}$$

If $L = 1\text{m}$, $A = 1\text{m}^2$, then $R = \rho$

Hence specific resistance of a material is the resistance offered by 1m length of wire of a material having an area of cross-section of 1m^2. The unit of resistivity is ohm-meter (Ω.m). The resistivity of a substance varies over a wide range. To give an idea to the reader, the following table may be referred.

Table 12.1: The resistivity's of some metals.

Metals	Resistivity(μΩm)
Copper (hard drawn)	0.0178
Aluminium	0.0285
Tin	0.114
Silver	0.0163
Brass	0.06-0.09
Iron	0.1
Lead	0.219

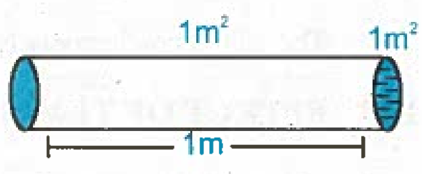


Fig: 12.4:cross-section area of a material

The resistivity of metals and alloys is very small. Therefore, these materials are good conductors of electric current. On the other hand, resistivity of insulators is extremely large. As a result, these materials hardly conduct any current. There is also an intermediate class of material known as semiconductors. The resistivity of these substances lies between conductors and insulators.

12.6 CONDUCTANCE

The Reciprocal of resistance of a conductor is called conductance (G) if a conductor has resistance R, then its conductance G is given by

$$G=\frac{1}{R} \qquad \text{...(12.4)}$$

The SI unit of conductance is mho (i.e. ohm spelt back ward) these days, it is a usual practice to use Siemen as the unit of conductance. It is denoted by symbol S.

12.7 CONDUCTIVITY

The reciprocal of resistivity of a conductor is called its conductivity. It is denoted by the symbol σ. If a conductor has resistivity ρ, then its conductivity is given by

$$\sigma=\frac{1}{\rho}=\frac{L}{RA} \qquad (12.5)$$

The unit of conductivity is mho m^{-1} or siemen meter^{-1} ($S.m^{-1}$)

12.8 EFFECT OF TEMPERATURE ON RESISTANCE

The resistance of a material changes with the change in temperature. The effect of temperature upon resistance varies according to the type of material.

i) Thus resistance of pure metal (e.g. copper, aluminum) increases with the increase of temperature. The change in resistance is fairly regular for normal range of temperatures.

ii) The resistance of electrolytes, insulators (e.g. glass, mica, rubber etc.) and semiconductors (e.g. germanium, silicon etc.) decreases with the increase in temperature.

12.8.1 TEMPERATURE CO-EFFICIENT OF RESISTANCE

Consider a conductor having resistance R_o at 0°C and R_T at T° C. It has been found that in the normal range of temperatures, the increase in resistance $(R_T - R_o)$ is

i) Directly proportional to the initial resistance i.e.

$$R_T - R_o \propto R_o$$

ii) Directly proportional to the rise in temperature i.e.

$$R_T - R_o \propto T$$

Combining the above two relations, we get

$$R_T - R_o \propto R_o T$$

$$R_T - R_o = \alpha R_o T \qquad (12.6)$$

Where α is the constant and is called temperature co-efficient of resistance. Its value depends upon the nature of material and temperature. Rearranging equation (12.6) we get:

$$R_T = R_o (1 + \alpha T) \qquad (12.7)$$

From equation (12.6) we get:

$$\alpha = \frac{R_T - R_o}{R_o T} \tag{12.8}$$

Temperature co-efficient of resistance α may be defined as *increase in resistance per ohm original resistance per degree rise in temperature.*

The unit of α, as derived from the definition is K^{-1}. Since the resistance of metals increases with the rise in temperature, they have positive temperature co-efficient of resistance, while co-efficient α is negative for semiconductor because they show decrease in resistance as the temperature raises.

12.8.2 VARIATION OF RESISTIVITY WITH TEMPERATURE

The resistivity or specific resistance of most materials increases linearly with increasing temperature. The equation, (12.7) also apply to ρ. At temperature 'T' the resistivity 'ρ_T' of a material is given by

$$\rho_T = \rho_o (1 + \alpha T) \quad \text{...(12.9)}$$

$$\alpha = \frac{\rho_T - \rho_o}{\rho_o T} \quad \text{...(12.10)}$$

Where α is called the temperature co-efficient of resistivity and may be defined as the fractional change in resistivity per Kelvin. The temperature co-efficient of resistivity is positive for metals. But the resistivity of Graphite (non-metal) decreases with the increasing temperature. Since at higher temperatures more electrons are "Shaken Loose" from the atoms and become mobile, hence the temperature co-efficient of resistivity of Graphite is negative. This same behavior occurs for semiconductors.

Example 12.3

A transmission line made of copper has a resistance of $100\,\Omega$ at $0C^{o}$. calculate the change in resistance between summer and winter, knowing that temperature varies from $+35C^{o}$ to $-30C^{o}$. Assume the temperature co-efficient of copper to be $0.00427/C^{o}$ at $0C^{o}$.

Solution:

$$R_T = R_o(1+\alpha T)$$
$$\Rightarrow R_{-30} = 100(1+0.00427(-30))$$
$$\Rightarrow R_{-30} = 87.2\,\Omega$$
$$\text{similarly } R_{+35} = 100(1+0.00427(35))]$$
$$R_{+35} = 115\Omega$$

Change in resistance = $115 - 87.2 = 27.8\,\Omega$

Example 12.4

Calculate the resistance of copper conductor having a length of 2 km and a cross-section of 22 mm^2. Assume the resistivity is $18 \times 10^{-9}\,\Omega$.m.

Solution:

The resistance of copper wire of length L, cross-sectional area A and resistivity ρ is given by

$$R = \frac{\rho L}{A}$$

L = 2 km = 2000 m

A = 22 mm^2 = 22×10^{-6} m^2

$\rho = 18 \times 10^{-9}\,\Omega$-m

Thus resistance of copper conductor is

$$R = 18\times10^{-9}\times\frac{2000}{22\times10^{-6}}$$

$$R = 1.64\Omega$$

12.9 Wire-Wound Variable Resistors

High stability and high accuracy resistors are always wire-wound. It is then enclosed in an insulating cover.

Generally, nickel chromium is used because of its very small temperature co-efficient of resistance. Wire wound resistors can safely operate at higher temperatures than carbon type resistors. Wire wound variable resistor can be used in two ways.

i) Rheostats ii) Potential divider

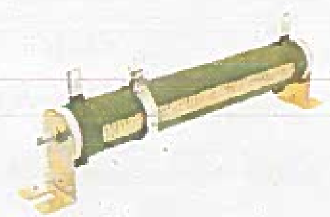


Figure 12.5 : Wire wound variable resistor

i) Rheostats

To use the variable resistor as a current control device one of the two fixed terminals say A, and the sliding terminal C are inserted in the circuit as shown in figure12.6. In this way the resistance of the wire between A and the sliding contact C is used. If the sliding contact is shifted away from the terminal A, the length and hence the resistance included in the circuit increase.

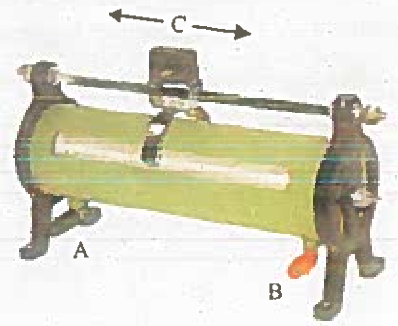


Figure 12.6 : Rheostats

If the sliding contact is moved towards A, the resistance decreases. Adjusting the resistance in the circuit controls the current in a circuit.

ii) Potential Divider

A potential divider provides a convenient way of getting a variable PD from a fixed potential difference. With the help of the battery, a potential difference V is applied across the ends A and B of the resistor. Let R be the resistance of wire AB. The current I passing through it is

$$I = V/R$$

If R_{BC} is the resistance of the portion of the wire BC and the current passing through BC is I. The PD between the points B and C is given by

$$V_{BC} = IR_{BC}$$

$$\Rightarrow \quad V_{BC} = \frac{V}{R} R_{BC}$$

$$\Rightarrow \quad V_{BC} = \frac{R_{BC}}{R} V$$

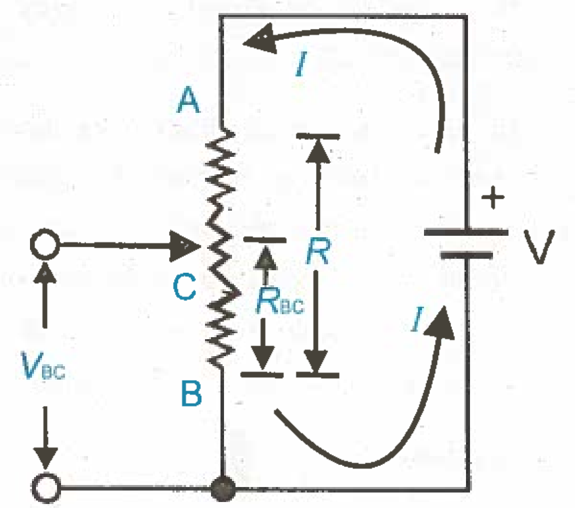


Figure 12.7 : potential divider

Depending on the position of the sliding contact C, the value of the fraction R_{BC}/R can be varied from 0 to 1. When the contact C, moving towards B, the length and hence the resistance R_{BC} of the portion of the wire decreases.

Thus V_{BC} decreases .On the other hand if the sliding contact C is moved towards the end A, the length and the resistance R_{BC} of the wire increases and hence the voltage V_{BC} increases.

12.10 Thermistor

A resistor made of semiconductors having resistance that varies rapidly and predictably with temperature is kn n as thermistor.

A thermistor (short for thermal resistor) is a heat sensitive device usually made of a semiconductor material whose resistance changes very rapidly with change of temperature. A thermistor has the following important properties.

i) The resistance of thermistor changes very rapidly with change of temperature.

ii) The temperature co-efficient of a thermistor is very high.

iii) The temperature co-efficient of a thermistor can be both positive and negative. Thermistors are made from semiconductor oxides of iron, nickel and cobalt. They are generally in the form of discs or rods. Pair of platinum leads are attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed.

APPLICATIONS

a) A thermistor with negative temperature co-efficient of resistance may be used to safeguard against current surges in a circuit where this could be harmful e.g. in a circuit where the heaters of the radio valves are in series as shown.

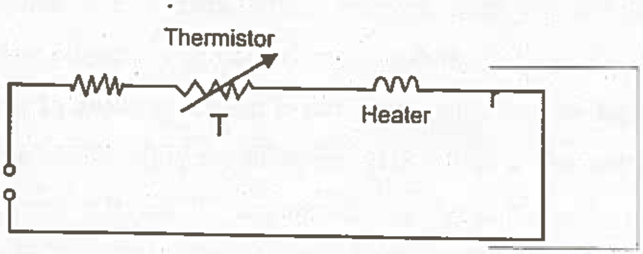


Figure 12.8 : thermistor

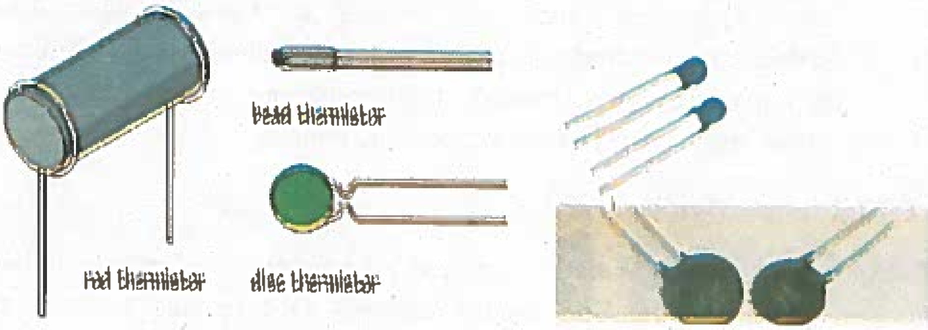


Figure12.9: types of thermistor

A thermistor T is included in the circuit. When the supply voltage is switched on the thermistor has a high resistance at first because it is cold. It thus limits the current to a moderate value. As it warm up, the thermistor resistance drops appreciably and an increased current then flow through the heaters.

Modern appliances, communication tools and accessories like mobile phones, computers, LCD displays, CPUs, rechargeable batteries, and medical and patient monitoring equipment are all equipped with thermistors so they can be used continuously without fear of overheating and appliance damage.

b) A thermistor with a negative temperature co-efficient (NTC) can be used to issue an alarm for excessive temperature of winding of motors, transformers and generators. When the temperature of windings is low, the thermistor is cool and its resistance is high. Therefore, only a small current flows through the thermistor. When the temperature of the windings is high, the thermistor is hot and its resistance is low. Therefore, a large current flows in the coil to close the contact.

c) Some appliances, such as washing machines, clothes dryers, refrigerators and freezers, as well as small appliances like hair dryers, curling irons, ovens, toasters, thermostats, air conditioners and fire alarms also have NTCs for temperature control

12.11 Electromotive Force

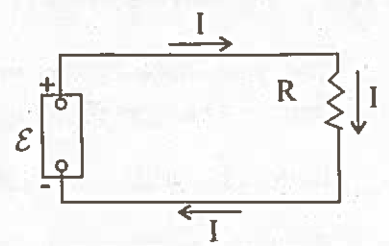
An external energy source is required by most electrical circuits to move charge through the circuit. The circuit therefore must include a device that maintains a potential difference between two points in the circuit, just as a circulating fluid requires as analogous device (pump).

Such a device which converts non-electrical energy into electrical energy is called a source of electromotive force (emf).

This reminds us a pump, which can cause water to move from a place of low gravitational potential to a place of high potential. The Fig.(12.10) shows the e.m.f. source $\mathcal{E}$, considered to be a battery connected to a resistor R. It maintains its upper terminal at a high potential and its lower terminal at a low potential, as indicated by the + and – signs. In the external circuit positive charge carriers would be driven in the direction shown by the arrows marked I.

We define emf $\mathcal{E}$ of a source equal to the work done in carrying 1 coulomb of charge through the source. Suppose q coulombs require an amount of work W joule to be transported through the source then $\mathcal{E}$ in volts is given by

$$\mathcal{E} = \frac{W}{q} \qquad \text{...(12.11)}$$



12.10:Electromotive force:

The potential at the ends of terminals of a battery when circuit is open is also called e.m.f.

The unit of emf is the joule/coulomb, which is the volt (abbreviation V):

$$1 \text{ volt} = 1 \text{ joule/coulomb}$$

For your Information

Just as a water fountain requires a pump, an electric circuit requires a source of electromotive force to sustain a steady current. In an electronic circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain. In this device a charge travels uphill's from lower to higher potential energy even though the electrostatic force is trying to push it from higher to lower potential energy. The direction of current in such a device is from lower to higher potential, just the opposite of what happens in an ordinary conductor. The influence that makes current flow from lower to higher potential is called electromotive force.

12.11.1 Sources of e.m.f

The work done by emf on charge carriers in its interior must be derived from a source of energy. There are many sources of e.m.f. A few examples are

1. Batteries or cells convert chemical energy into electrical energy.
2. Electrical generators convert mechanical energy into electrical energy.
3. Thermocouples convert heat energy into electrical energy.
4. Radiant (a solar cell) converts sunlight directly into electrical energy.

12.12 Internal Resistance of a Supply.

All supplies (e.g. a cell) must have some internal resistance, however small. When the cell is delivering no current i.e. no load, the P.D across the terminals will be equal to e.m.f ($\mathcal{E}$) of the cell. When some load resistance 'R' is connected across the terminals of the cell, the current I starts flowing in the circuit as shown in Fig 12.11. This current causes a voltage drop across internal resistance r of the cell so that terminal voltage V available will be less than $\mathcal{E}$. The relationship between $\mathcal{E}$ and V can be easily established.

$$I = \frac{\mathcal{E}}{R+r}$$

$$\Rightarrow \quad IR = \mathcal{E} - Ir$$

But $IR = V$, the terminal voltage of the cell

$$V = \mathcal{E} - Ir$$

Now, if

$$r = 0,$$

(Open Circuit); $I = 0$

Then $V = \mathcal{E}$

But, when the circuit is closed, the current move through it experiences an associated drop in potential equal to Ir. Thus when a current is flowing through a source, the Potential difference V between the terminals of the source is $V = \varepsilon - Ir$ the Potential V called the terminal voltage, is less than the emf ε because of the term Ir representing the Potential drop across the internal resistance r.

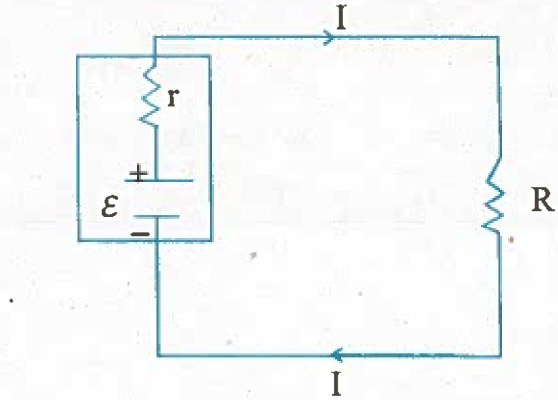


Figure 12.11: internal resistance of a battery

12.13 Electric Power

The rate at which work is done in an electric circuit is called electric power i.e.

$$\text{Electric Power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, (Fig 12.12). It causes current to flow through the resistance R in time t.

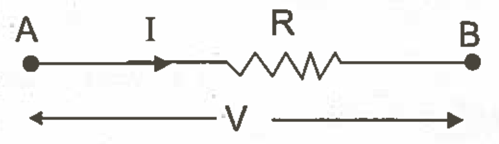


Figure 12.12 : work done on a charge

Clearly work is being done in moving the charges in the circuit. This work done in moving the charges in a unit time is called the electric power.

Thus total charge Q flows in t second is $Q = I \times t$ and $V = \frac{W}{Q}$

$$W = QV = VIt$$

$$\text{Electric power } = P = \frac{W}{t} = \frac{VIt}{t}$$

$$= VI \qquad ...(12.12)$$

$$or \quad P = I^2R \qquad ...(12.13)$$

$$\therefore V = IR$$

$$or \quad P = \frac{V^2}{R} \qquad ...(12.14)$$

$$\therefore I = \frac{V}{R}$$

The above three formulas are equally valid for calculation of electric power in a d.c. circuit. Which one is to be used depends simply on the known quantities.

Unit of electric Power: The unit of power in SI is watt . *A power of 1 watt is said to be consumed in a circuit if a potential difference of 1 volt causes a current of 1 ampere to flow through it.*

$$1 \text{ Watt} = 1\text{V} \times 1\text{A}$$

The bigger units of power are kilowatt (kW) and mega watt (MW)

$$1 \text{ kW} = 1000 \text{ watts} = 10^3 \text{ watts}$$

$$1 \text{ MW} = 10^6 \text{ watts}$$

Kilowatt-hours:

Electric energy is commonly consumed in very large quantity for the measurement of which Joule is a very small unit. Hence a large unit of energy is required which is called kilowatt-hour. On kilowatt-hour is commonly termed as one commercial unit of electrical energy. It is the amount of energy obtained by a power of 1 kilowatt in one hour.

$$1 \text{ kWh} = 1000 \text{ W} \times 3600\text{s} = 3.6 \times 10^6 \text{ W s}$$

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J} \qquad \therefore 1\text{J} = 1\text{W} \times 1\text{s}$$

Example 12.5

A heating coil has a resistance of 20Ω. It is designed to operate on 220 V. What electric energy in joules is supplied to the heater in 10 s?

Solution:

$$R = 20\ \Omega, \quad V = 220\,\text{V}$$
$$t = 10\,\text{s}, \quad W = ?$$

From Ohm's law $V = IR$ or $I = \dfrac{V}{R}$

$$I = \frac{220\ \text{V}}{20\ \Omega} = 11\,\text{A}$$

Now applying the formula

$$W = I^2 Rt$$
$$W = (11\ \text{A})^2(20\Omega)(10\ \text{s})$$
$$W = 24200\ \text{J}$$

Example: 12.6

The following are the details of load and a circuit connected through a supply meter.

(i) Six lamps of 40 watts each working for 4 hours per day.

(ii) Two fluorescent tubes 125 watts each working for 2 hours per day.

(iii) 1000 watts heater working for 3 hours per day.

Find the cost of electrical energy consumed in 30 days of a month when the rate of electricity i.e, energy costs is Rs 7.0 per unit?

Solution

Total watts for lamps = $40 \times 6 = 240\,watts$

Total watts for tubes = $125 \times 2 = 250\,watts$

Heater = 1000 watts

Energy consumed per day =

= $(240 \times 4) + (250 \times 2) + (1000 \times 3)$

= 4460 watt hours

= 4.46 Kwh

Total energy consumed in the month of June

= $4.46 \times 30 = 133.8\,kwh$

Bill for the month of June = 7×133.8

= $936.6\,Rs$

Example: 12.7

Determine the resistance of a resistor which must be placed in series with a 75 Ω resistor across 120 V source in order to limit the power dissipation in the 75Ω ,resistor to 90 watts.

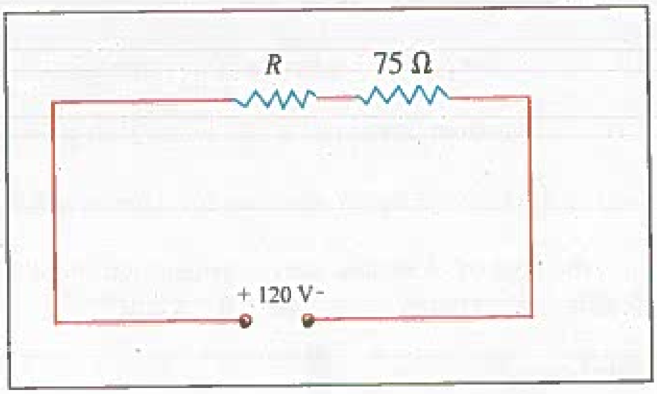
Solution

$$I^2 \times 75 = 90$$

$$I = \sqrt{\frac{90}{75}}$$

$$I = 1.095A$$

$$Now \quad I = \frac{120}{R+75}$$



$$1.095 = \frac{120}{R+75}$$

$$\Rightarrow \quad R+75 = \frac{120}{1.095}$$

$$R = 34.6\Omega$$

12.14 Maximum Power output

In many electronic circuits and systems it is important to have maximum transfer of power from the source to the load. For example, in radio or TV transmitting systems, it is desired to transfer the maximum power possible from the transmitting medium to the antenna systems. We want maximum power transfer from amplifier to speaker system. This is accomplished by proper matching of load resistance *R* and source resistance *r*.

If the load resistance is less or greater than the source resistance, then the power delivered to the load will be minimum.

Consider the circuit of fig:12.13, as the current *I* flows through, the load R the charges flow from a point of higher potential to a point of lower potential. In this process, they lose potential energy. If *V* is the P.D. across *R*, the loss of potential energy per second is known as power delivered to *R* by the current *I*.

From electrical power

$$P_{out} = IV$$

$$P_{out} = I^2R$$

$$P_{out} = \frac{\varepsilon^2 R}{(R+r)^2} \qquad \therefore I = \frac{\varepsilon}{(R+r)}$$

$$P_{out} = \frac{\varepsilon^2 R}{(R-r)^2 + 4Rr}$$

When $R = r$, the denominator of the expression for P_{out} is minimum and so P_{out} is maximum.

Thus it can be concluded that *maximum power is delivered to a load R when the internal resistance of the source of emf is equal to the load resistance*, also called maximum power transfer theorem.

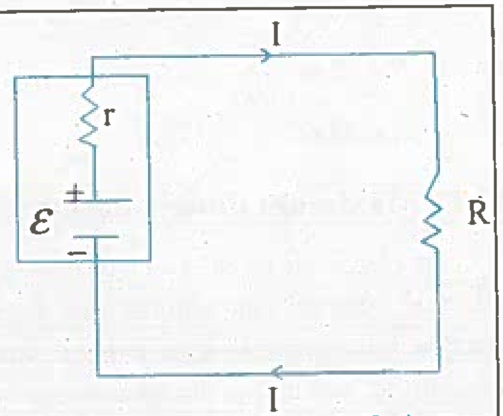


Figure 12.13: internal resistance of a battery

The value of the maximum output power is

$$(P_{out})_{max} = \frac{\varepsilon^2}{4r} = \frac{\varepsilon^2}{4R} \qquad ...(12.15)$$

12.15 Thermocouples

In early 1821, Thomas Seebeck searched experimentally for a relation between electricity and heat. He joined two wires of two dissimilar metals to form a circuit. He discovered that if one junction is heated to a high temperature, and the other junction remained at a cooler temperature then the galvanometre connected at their ends shows a deflection. This is known as Seebeck Effect. The e.m.f. generated in the circuit is called thermoelectric e.m.f. The resulting current is known as thermoelectric current.

The two junction circuit is called a thermocouple. In this process heat energy is directly converted into electrical energy. The wire pairs can be composed of noble metals (such as platinum, iridium, silver, osmium, gold, and rhodium) or base metals, (such as copper, iron or nickel-copper alloy, lead and zinc).

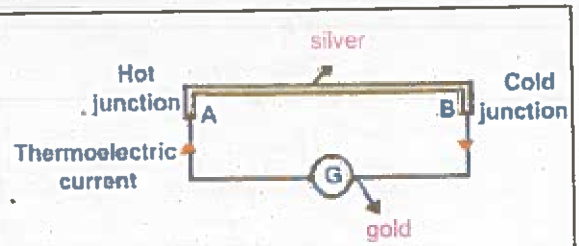


Figure12. 14: thermocouple

Thus it remains true for any pair of metals. Thermocouples are the most widely used temperature sensors in industry due to their low cost, simplicity, size and useable temperature range. The electromotive force $\mathcal{E}$ is a function of the temperature gradient.

The thermo- e.m.f. produced is very small, of the order of mV per every degree of temperature difference. The Seebeck effect is reversible, i.e., if the hot and cold junctions are interchanged, the direction of e.m.f. (and hence current) reverses. The greater the separation of the metals forming the thermocouple in the series, greater is the thermo e.m.f. produced. The thermo e.m.f. of many thermocouples has been measured as a function of the temperature T of the hot junction, when the cold junction is maintained at 0^{o}C. Its temperature dependence is given by

$$\varepsilon = \alpha T + \frac{1}{2}\beta T^2 \qquad ...(12.16)$$

Where α and β are constants (called thermoelectric coefficients) which depends on the nature of the metals.

12.15.1 Variation in thermoelectric e.m.f. with Temperature

Fig:12.15(a) shows an arrangement to study the effect of temperature difference between the two junctions in a Cu-Fe thermocouple. Keeping the junction B at 0^{o}C, the temperature of junction A is increased. When both the junctions are at the same temperature, there is no thermo e.m.f.

The thermo e.m.f. increases with temperature and reaches a maximum value at a certain temperature, called the neutral temperature T_n.

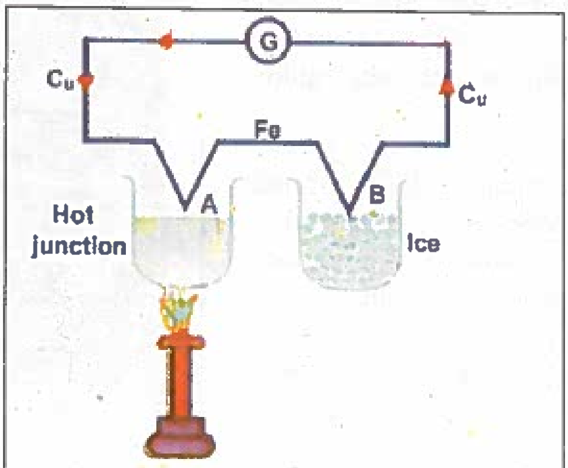


Figure 12.15 (a) : variation of thermo emf with temperature

The value of the neutral temperature is constant for a thermocouple, depends on the nature of materials and is independent of the temperature of the cold junction. As the temperature of the hot junction is increased, the thermo e.m.f. starts decreasing instead of increasing. The particular temperature at which, the thermo e.m.f. becomes zero is called the inversion temperature. The graph shows the variation of the thermo e.m.f. with the temperature of hot junction, with the cold junction.

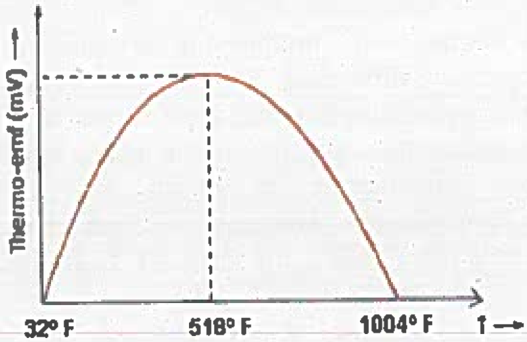


Figure 12.15 (b): variation of thermo emf with temperature *T* for copper-iron thermocouple

12.16 Resistance thermometers

Resistance thermometers, also called resistance temperature detectors ('*RTD*'s), are sensors used to measure temperature by correlating the resistance of the RTD element with temperature.

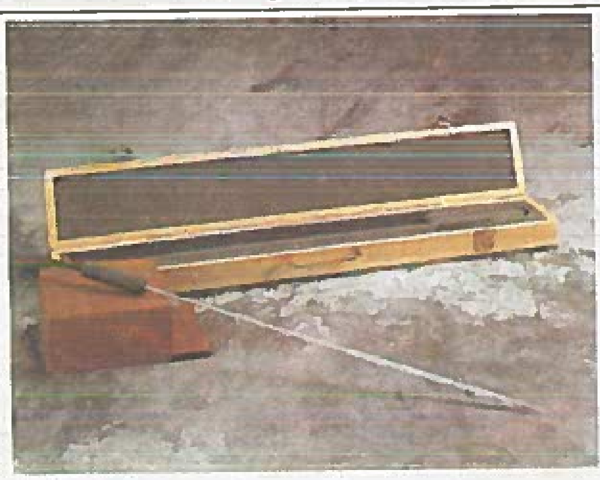


Figure 12.15 (c): Resistance thermometers

Common RTD sensing elements constructed of platinum, copper or nickel have a unique, and repeatable and predictable resistance versus temperature relationship (R vs T) and operating temperature range.

The R vs T relationship is defined as *the amount of resistance change of the sensor per degree of temperature change.* The relative change in resistance (temperature coefficient of resistance) varies only slightly over the useful range of the sensor.

Platinum is the best metal for RTDs because it follows a very linear resistance-temperature relationship and it follows the R vs T relationship in a highly repeatable manner over a wide temperature range. The unique properties of platinum make it the material of choice for temperature standards over the range of -185 °C to 630 °C. They are slowly replacing the use of thermocouples in many industrial applications below 600 °C, due to higher accuracy and repeatability.

12.17 Kirchoff's Law

Sometimes we encounter circuits where simplification by series and parallel combinations is impossible consequently. Ohm's law cannot be applied to solve such circuits. This happens when there is more than one emf in the circuit or when resistors are connected in a complicated manner. Such circuits are called complex circuits. Fig 12.16(a) shows a circuit containing two sources of emf $\mathcal{E}_1$ and $\mathcal{E}_2$ and three resistors. This circuit cannot be solved by series parallel combinations. Are resistors R_1 and R_3 in parallel? Not quite, because there is an e.m.f source $\mathcal{E}_1$ between them. Are they in series? Not quite, because same current does not flow between them.

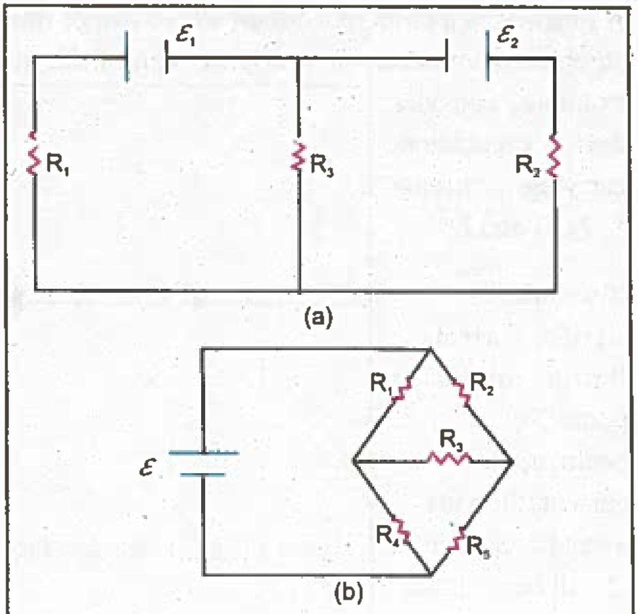


Figure 12.16 : complex circuit

Figure:12.16b shows another circuit which cannot be solved by series–parallel combinations. Though this circuit has one source of e.m.f ($\mathcal{E}$), it cannot be solved by using series and parallel combinations. These resistors R_1 and R_2 are neither in series nor in parallel.

In order to solve such complex circuits, German physics Gustar Robert Kirchhoff's (1824-1887) gave two laws, known as Kirchhoff's laws

Kirchoff's current law (KCL)

This law is related to the currents at the junctions of an electric circuit and may be stated as under:

The algebraic sum of all the currents meeting at a junction in an electrical circuit is zero. $\sum I = 0$

A junction is a point in a circuit where two or more components are connected. As algebraic sum is one in which the sign of the quantity is taken into account. For example, consider four conductors carrying currents I_1, I_2, I_3 and I_4.

If we take the signs of currents flowing towards point O as positive, then currents flowing away from point O will be assigned negative sign.

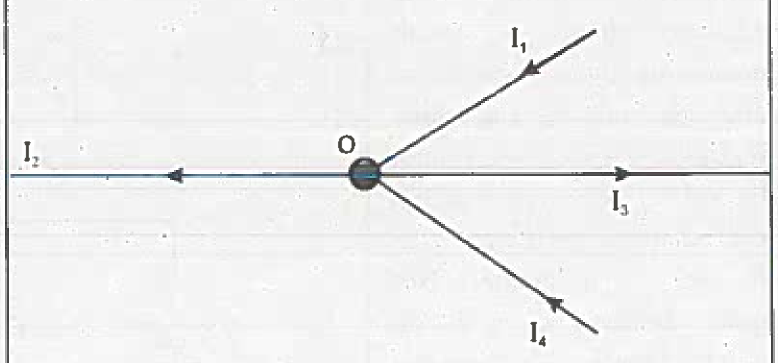


Figure 12.17: algebraic sum of the currents with junction at O.

Thus applying Kirchhoff's current laws to the junction

$$(I_1)+(I_4)+(-I_2)+(-I_3)=0$$

Or $$I_1+I_4=I_2+I_3 \quad ...(12.17)$$

i.e. sum of incoming currents = sum of outgoing current.

Hence, Kirchhoff's current law may also be stated as *the algebraic sum of currents flowing towards the junction in an electrical circuit is equal to the algebraic sum of currents flowing away from that junction.*

Kirchoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge. Hence Kirchoff's current law is based on the law of conservation of charge.

Kirchoff's Voltage Law (KVL)

This law refers to e.m.f and voltage drops in a closed circuit or closed loop and may be stated as under:

In any closed electrical circuit, the algebraic sum of all the electromotive force, (e.m.f) and voltage drops in resistor is equal to zero i.e.

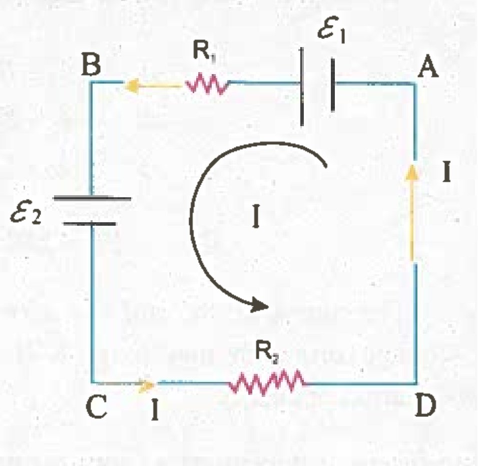


Figure 12.18 Kirchoff's voltage law

Algebraic sum of emf + algebraic sum of voltage drops = 0.

The validity of Kirchoff's voltage law can be easily established by referring to the closed loop ABCDA shown in Fig. 12.18

Starting from any point (say point A) in this closed circuit and go back to this point (i.e. point A) after going around the circuit, then there is no increase or decrease in potential.

Consider the circuit shown in figure. The following convention is adopted to solve the circuit.

i) A rise in potential is taken positive.
ii) A drop in potential is taken negative.
iii) For batteries, the positive end is always at higher potential.
iv) Current flows from high potential to the low potential.
v) Assume the direction of current either clock wise or counter clockwise.

Let's take counter–clockwise round. Consider current from +ve terminal of $\mathcal{E}_1$, we take the rise of potential as + $\mathcal{E}_1$, through R_1, the voltage drop is – IR_1. Across $\mathcal{E}_2$, from the +ve to –ve side there is a fall of potential $-\mathcal{E}_2$. Through R_2, there is a voltage drop of $-IR_2$, apply KVL, we get

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

$$\Rightarrow \quad \mathcal{E}_1 - \mathcal{E}_2 = IR_1 + IR_2$$

$$\Rightarrow \quad \mathcal{E}_1 - \mathcal{E}_2 = IR_1 + IR_2$$

$$\text{Or} \quad \Sigma\mathcal{E} = \Sigma IR \qquad \text{...(12.18)}$$

The sum of all the emf $\mathcal{E}$ in a loop in a circuit is equal to the sum of all the IR voltage drops in that loop. KVL is in fact, a statement of the law of conservation of energy.

Procedures of Kirchhoff's Laws for problem solution

In order to solve problems by using Kirchhoff's Laws consider the circuit shown in Fig12.19

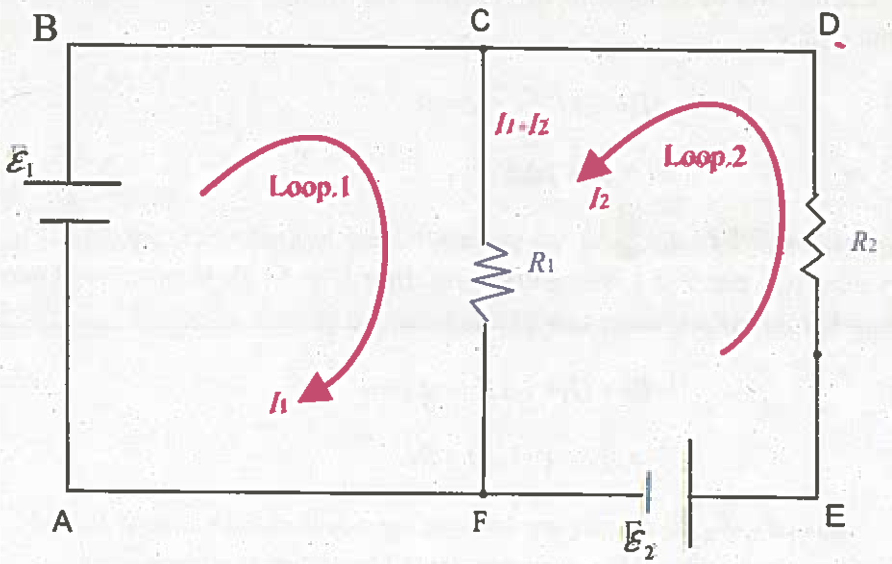


Figure 12.19 : Kirchhoff's law procedure

Let us take two closed loops in Fig.12.19 loop.1 (ABCFA) and loop.2 (CDEFC). The choice of loops is quite arbitrary, but it should be such that each resistance is included at least once in the selected loop. Kirchhoff's voltage law can be applied to these closed loops to get the desired equations.

Suppose a current I_1 is flowing in loop.1 (ABCFA) and current I_2 in loop.2 (CDEFC). Thus at junction C in Fig. the incoming currents to the junction are I_1 and I_2. Obviously, the current in branch CF will be $I_1 + I_2$. The direction in which currents are assumed to flow is un-important, since if wrong direction is chosen, it will be indicated by a negative sign in the result.

Loop:1(ABCFA). In this loop, e.m.f. $\mathcal{E}_1$ will be given positive sign. It is because as we consider the loop in the order *ABCFA*, we go from –ve terminal to the positive terminal of the battery in the branch *AB* and hence there is a rise in potential. The voltage drop in branch *CF* is $(I_1 + I_2)\, R_1$ and shall bear negative sign. It is because as we consider the loop in the order *ABCFA*, we go with current

in branch CF and there is a fall in potential. Applying Kirchhoff's voltage law to the loop $ABCFA$,

$$-(I_1+I_2)R_1+\mathcal{E}_1=0$$

or $$\mathcal{E}_1=(I_1+I_2)R_1$$

Loop:2 (CDEFC). As we go around the loop in the order $CDEFC$, drop I_2R_2 is positive, e.m.f. $\mathcal{E}_2$ is negative and drop $(I_1+I_2)R_1$ is positive. Therefore, applying Kirchhoff's voltage law to this loop, we get,

$$I_2R_2+(I_1+I_2)R_1-\mathcal{E}_2=0$$

Or $$\mathcal{E}_2=I_2R_2+(I_1+I_2)R_1$$

Since $\mathcal{E}_1$, $\mathcal{E}_2$, R_1 and R_2 are known, we can find the value of I_1 and I_2 from the above two equations. Hence currents in all branches can be determined.

Example 12.8

The emfs of two batteries are 6V and 2V and internal resistances of 2Ω and 3Ω respectively which are connected in parallel across a 5Ω resistor. Calculate (a) current through each battery and (b) terminal voltage.

Solution:

Fig. 12.20 shows two loops having two unknown quantities I_1 and I_2. The direction of current are marked in the various branches. Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be the potential difference of these batteries.

(a) Loop *HBCDEFH*:

Applying Kirchhoff's voltage law to the loop $HBCDEFH$, we get,

$$-\mathcal{E}_1+\mathcal{E}_2+I_1R_1-I_2R_2=0$$

$$2I_1-6+2-3I_2=0$$

Or $$2I_1-3I_2=4 \qquad (1)$$

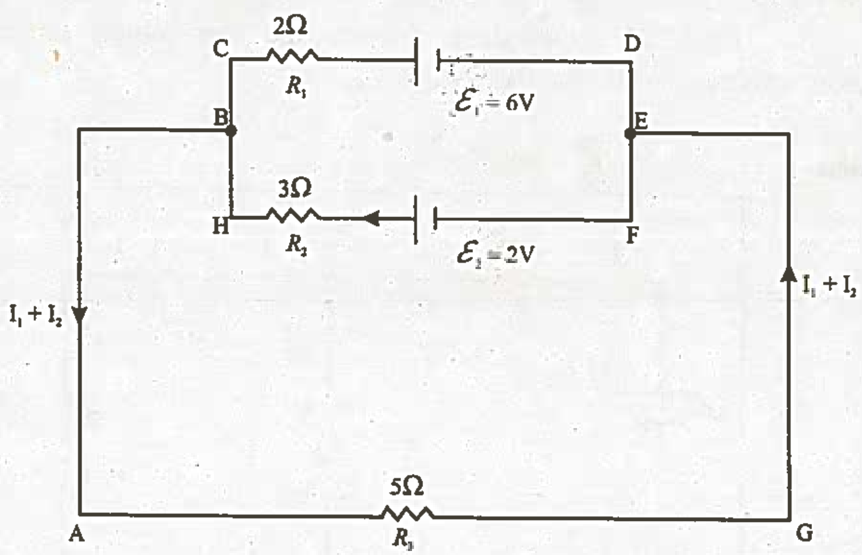


Figure 12.20 : complex circuit solution through Kirchhoff's rule

(b) Loop *ABHFEGA*:

Applying Kirchhoff's voltage law to the loop *ABHFEGA*, we get,

$$R_2 I_2 - \mathcal{E}_2 + R_3 (I_1 + I_2) = 0$$

$$3I_2 - 2 + 5 (I_1 + I_2) = 0$$

Or $\quad 5I_1 + 8I_2 = 2 \qquad (2)$

Multiplying eq. (1) by 8 and eq. (2) by 3 and then adding them, we get,

$$31\ I_1 = 38 \quad \text{or} \quad I_1 = 38/31 = 1.23\text{A}$$

i.e. battery $\mathcal{E}_1$ is being discharged at 1.23A. Substituting $I_1 = 1.23$ A in eq. (1), we get, $I_2 = 0.52$A i.e. battery $\mathcal{E}_2$ is being charged.

(ii) Terminal voltage $= (I_1 + I_2)\ 5 = (1.23 - 0.52)\ 5 = 3.55$V

Example 12.9

Fig:12.21 shows three resistors and two voltage sources. Use Kirchhoff's voltage law to find the voltage V_{ab}.

Solution

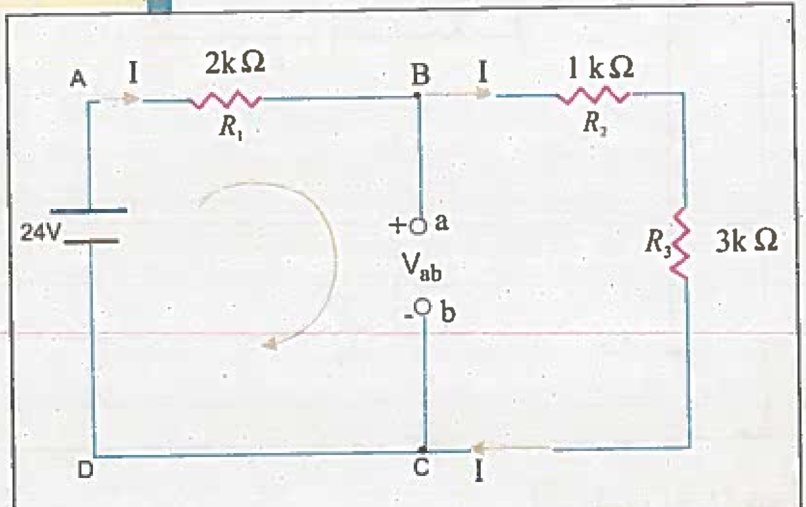


Figure 12.21: complex circuit solution through Kirchhoff's rule

Total circuit resistance, $R_T = 2 + 1 + 3 = 6k\Omega$

Circuit current, $I = \frac{V}{R_T} = \frac{24V}{6k\Omega} = 4mA$

Applying Kirchhoff's voltage law to loop *ABCDA*, we have

$$\mathcal{E}_1 - IR - V_{ab} = 0$$

$$24 - 4\text{ mA} \times 2k\Omega - V_{ab} = 0$$

$$24 - 8 - V_{ab} = 0 \qquad \therefore \qquad V_{ab} = 24 - 8 = 16V$$

12.18 WHEATSTONE BRIDGE

This bridge was first proposed by Wheatstone an English telegraph engineer for measuring accurately the value of an unknown resistance.

It consist of four resistors (two fixed resistances P and Q which are known,one variable resistance R which is also known and one unknown resistance X whose value is to be found) as shown in fig12.22. Across one pair of opposite junctions (A and C) battery is connected and across the other opposite pair of junctions (B and D), a galvanometer is connected.

WORKING

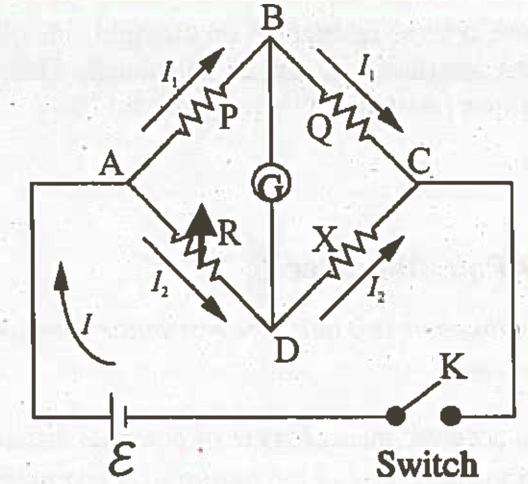


Figure 12.22 ; wheat stone bridge

The value of P and Q are properly fixed. When the key is switched on, the current I divide unequally between the two branches and the galvanometer shows current. The value of resistance R is varied until the galvanometer shows zero current. Under such conditions, the bridge is said to be balance. The point at which the bridge is balanced is called null point. Let I_1 and I_2 be the currents through P and R respectively when the bridge is balanced. Since there is no current through Galvanometer, the current in Q and X are also I_1 and I_2 respectively. As the Galvanometer reads zero, points B and D are at the same potential. This means that voltage drops from A to B and A to D must be equal. Also voltage drops from B to C and D to C must be equal. Hence

For balanced bridge the potential drop across AB= the potential drop across AD

$$I_1 P = I_2 R \qquad (i)$$

similarly the potential drop across BC= the potential drop across DC

and $$I_1 Q = I_2 X \qquad (ii)$$

Dividing Eq:(i) by Eq:(ii), we get

$$\frac{P}{Q} = \frac{R}{X}$$

$$PX = RQ$$

$$\Rightarrow \quad X = \frac{RQ}{P} \qquad ...(12.19)$$

Hence, if three resistances on the right side of equation (12.19) are known, the fourth resistance X can be calculated. This is the principle of determining unknown resistance by Wheatstone bridge.

12.19 Potentiometer

A potentiometer is a null type resistance network device for measuring potential differences.

For the accurate measurement of potential difference, current and resistance the potentiometer is one of the most useful instruments. Its principle of action is that an unknown emf or P.D. is measured by balancing it, wholly or in part, against a known potential difference.

Construction

: Asimplest potentiometer consists of wire LM of uniform cross-section, stretched alongside a scale and connected across battery of potential V as shown in fig12.23. A standard cell of known emf $\mathcal{E}_l$ is connected between L and terminal 1 of a two way switch S.

Working:

Slider N is pressed momentarily against wire LM and its position is adjusted until the galvanometer deflection is zero when N is making contact with LM. Let l_l be the corresponding distance between L and N.

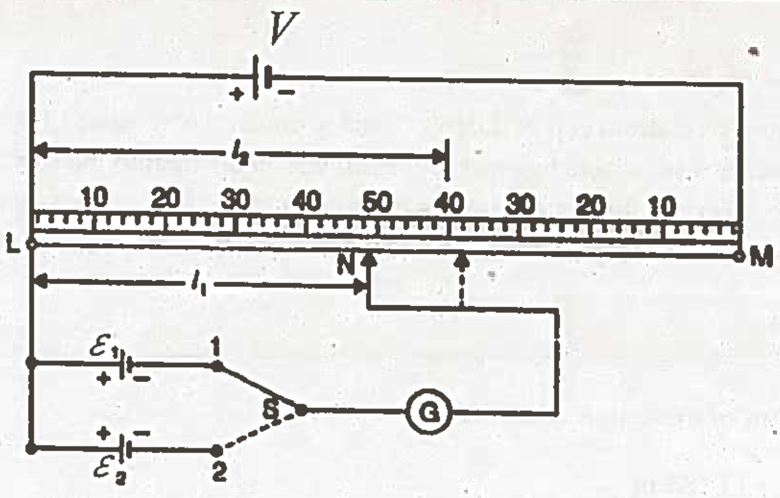


Figure 12.23 : simplest potentiometer circuit

The fall of potential over length l_1 of the wire is then the same as the emf $\mathcal{E}_1$. Then move the switch to 2, thereby replacing the standard cell by another cell, the e.m.f $\mathcal{E}_2$ of which is to be measured. Adjust the slider N again to give zero deflection on G. If l_2 be the new distance between L and N, then

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}$$

$$\mathcal{E}_2 = \frac{l_2}{l_1} \times \mathcal{E}_1 \qquad \ldots(12.20)$$

Applications of potentiometer: following are the applications of potentiometer.

1. Measurement of small e.m.f (upto 2V)
2. Comparison of e.m.f of two cells.
3. Measurement of high e.m.f (say 250 V).
4. Measurement of resistance.
5. Measurement of current.
6. Calibration of ammeter.
7. Calibration of voltmeter.

Example: 12.10

Using a weston cadium cell of 1.0183 V and a standard resistance of 0.1Ω a potentiometer was adjusted so that 1.0183 m was equivalent to the e.m.f of the cell: when a certain direct current was flowing through the standard resistance, the voltage across it corresponds to 150 cm. What was the value of current?

Solution:

Emf of the cell, $\mathcal{E}_1$=1.0183 V

L_1=1.0183 m

L_2=150 cm = 1.5 m

Resistance R= 0.1Ω

$\mathcal{E}_1$= ?

$$\frac{\mathcal{E}_2}{\mathcal{E}_1}=\frac{\ell_2}{\ell_1}$$

$$\mathcal{E}_2=\frac{\ell_2}{\ell_1}\times\mathcal{E}_1=\frac{1.5}{1.0183}\times 1.0183=1.5\,\text{V}$$

current flowing through the standard resistance,

$$I_2=\frac{\mathcal{E}_2}{R}=\frac{1.5}{0.1}=15\text{A}$$

Key points

- The continuous flow of free electrons is called steady current.
- Current is flow of electrons and electrons are the constituents of matter.
- Those substances which have larger number of free electrons will permit current flow easily. Such substances are called conductors, for example copper, Zinc, Silver, aluminum etc.
- *The current through the cross-section area A is the net charge flowing through the area per unit time.* $I = Q/t$
- *The average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called drift velocity* ($\vec{v}_d$).
- Ohm law states that *the magnitude of the current in metals is proportional to the applied voltage as long as the temperature of the conductor is kept constant.* $V = IR$
- *Resistance is the opposition offered by the substance to the flow of free electrons.*
- The resistance R of a conductor is directly proportional to its length and inversely proportional to its area of cross-section and is given by

$$R = \rho L/A$$

- The Reciprocal of resistance of a conductor is called conductance (G).
- The increase in resistance $(R_T - R_o)$ is directly proportional to the initial resistance, rise in temperature and nature of material and is given by

$$R_T - R_o = \alpha R_o T$$

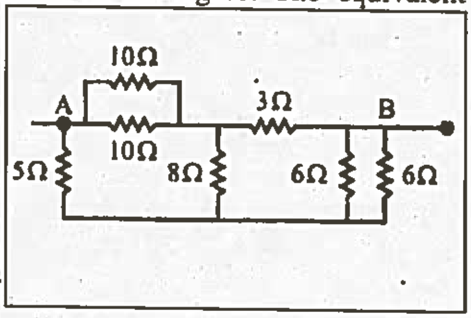
- Temperature co-efficient of resistance α may be defined as *Increase in resistance per ohm original resistance per degree rise in temperature.*
- *The temperature co-efficient of resistivity α may be defined as the fractional change in resistivity per unit resistivity per Kelvin.*
- High stability and high accuracy resistors are wire-wound. Wire wound variable resistor can be used in two ways. i) Rheostats ii) Potential divider
- A thermistor is a heat sensitive device usually made of a semiconductor material whose resistance changes very rapidly with change of temperature.
- *Such a device which converts non-electrical energy into electrical energy is called a source of electromotive force (emf).*
- Batteries or cells, Electrical generators, Thermocouples etc, are the sources of emf.
- *The rate at which work is done in an electric circuit is called electric power.*
- Maximum power will be delivered to a load R when the internal resistance of the source of emf equals the load resistance.
- When the circuit has two sources of emf or it has many resistors neither connected in series nor in parallel to solve such complex circuit we use Kirchoff's law. Which states that *the algebraic sum of the currents meeting at a junction in an electrical circuit is zero.* $\sum I = 0$

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. Seven resistances are connected as shown in the figure. The equivalent resistance between A and B is



a) 3 Ω b) 4 Ω

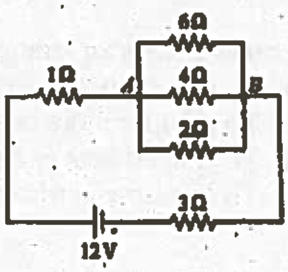
c) 4.5 Ω d) 5 Ω

2. Three resistors of resistance R each are combined in various ways. Which of the following cannot be obtained?

(a) $3\,R\,\Omega$ (b) $\frac{2R}{4}\Omega$

(c) $\frac{R}{3}\Omega$ (d) $\frac{2R}{3}\Omega$

3. Calculate current in 2Ω resistor.



a) 1 A b) 1.29A

c) 0.73A d) 1.43A

4. 10^6 electrons are moving through a wire per second, the current developed is

a) 1.6×10^{-19}A b) 1 A

c) 1.6×10^{-13} A d) 10^6 A

5. When a wire is stretched and its radius becomes $r/2$, then its resistance will be

(a) $16\,R$ (b) $4\,R$

(c) $2R$ (d) 0

6. A wire of uniform cross-section, A length L and resistance R is cut into two equal parts. The resistivity of each part will be

   (a) doubled (b) halved

   (c) remains the same (d) one fourth.

7. The resistivity of two wires is ρ_1 and ρ_2 which are connected in series. If there dimensions are same then the equivalent resistivity of the combination will be

   (a) $(\rho_1+\rho_2)$ (b) $\frac{1}{\rho_1}+\frac{1}{\rho_2}$

   (c) $\frac{\rho_1+\rho_2}{2}$ (d) $\frac{\rho_1}{\rho_2}$

8. The powers of two electric bulbs are 100w and 200 w. Which are connected to power supply of 220 V. The ratio of resistance of their filament will be,

   (a) 1:2 (b) 2:1 (c) 1:3 (d) 4:3

9. Thermocouple is an arrangement of two different metals
   a) To convert heat energy in to electrical energy
   b) To produce more heat
   c) To convert heat energy into chemical energy
   d) To convert electric energy in to heat energy

Comprehensive questions

1. Describe the concept of steady state current as a flow of positive negative or both. Define the unit of current.
2. Explain the electronic current in a metallic wire as due to the drift of free electrons in the wire.

3. State Ohms law. Discuss its scope and validity (a) discuss resistivity and conductivity of a material (b) how does resistance change with temperature?
4. Explain the term: emf, internal resistance and terminal potential difference of a battery.
5. Explain the construction of rheostat. How is it used as a, (a) control the current, (b) as a potential divider?
6. What is wheat stone bridge? Explain with diagram the balancing, the principle and the working of this bridge?
7. What is a potentiometer? Explain its principle, construction, and working. What is the advantage of using potentiometer for measuring potential difference?
8. What is thermocouple? Explain the working of thermocouple by drawing its diagram. By sketching curve explain the variation in thermoelectric e.m.f. with Temperature.
9. Why we use Kirchoff's law for circuit problems solution. Explain its rules for circuit solution by giving proper example.
10. What is EEG. How it is used to measure brain electrical activity through electrical signals?

Conceptual questions

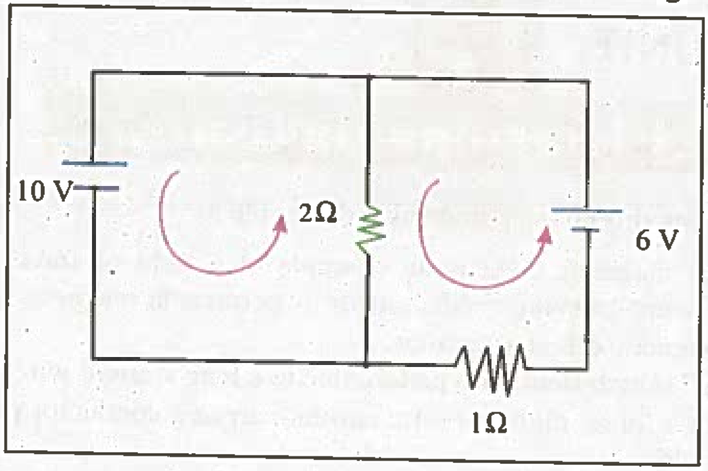
1. Why it is not possible to measure the drift speed for electron by timing their travel along a conductor?
2. The relationship $R = V/I$ tells us that the resistance of a conductor is directly proportional to the potential difference applied to it. What do you think of this proportion?
3. A heavy duty battery of a truck maintains a current of 3A for 24 hour. How much charge flows from the battery during this time?
4. While analysing a circuit the internal resistance of e.m.f. sources are ignored why?
5. Under what circumstances can the terminal P.D. of a battery exceed its e.m.f.?
6. What is the difference between an e.m.f. and a P.D.?

7. The loop rule is based on the conservation of energy principle and the junction rule on conservation of charge principle. Explain just how these are based on these principles?
8. Why rise in temperature of a conductor is accompanied by a rise in the resistance?
9. Does the direction of e.m.f. provided by a battery depend on the direction of current flow through the battery?

Numerical Problems

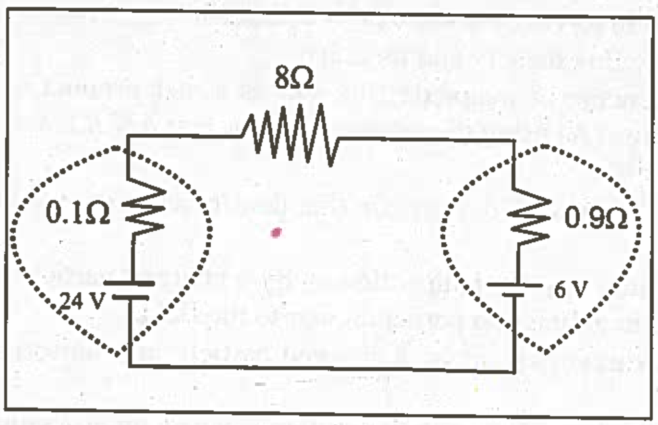
1. A battery has an e.m.f. of 12.8V and supplies a current of 3.2A. What is the resistance of the circuit? How many coulombs leave the battery in 5 minute? (4 Ω, 1200 C)
2. A carbon electrode has a resistance of 0.125 Ω at 20 C°. The temperature co-efficient of carbon is -0.0005 at 20C°. What will be the resistance of the electrode at 85 C°. (0.12 Ω)
3. Calculate the resistance of wire 10 m long that has a diameter of 2mm and resistivity of 2.63×10^{-2} Ω m. (0.0838 Ω)
4. A typical 12V automobile battery has a resistance of 0.012 Ω. What is the terminal voltage of this battery when the starter draws a current of 100A? calculate the R, $P_{\mathcal{E}}$, P_R and P_r.
(0.108 Ω , 1200W, 1080W, 120 W)
5. A 10 watt resister has a value of 120 Ω. What is the rated current through the resister? (0.2886 A)
6. A resistor of 50 Ω has a P. D. of 100V, D.C. across 1 hour. Calculate (a). Power and (b). Energy. ((a) 200W, (b) 0.72 MJ)
7. Calculate the current through a single loop circuit if $\mathcal{E}$ = 120V, R = 1000 Ω and internal resistance r = 0.01 Ω. (120 mA)

9. Find the current flowing through the resistors of the fig:



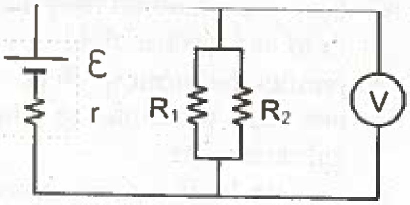
(-4 A, -9A)

10. Find the terminal potential difference of each cell in circuit of figure.



(23.8 V, 7.8 V)

11. The voltmeter in the circuit at the right may be considered to be ideal. Values are, $\mathcal{E}$= 15.0 V; internal resistance, r = 5.00 Ω; R_1= 100 Ω; R_2= 300 Ω. Calculate the current in R_1.



(0.01406A)

UNIT 13

.....Electromagnetism......

After studying this chapter students will be able to:

- explain that magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets.
- describe magnetic effect of current.
- describe and sketch field lines pattern due to a long straight wire.
- explain that a force might act on a current-carrying conductor placed in a magnetic field.
- Investigate the factors affecting the force on a current carrying conductor in a magnetic field.
- solve problems involving the use of $F = BIL \sin \theta$.
- define magnetic flux density and its units.
- describe the concept of magnetic flux (Φ) as scalar product of magnetic field (B) and area (A) using the relation $\Phi_B = B.A = B \perp A$.
- state Ampere's law.
- apply Ampere's law to find magnetic flux density around a wire and inside a solenoid.
- describe quantitatively the path followed by a charged particle shot into a magnetic field in a direction perpendicular to the field.
- explain that a force may act on a charged particle in a uniform magnetic field.
- describe a method to measure the e/m of an electron by applying magnetic field and electric field on a beam of electrons.
- predict the turning effect on a current carrying coil in a magnetic field and use this principle to understand the construction and working of a galvanometer.
- explain how a given galvanometer can be converted into a voltmeter or ammeter of a specified range.
- describe the use of avometer / multimeter (analogue and digital).

A magnet (from Greek "Magnesian stone") is a material that produces a magnetic field. This magnetic field is invisible but is responsible for the most notable property of a magnet: a force that pulls on other ferromagnetic materials, such as iron, and attracts or repels other magnets. A permanent magnet is an object made from a material that is magnetized and creates its own persistent magnetic field. An everyday example is a refrigerator magnet used to hold notes on a refrigerator door.

For your information

Magnetic recording media: VHS (Video Home System) tapes contain a reel of magnetic tape. The information that makes up the video and sound is encoded on the magnetic coating on the tape. Common audio cassettes also rely on magnetic tape. Similarly, in computers, floppy disks and hard disks record data on a thin magnetic coating.

Credit, debit, and ATM cards: All of these cards have a magnetic strip on one side. This strip encodes the information to contact an individual's financial institution and connect with their account(s).

13.1 Magnetic Field

The study of magnetism started with the discovery of the mineral called loadstone, which was found to attract iron and other magnetic materials. Today, much is known of this mineral, also called magnetic iron ore, or iron oxide. From these early discoveries, interest was developed in the study of the properties of magnetism.

By the end of the nineteenth century, scientists had tested all the known elements and compounds for their magnetic properties. As a result, these materials were grouped into categories based on their magnetic behavior. With the discovery of electricity it was soon realized that a steady current through a conducting wire creates a magnetic field around the wire as shown in figure 13.1. The direction of such field is determined by right hand rule.

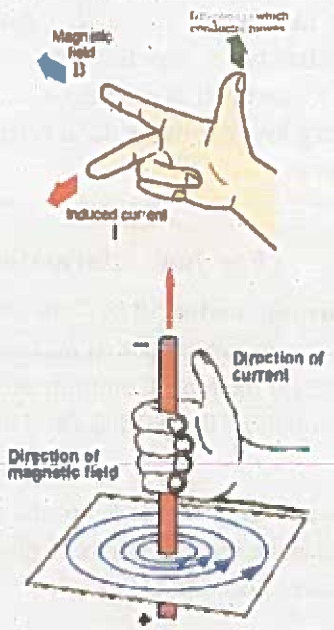


Figure 13.1 : direction of current

Curl of the fingers show the direction of magnetic field while thumb indicates the direction of the current *I*. The direction of the magnetic field can be verified by placing compasses on a card near a current carrying wire and observing their direction. The magnetic field is the region around a magnet or a current carrying wire in which it can attract or repel other magnetic materials. Magnetic field is a vector quantity and represented by a vector **B** called magnetic induction.

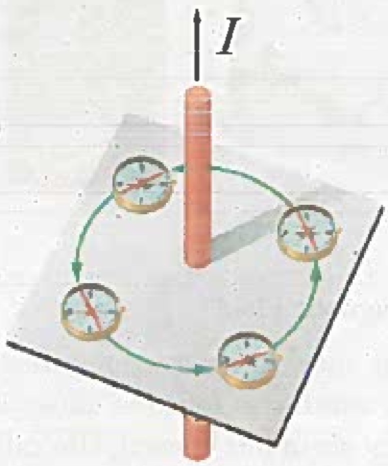


Figure 13.2: Compasses show direction of magnetic field.

13.2 Force on a current carrying conductor

When a current carrying wire is placed at right angles to a uniform magnetic field, the magnetic field of the wire and external uniform field interact, resulting in a force F on the wire. This force depends on the current I in the wire, and the length L of the wire that lies in the field figure 13.3.

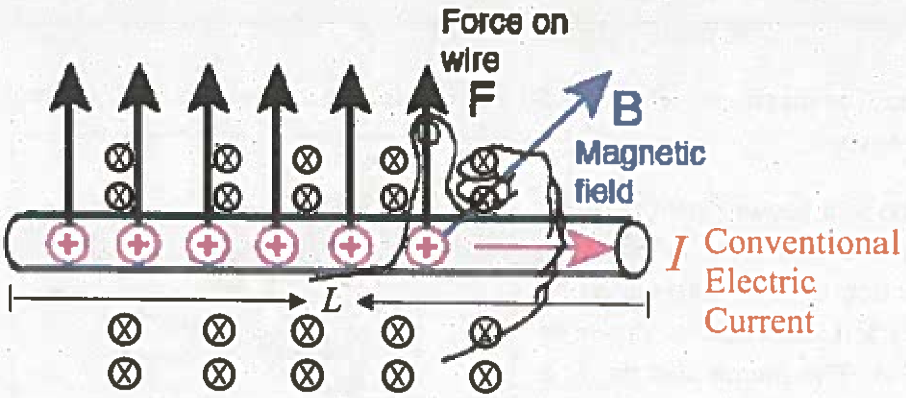


Figure 13.3: conductor in a magnetic field

$$F \propto IL$$

The strength of the uniform field, which is called magnetic induction (or magnetic flux density and will be defined later) B, is the constant of proportionality.

$$F = BIL$$

If the wire is place in the field B at some angle θ with it, then the force is given by the relation

$$F = BIL \sin\theta \qquad (13.1)$$

Or in vector form

$$\mathbf{F} = I\,(\mathbf{L} \times \mathbf{B}) \qquad (13.2)$$

Equation 13.1 shows that the magnitude of the force F on a current carrying wire of length L depends upon the following factors.

B: Magnitude of magnetic induction

I: amount of current flowing in the wire

L: Length of the wire lying in the field

θ: angle between B and L

This force F is maximum when θ is 90 and is minimum when θ is 0, if we keep B, I and L constant.

As force is a vector quantity so its direction should also be determined. The direction of F is determined by Fleming's left hand rule as shown in figure 13.4. The thumb and the first two fingers of the left hand are set at right angles to each other.

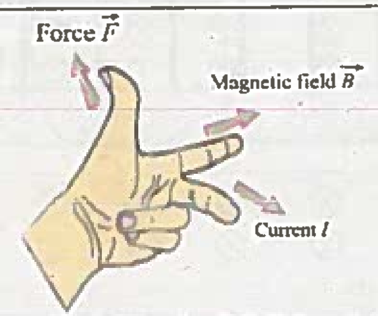


Figure 13.4: Fleming's left hand rule

With the first finger pointing in the direction of the field and second finger pointing in the direction of current, the thumb gives the direction of the force. The S.I. unit for magnetic induction B is tesla (T) and it can be defined as

$$B = \frac{F}{IL}$$

$$1\ \text{T} = 1\ \text{N}\ \text{A}^{-1}\text{m}^{-1}$$

If the force experienced by 1 m of a wire carrying 1 A current placed perpendicularly in magnetic field is one newton, then the magnetic induction is one tesla.

Another unit used for B is Gauss, which is given by

$$1\ \text{G} = 10^{-4}\ \text{T}$$

Or $$1\ \text{T} = 10^{4}\ \text{G}$$

Example 13.1:

A wire carrying 2 A current and has length of 10 cm between the poles of a magnet is kept at an angle of 30^0 to the uniform field of 0.6 T. Find the force acting on the wire?

Solution:

Using equation 13.1

$$F = BIL \sin\theta$$

$$F = 0.6 \text{ T} \times 2 \text{ A} \times 0.1 \text{ m} \times \sin 30^0$$

$$F = 0.06 \text{ N}$$

13.3 Magnetic Flux

Magnetic induction tells us how close together magnetic field lines are, as it tells us the strength of the magnetic field. Now we define another quantity magnetic flux which is the dot product of magnetic induction B and vector area element ΔA.

Magnetic flux denoted by symbol Φ, is given by

$$\Delta\Phi = B \cdot \Delta A \qquad (13.3)$$

$$= B\, \Delta A \cos\theta$$

Where $\Delta\Phi$ represents the magnetic field lines passing through the vector area element ΔA placed perpendicular to the field. Direction of the vector area element ΔA is normal to the surface area.

The total flux through surface area A is given by

$$\Phi = \sum B \cdot \Delta A$$

$$= B \cdot A$$

$$= \text{B A} \cos\theta$$

The flux Φ through the area A will be maximum if the surface is perpendicular to the field, because in this case normal to the surface will be parallel to B as shown in Fig 13.5 (b). Similarly the flux will be zero when normal to the surface become perpendicular to B Fig 13.5 (c).

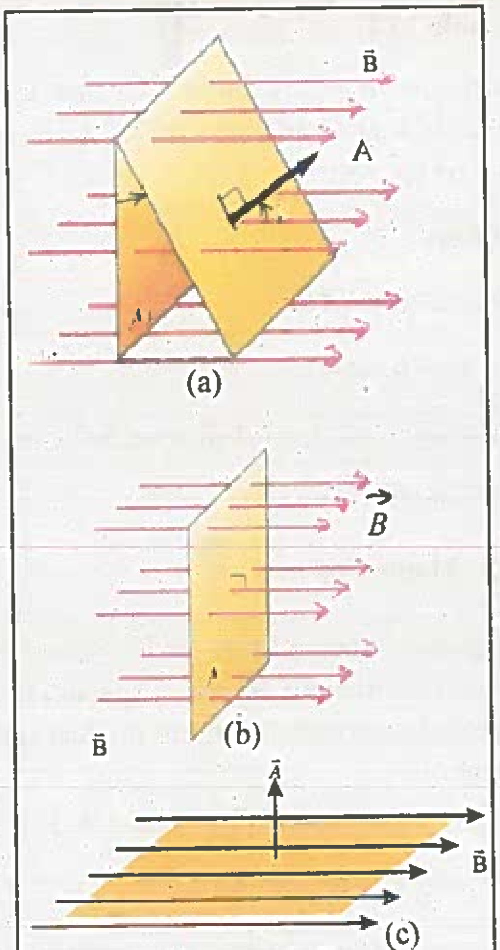


Figure 13.5: magnetic flux

The unit of magnetic flux is weber. One weber is given by

$$1\ Wb = 1\ N\ m\ A^{-1}$$

Magnetic Flux Density

Using equation 13.3 we can see that

$$B = \frac{\Phi}{A}$$

So magnetic induction B can also be defined as *magnetic flux per unit area, and it is called magnetic flux density.* So

$$1\ T = 1\ Wb\ m^{-2}$$

Quiz?

Can you explain how bullet train and a circuit breaker work on the magnetic effect of a current?

13.4 Ampere's Law

Ampere's circuital law, discovered by Andre Marie Ampere in 1826, relates the integrated magnetic field in a loop around a current carrying wire to the current passing through the wire. We know that there is a magnetic field around a current carrying wire. If we consider a closed path around the wire in the

form of a circle having the wire at the center, then the magnitude of magnetic flux density B changes with the current I in the wire and the distance r from the wire. figure 13.6.

So

$$B \propto I$$

$$\& \quad B \propto \frac{1}{r}$$

$$\text{or} \quad B \propto \frac{I}{r}$$

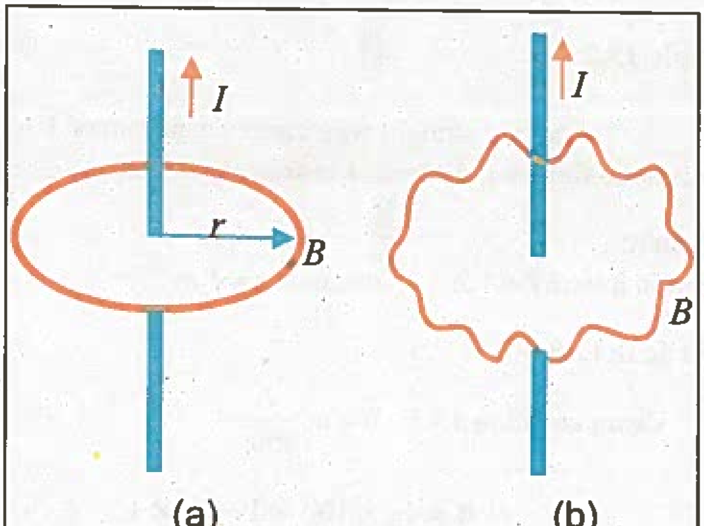


Figure 13.6: Ampere's circuital law

Summing up all around the circular path

$$B = \mu_o \frac{I}{2\pi r}$$

μ_o = Permeability of free space ($4\pi \times 10^{-7}$ Wb A^{-1} m^{-1})

$$B\ 2\pi r = \mu_o I \qquad (13.4)$$

Now consider a closed path around the wire figure 13.8. For any path element ΔL we can write

$$B \cdot \Delta L = \mu_o I$$

As B and ΔL are parallel, so $B \cdot \Delta L = B\Delta L = \mu_o I$

Now summing over the entire closed path,

$$\sum B \cdot \Delta L = \mu_o I, \qquad (13.5)$$

which is Ampere's Law. The closed path is called the amperian path. In general this law can be applied to any closed path around a uniform magnetic field.

Example *13.2*

A long straight wire carries a current of 1 A. Find the magnitude of the magnetic field at a distance 1 m from it.

Solution:

Current in a wire $I = 1$ A distance $r = 1$ m

Magnetic field $B = ?$

Using equation 13.5, $B = \mu_o \dfrac{I}{2\pi r}$

$$B = 4\pi \times 10^{-7} \times 1 / 2\pi \times 1$$

$$B = 2 \times 10^{-7}\ \text{T}$$

Example 13.3

Two long parallel wires 6cm apart carry current of 8 A and 2 A. What is magnitude of magnetic field midway between them?

Solution:

current due to 1st wire $= I_1 = 8$ A

current due to 2nd wire $= I_2 = 2$ A

The midway between wires is $= r_1 = r_2 = 3\text{cm} = 3 \times 10^{-2}\text{m}$

$$B_{net} = B_1 + B_2$$
$$= \mu_o \frac{I_1}{2\pi r_1} - \mu_o \frac{I_2}{2\pi r_2}$$

negative sign is there as fields are opposite in the middle,

$$B = 4\pi \times 10^{-7}\left(\frac{8-2}{2\pi \times 3 \times 10^{-2}}\right) = 4 \times 10^{-5}\,\mathrm{T}$$

$$B = 4 \times 10^{-5}\,\mathrm{T}$$

13.5 Magnetic Field due to a current carrying solenoid

A solenoid is long spring like coil, of length many times its diameter, with many turns every centimeter. A current I in the solenoid produce a magnetic field B along its axis as shown in figure 13.7. The magnetic field of the solenoid is strong along its axis and weaker, rather negligible outside.

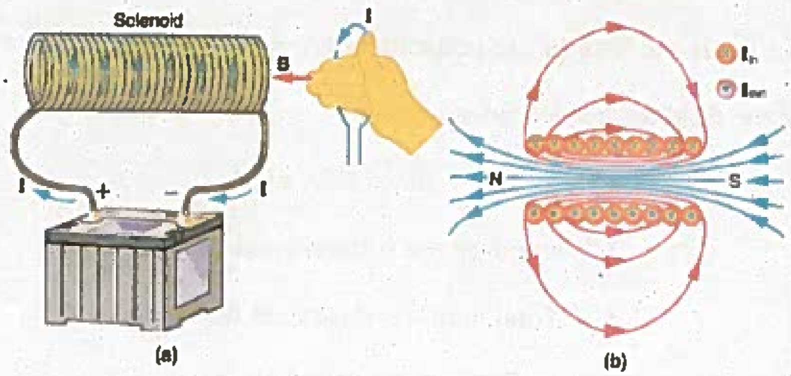


Figure 13.7 : Magnetic Field due to a current carrying solenoid

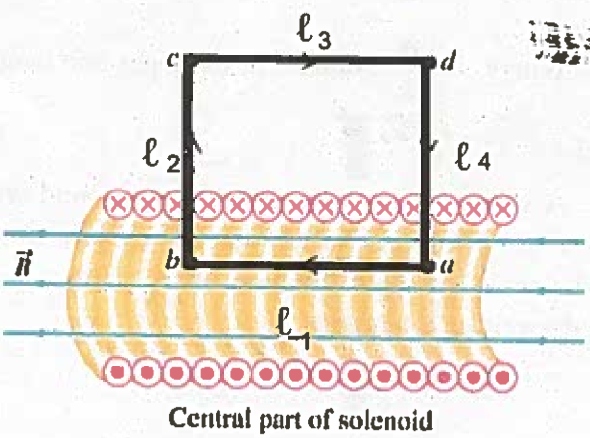


Figure 13.7(c): amperian loop in a solenoid

To determine the value of B of a solenoid let us consider an amperian path abcd with lengths ℓ_1 and ℓ_3 much longer as compare to other two lengths. Now applying Ampere's law

$$\sum B \cdot \Delta L = \mu_o I,$$

$$B \cdot \ell_1 + B \cdot \ell_2 + B \cdot \ell_3 + B \cdot \ell_4 = \mu_o I$$

Now inside the solenoid, B and l_1 are parallel, so $B \cdot \ell_1 = B\,\ell_1$

Outside $B = 0$, so $B \cdot \ell_3 = 0$

For ℓ_2 and ℓ_4, B and lengths are perpendicular, so $B \cdot \ell_2 = 0$ and $B \cdot \ell_4 = 0$

Therefore the field for a solenoid is given by

$$\sum B \cdot \Delta L = B\,\ell_1 = \mu_o I,$$

Now if L = length of the solenoid and

N = Total number of turns in the solenoid, then

$$\sum B \cdot \Delta L = B\,L = N\mu_o I,$$

So $$B = n\,\mu_o I, \qquad (13.6)$$

Where $n = \dfrac{N}{L}$ Number of turns per unit length.

Example 13.4

A solenoid is 10 cm long and is wound with two layers of wire. The inner layer has 50 turns and the outer layer has 40 turns. A current of 3A flows in both layers in the same direction. What is the magnitude of magnetic flux density along the axis of solenoid?

Solution:

Length of a solenoid is = L= 10 cm

Inner layer = n_1= 50 turns
Outer layer = n_2 = 40 turns
Flow of current = I= 3A
magnetic flux density = B = ?

$$B = n_1 \mu_o I_1 + n_2 \mu_o I_2$$
$$= 3.4 \times 10^{-3}\ \text{T}$$

13.5.1 Applications of magnetic field

The magnetic strength of an electromagnet depends on the number of turns of wire around the electromagnet's core, the current through the wire and the size of the iron core. Increasing these factors can result in an electromagnet that is much larger and stronger than a natural magnet. For example, there is no known natural magnet that is able to pick up a large steel object such as a car, but industrial electromagnets are capable of such a task. Also, if the core of the electromagnet is made of soft iron, its magnetic force can be turned off by turning off the electricity to the electromagnet.

Cranes

Strong electromagnets are often used in cranes to move large pieces of iron or steel.

Figure 13.8(a) : Crane uses electromagnet

Electromagnetic lock

An electromagnetic lock be used to lock a door by creating a strong field in an electromagnet that is in contact with a magnetic plate. As long as there is current through the electromagnet, the door remains closed and locked.

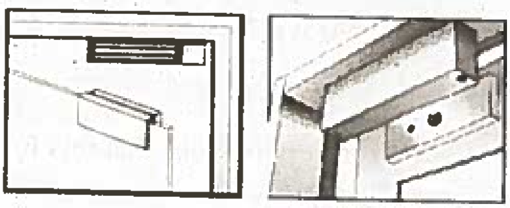


Figure 13.8(b) : electromagnetic door lock

Another type of electromagnetic lock uses an electromagnet to extend a plunger between the doors, making it nearly impossible to open the door until the electromagnet releases the plunger.

Doorbell ringer:

An old-fashioned doorbell used an electromagnet that was rapidly turned on and off to pull a clangor against a bell.

Quiz?

1. Why wheat flour is usually passed near a magnet before being packed?
2. Why a steel ship becomes magnetized as it is constructed?

13.6 Motion of a charged particle in uniform magnetic field

In our study of electrostatics, we saw that a charged particle in an electric field experiences a force in the direction of the field, or against the field, depending on the sign of the charge. Similarly, a charged particle in a magnetic field would experience a force. In case of the magnetic field, however the charge must be moving. The force acting on the charged particle results from the interaction of the external magnetic field and the magnetic field created by the moving charge.

For a positive charge q moving with velocity v in a magnetic field of flux density B, the force acting on the charge is given by the expression

$$\vec{F} = q(\vec{v}\times\vec{B}) \qquad (13.7)$$

$$= q\ v\ B\ \text{Sin}\theta\ \hat{n}$$ where θ is the angle between the velocity and magnetic field.

It is very obvious that this force is maximum when charge particle moves perpendicularly to the magnetic field and minimum when the charge moves parallel to the field.

$$F_{max} = qvB \qquad \theta = 90^0$$

$$F_{min} = 0 \qquad \theta = 0^0$$

The direction of this force is determined by Fleming's left hand rule as shown in figure 13.9. The thumb and the first two fingers of the left hand are set at right angles to each other. With the first finger pointing in the direction of the field and second finger pointing in the direction of velocity of the charged particle, the thumb gives the direction of the force.

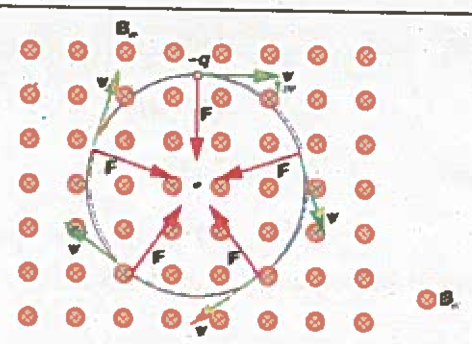


Figure 13.9: a charged particle in uniform magnetic field

Now as the force is always perpendicular to the direction of velocity, so the charged particle follows a circular path as shown in figure 13.10. The symbol ⊗ shows that the magnetic field is acting into the plane of the paper.

Fig 13.10 shows that charge particle move on spiral path when angle θ is between 0 & 90°.

So the centripetal force provided for this motion of a charged particle entering perpendicularly to the magnetic field is given by

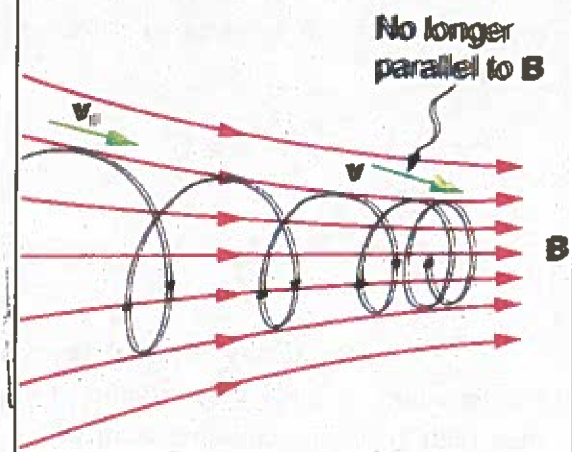


Figure 13.10: charges particle motion on a spiral path

$$\frac{mv^2}{r} = mr\omega^2 = qvB \qquad \therefore \quad F_c = qvB$$

As $v = r\omega$ so,

$$\omega = qB/m \qquad (13.8)$$

and $\quad T = 2\pi/\omega$

$$= 2\pi m/qB \qquad (13.9)$$

And $\quad f = qB/2\pi m$

Where T is the time period of the charged particle and f is the cyclotron frequency in above equations.

Determination of e/m for an electron

The circular motion of an electron shot perpendicularly into a magnetic field can be used to determine its charge to mass ratio. Using the equation

$$\frac{mv^2}{r} = evB$$

$$\frac{e}{m} = \frac{v}{Br} \qquad (13.10)$$

Practically we shot beam of electrons into a magnetic field of known value, so B is known. Radius of the electrons can be measured by making their path visible by colliding them with a gas like hydrogen or helium in a tube place in uniform magnetic field. Electrons excite the atoms of the gas and their de-excitation causes emission of visible blue light. So the path is visible. For the velocity of the electrons their kinetic energy is measured by passing them through a potential difference of known value.

So $\frac{1}{2}mv^2 = eV,$ (13.11)

Using equations 13.10 and 13.11

$$\frac{e}{m} = \frac{2V}{B^2r^2} \quad (13.12)$$

Example 13.5

The path of an electron in a uniform magnetic field of flux density 1.0×10^{-2} T in a vacuum is a circle of radius 1 cm. Calculate the period of its orbit?

Solution:

Magnetic field density= $B = 1.0 \times 10^{-2}$ T
Radius of circle = r = 1 cm
Period of circular orbit = T= ?
Mass of electron= m_e=9.109×10^{-31} kg
Charge on electron = e=1.6×10^{-19}C

Using equation $T = \dfrac{2\pi m}{qB}$

Putting values

$$= \frac{2\times3.14\times9.109 \times 10^{-31}\text{kg}}{1.6\times10^{-19}\times1.0 \times 10^{-2}\text{T}}$$

$$T = 3.57 \times 10^{-9}\text{ s}$$

Velocity Selector:

If a charged particle is passed through a region where both electric and magnetic fields are acting such that two forces may balance each other,

$$F_E = F_M$$

$$q\,E = q\,v\,B$$

$$v = \frac{E}{B}$$

Such arrangement is called velocity selector because charges with velocity in the ratio of E to B will come out un-deflected as shown in figure 13.11.

In a beam all charged particles do not move with same velocity so we can separate charges with desired velocity.

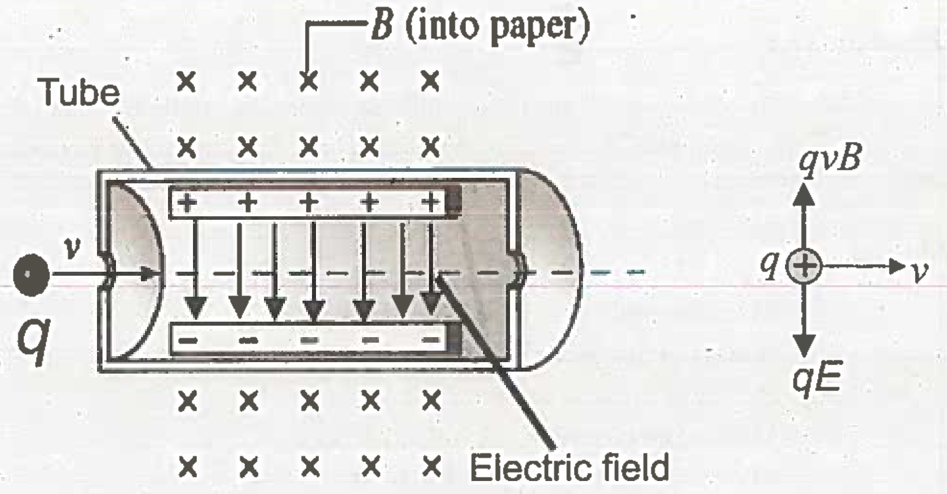


Figure 13.11: charge particle motion in both electric and magnetic fields

13.7 Torque on a current carrying loop / coil

If a current carrying rectangular coil is placed in a uniform magnetic field, it experience force and torque. Let us consider a rectangular coil having N turns carrying current I in a uniform magnetic field of flux density B. Each side PS and QR of the coil experiences a force $\vec{F}$ as shown in figure 13.12. The effect of these forces is to try to compress the coil. Since the coil is rigid, no distortion of the coil occurs. The force F acting on the sides PQ and RS due to the magnetic field are in opposite directions and normal to the magnetic field and the sides PQ and RS.

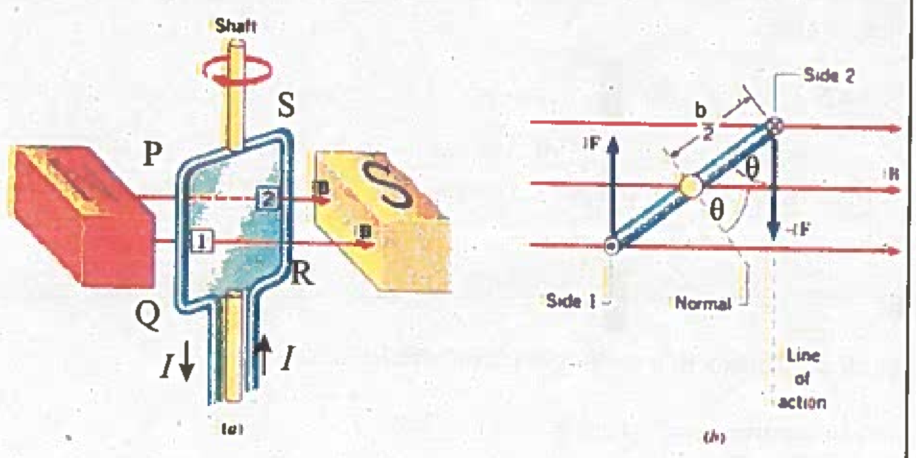


Figure 13.12(a) : a coil in a magnetic field (b): torque on a coil

The magnitude of this force on sides PQ and RS is

$$F_1 = F_2 = F = N\,B\,I\,a \qquad \ldots(13.13)a$$

Where, '*a*' is the length of sides PQ and RS. Fig:13.12(b) shows the directions of the two forces as seen from the top. The effect of this pair of forces is a couple which has a torque, given by

$$\tau = F\,(\,b\cos\theta) \qquad \ldots(13.13)b$$

where θ = angle between the plane of the coil and the magnetic field, and '*b*' is the moment arm of the couple. From equation 13.13, we have

$$\tau = N\,B\,I\,a\,(\,b\cos\theta)$$

$$= NBI(ab)\cos\theta$$

$$= NBIA\cos\theta, \qquad \ldots(13.14)$$

where *A*= area of the coil. The maximum torque is *BINA* it occurs when the angle θ = 0, that is when the plane of the coil is parallel to the magnetic field or normal to the plane is perpendicular to the field as shown in Fig:13.12. When the plane of the coil is perpendicular to the magnetic field, or normal to it is parallel to the field then torque is zero.

coil is perpendicular to the magnetic field, or normal to it is parallel to the field then torque is zero.

Example 13.6

A rectangular coil of 100 turns and area 200 cm^2 carrying 2A current is place in a uniform magnetic field of 2 T. Calculate the maximum torque on the coil.

Solution:

Number of turns in a rectangular coil= N = 100 turns

Area of a coil= $A = 200\ cm^2 = 200\times10^{-4}\ m^2$.

Current = I = 2A

Torque on the coil= τ =?

For maximum torque, $\theta = 0$

Using equation, $\tau = NBIA\cos\theta$.

$$\tau = 100\times2\times2\times200\times10^{-4} = 8\ Nm$$

MRI

Magnetic Resonance Imaging (MRI) is one of the most advanced diagnostic tools available. MRI uses a combination of a strong magnetic field and radio waves to produce detailed high resolution images of the inside of the body.

Principle:

Magnetic Resonance Imaging (MRI) systems are able to generate high-quality diagnostic images through the use of magnetic field. The human body is composed primarily of fat and water, it is made up mostly of hydrogen atoms. By applying short radio frequency (RF) pulses to a specific anatomical slice, the protons in the slice absorb energy at this resonant frequency, causing them to spin perpendicular to the magnetic field.

As the protons relax back into alignment with the magnetic field, a signal is received by an RF coil. This signal acts as an antenna, and is processed by a computer to produce diagnostic images of areas of the body. The brain consists of two distinct regions: white matter, which is composed of myelinated nerve fibres, and grey matter, which consists of nerve cell bodies.

These two regions interact to perform critical information processing and damage to either region causes brain dysfunction. Notably, the white matter is referred to as being 'white' due to the light colour of the myelin insulation covering the nerve fibres. Now, magnetic resonance imaging (MRI) is used widely to study brain function, where damage to the white matter is seen as bright areas.

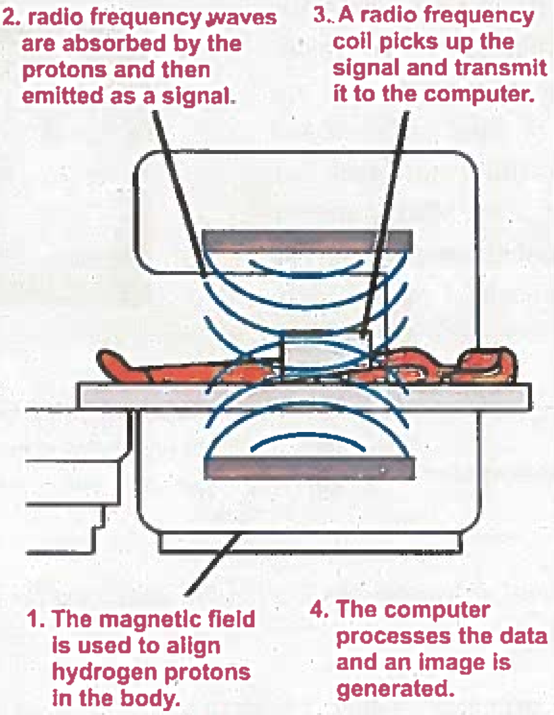


Figure 13.13(a).MRI Scanning process

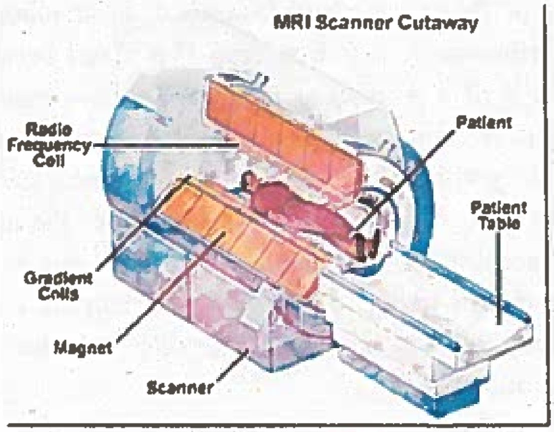


Figure 13.13(b).

MRI is also capable of imaging such as the movement of the wall of the heart and the injection of fluid into a blood vessel in order to reach an organ or tissues. MRI is able to create detailed images to assist in the diagnosis of cancer, heart disease.

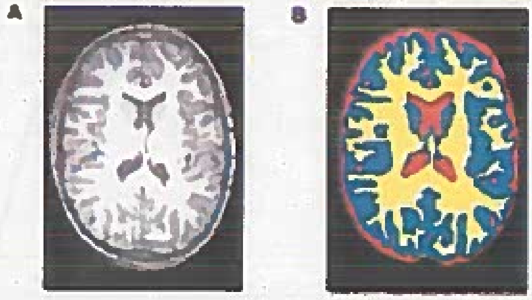


Figure 13.13(c). Example of an MRI image of the brain, showing gray matter (blue), white matter (yellow), and cerebral spinal fluid (red).

13.8 Galvanometer

A moving coil galvanometer is an instrument used for detection and measurement of small electric currents.

Its principle is that a current carrying loop placed in a magnetic field experiences a torque. A simplified version of a galvanometer is shown schematically in figure 13.14. Most modern galvanometers are of the moving-coil type and are called d'Arsonval galvanometers. It mainly consists of a rectangular coil ABCD of fine wire wrapped around an aluminum frame is suspended by conducting ribbons. A soft iron core F is fixed between cylindrically concave poles N and S of a permanent magnet. The suspension wire T_1 is used as one current lead to the coil and the other terminal of the coil is connected to a loosely wound spiral T_2 which serves as the second current lead. A cylinder of soft iron is placed at the centre of the coil which intensifies the magnetic field and makes it radial by concentrating the magnetic field lines due to its high permeability (and gives more inertia to the coil). When a current I flows through the coil, a magnetic field B is set up which interacts with that of the permanent magnet producing a torque τ.

$$=NIAB\cos\alpha$$

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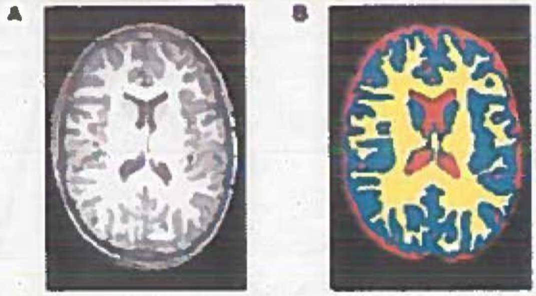

Figure 13.13(c). Example of an MRI image of the brain, showing gray matter (blue), white matter (yellow), and cerebral spinal fluid (red).

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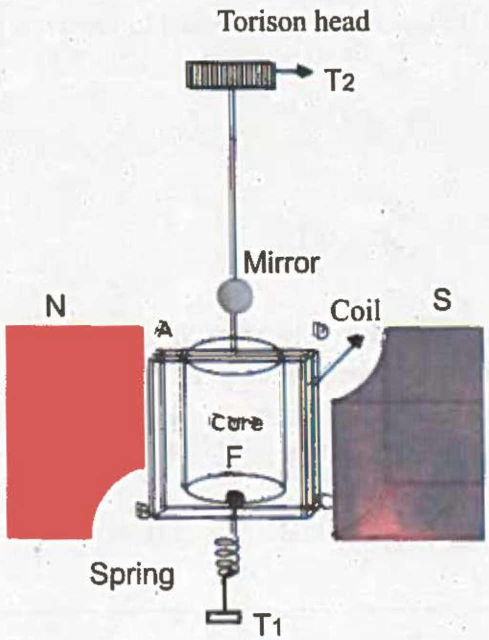
$$=NIAB \cos \alpha$$

In this expression,

N = number of turns in the coil, A = area per turn of the coil,

B = magnetic induction of the radial magnetic field,

α= angle between the plane of coil and the direction of $\vec{B}$.



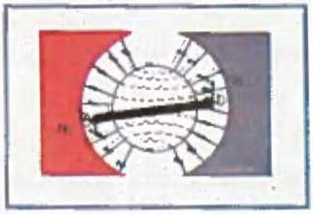


Figure 13.14 : (a) galvanometer- 13.14 (b) radial field

Since the magnetic field is radial the plane of the coil is parallel to the magnetic field B, so

$$\tau = NBIA \qquad ...(13.14)$$

The torque rotates the coil and twists the suspension ribbon, until it is fully resisted by the suspension. As a result a restoring torque comes into play trying to restore the coil back to original position.

If θ be the twist produced in the strip and C be the restoring torque per unit twist then:

$$\text{Restoring torque} = \tau = C\,\theta. \qquad ...(13.15)$$

When the coil is in equilibrium then the deflecting torque is equal to the restoring torque. Then from Eq: (13.14) & (13.15)

$$NBIA = C\,\theta$$

$$\text{Or} \quad I = \frac{C\theta}{NAB} \qquad ...(13.16)$$

From Eq: 13.16 it is clear that galvanometer may be made more sensitive by making deflecting angle θ large for a certain small value of current I. It may be achieved by making C/NAB small. A sensitive galvanometer is one which produces large deflection for small value of current. Where, C/NAB is a constant.

The angular displacement θ produced being proportional to the current I.

$$I \propto \theta$$

The result is read from a scale onto which a light beam is reflected from a mirror M carried on the suspension ribbons. There are two methods of observing the angle of deflection of the coil.

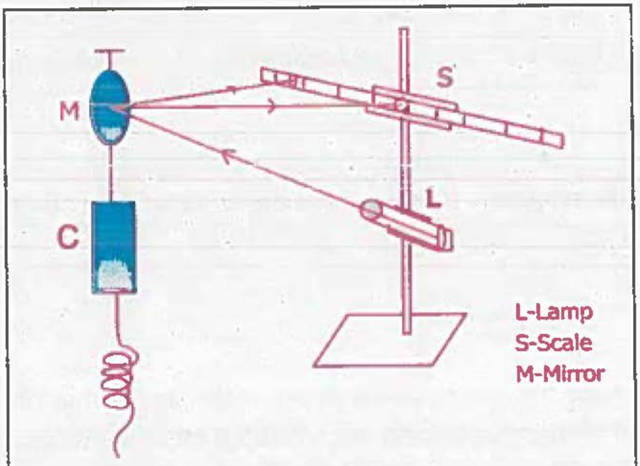


Figure 13.15 : Lamp and scale arrangement for measuring the angle of deflection.

1) Lamp Scale method:

In sensitive galvanometer the large angle is observed by means of small mirror attached with coil along with lamp and scale arrangement. A beam of light from the lamp is directed towards the mirror of the galvanometer. After reflection from the mirror it produces a spot on a translucent scale placed at a distance of one meter from the galvanometer. As the coil along with the mirror rotates the spot of light moves along the scale.

The displacement of the spot of light on the scale is proportional to the angle of deflection (provided the angle of deflection is small).

2) Pivoted coil galvanometer:

In second type of observing angle θ is used in the pivoted coil galvanometer.

In less sensitive galvanometer the coil is pivoted between two jeweled bearings. The restoring torque is provided by two hair springs which also serves as current lead.

A light aluminum pointer is attached to the coil which moves over the scale thus giving the angle of deflection of the coil.

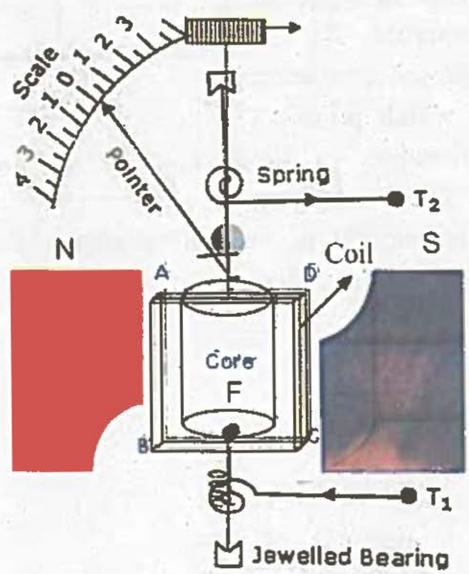


Figure 13.16 : pivoted type moving coil galvanometer.

13.9 Conversion of Galvanometer into Ammeter

Galvanometer very sensitive instrument, a large current cannot be passed through it, as it may damage the coil. The conversion of a galvanometer into an ammeter is done by connecting a low resistance in parallel with it. As a result, when large current flows in a circuit, only a small fraction of the current passes through the galvanometer and the remaining larger portion of the current passes through the low resistance. The low resistance connected in parallel with the galvanometer is called shunt resistance. The scale is marked in ampere.

An ammeter is a measuring instrument used to measure the electric current in a circuit.

The value of shunt resistance depends on the fraction of the total current required to be passes through the galvanometer. Let R_s represent the shunt resistance, R_g the resistance of the galvanometer, I_g the current which produces full scale deflection in the galvanometer. Since the shunt is connected in parallel to the galvanometer, the potential difference across galvanometer = potential difference across shunt.

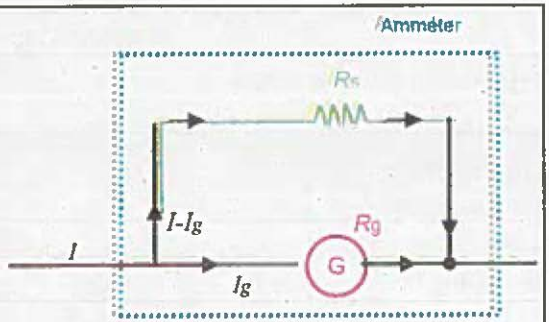


Figure 13.17 : ammeter circuit

$$V = R_g I_g$$

And $$V = R_s(I - I_g) \quad \ldots(13.17)$$

Therefore, $$R_s(I - I_g) = R_g I_g$$

$$R_s = \frac{R_g I_g}{(I - I_g)} \quad \ldots(13.18)$$

An ideal ammeter has a zero resistance.

13.10 Conversion of Galvanometer to Voltmeter

A Galvanometer can be converted into a voltmeter by connecting a high resistance in series with a galvanometer as shown in fig: 13.18.

The value of this resistance depends upon the range of the voltmeter. In series connection the current through the galvanometer is same as that due to the resistance.

Suppose a galvanometer has resistance "R_g" and current "I_g" is passing through it of potential "V_g" across it. And the high resistance also draws same current "I_g", and potential "V_h" across resistor "R_h".

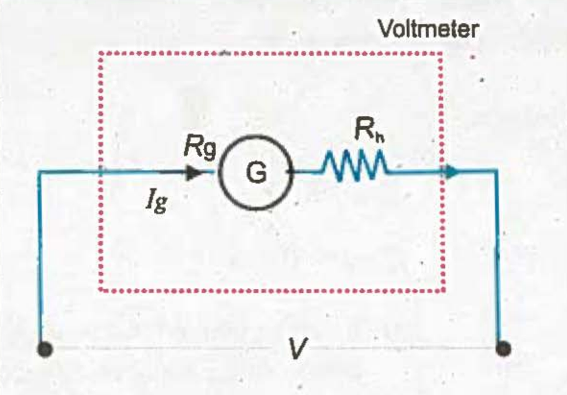


Figure 13.18: voltmeter circuit

The desired potential "V" is to be determined.

Hence, $$V = V_g + V_h$$

$$V = R_g I_g + R_h I_g$$

$$V = I_g (R_g + R_h)$$

$$\frac{V}{I_g} = (R_g + R_h)$$

$$R_h = \frac{V}{I_g} - R_g \qquad \ldots(13.19)$$

This works as a voltmeter of range 0 to V volt. Since the value of R_h is high, the effective resistance also has a higher value. Voltmeters have a high resistance. The reason is that the voltmeter must not draw current from the

circuit otherwise; the P.D. which the voltmeter is required to measure will change. An ideal voltmeter has an infinite resistance.

Example 13.7

A galvanometer has a resistance of 100 ohms and gives full scale deflection on 1 mA current. How it can be converted into a) an ammeter of range 10 A, b) voltmeter of range 10 V

Solution:

$R_g = 100\ \Omega$

$I_g = 1 \times 10^{-3}$ A

a) it can be converted into an ammeter of range I =10A by connecting a shunt resistance in parallel

$$R_s = \frac{R_g\ I_g}{(I - I_g)}$$

$R_S = 0.010\ \Omega$

b) it can be converted into a voltmeter of range V = 10 V by connecting a high resistance R in series

$$R_h = \frac{V}{I_g} - R_g$$

$R = 9900\ \Omega$

13.11 AVOMETER-MULTIMETER

It is an instrument to measure current, voltage and resistance. So it is Amperemeter, Voltmeter and Ohmmeter (AVO). It can measure direct as well as alternating current and voltage. It is a galvanometer having a series of combination of resistors, all enclosed in a box. It has different scales graduated in

such a manner that all the three quantities can be measured. It has its own battery for its function and for operating the electrical circuits.

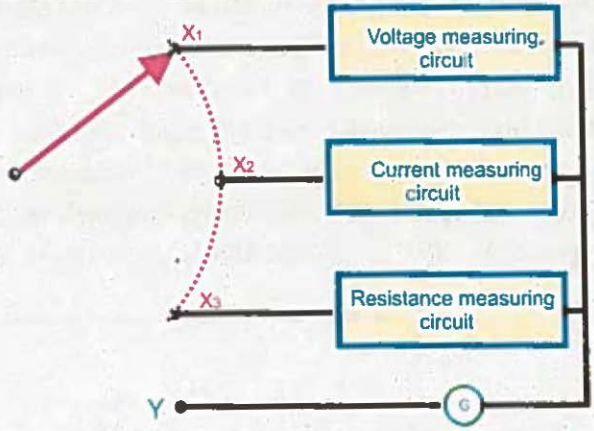


Figure 13.19 : AVO meter circuit

The quantity to be measured and its range can be selected by a selector switch which connects the particular electrical circuit to the galvanometer.

Current Measurement

For the measurement of current the selector switch is turned to X_2. The proper scale is selected. This circuit is a series combination of shunt resistances R_1, R_2, and R_3 is called Universal shunt as shown in Fig:(13.20). Any one of the shunts can be used for measurement of current in different ranges. This circuit provides a safe method of switching between current ranges without any danger of excessive current through the meter.

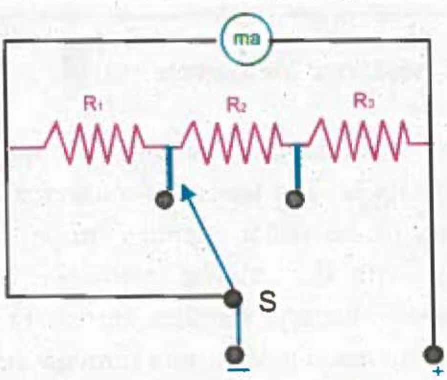


Figure 13.20 : AVO meter switch turned to current measurement

Voltage Measurement

For measuring voltage the voltage selector switch of the AVO meter changes the circuit to the type shown in Fig:(13.21). This circuit allows selecting any range and the corresponding high resistance to be connected in series with the galvanometer. The added high resistance converts the galvanometer to a voltmeter of specific range. For example, connections at A and C, selected by the multiple switch arrangement gives a range of 10-V. Similarly, connections at A and at B, D, E or F give the ranges 2.5V, 50V, 250V and 1000V respectively, Fig.(13.21).

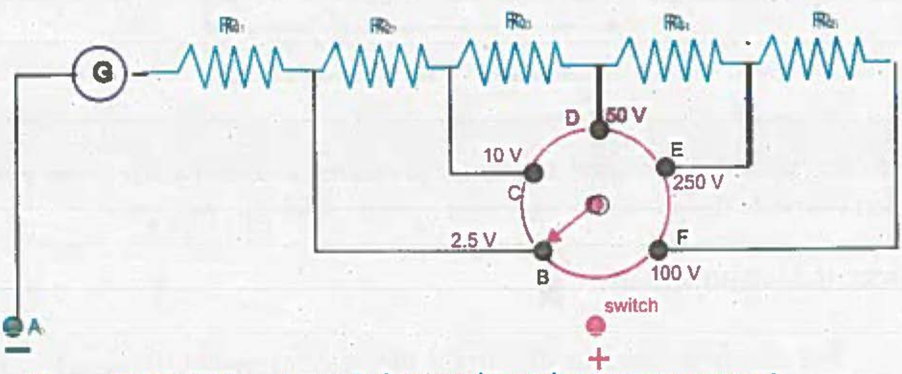


Figure 13.21: AVO meter switch turned to voltage measurement

Resistance Measurement:

For resistance measurement the selector switch uses the circuit as shown in fig: 13.22. The leads are connected across the resistance to be measured. The battery of the meter supplies current to the meter for deflection which in turn varies with the external resistance and can be calibrated. In this case the ohmmeter battery, supplies current to the galvanometer for deflecting its coil. Since the amount of current through the galvanometer depends upon the external resistance, we can calibrate the scale in ohms. The amount of deflection on the ohms scale indicates directly the magnitude of the resistance.

The ohmmeter reads up-scale regardless of the polarity of the leads because the polarity of the internal battery determines the direction of the current through the galvanometer. Commercial AVO meters provide resistance measurements from less than one ohm up to many megaohms.

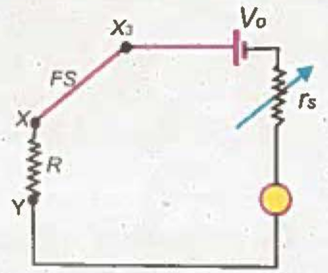


Figure 13.22 : AVO meter switch turned to resistance measurement

Digital Multimeters

Modern multimeters are often digital due to their accuracy, durability and extra features. In a digital multimeter the signal under test is converted to a voltage and an amplifier with electronically controlled gain preconditions the signal. A digital multimeter displays the quantity measured as a number, which eliminates parallax errors.

Modern digital multimeters may have an embedded computer, which provides a variety of convenience features.

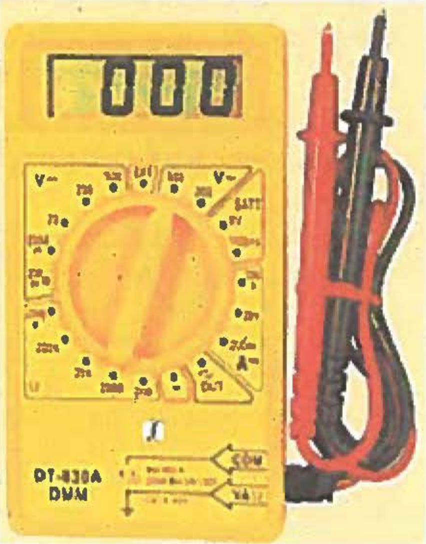


Figure 13.23 : digital Multimeters

KEY POINTS

- A Current carrying wire has a magnetic field around it.
- The force on the current carrying wire in a uniform magnetic field is given by $\boldsymbol{F} = \boldsymbol{I}\,\boldsymbol{L} \times \boldsymbol{B}$
- The magnetic flux is defined as $\Phi = B \cdot \Delta A$
- Ampere's law relates the integrated magnetic field in a loop around a current carrying wire to the current passing through the wire.

  $$B \cdot \Delta L = \mu_0 I$$
- The magnetic field of a solenoid is given by $B = n\,\mu_0 I$
- The force on a charged particle shot at right angles to the magnetic field is $F = qvB$
- Torque on a current carrying coil is $\tau = NIAB \cos\theta$
- The shunt resistance for an ammeter is

  $$R_s = \frac{R_g\, I_g}{(I - I_g)}$$
- The high resistance for a voltmeter is

  $$R_h = \frac{V}{I_g} - R_g$$

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1 A moving charge is surrounded by
a) 2 fields
b) 3 fields
c) 4 fields
d) None of these

2 A photon while passing through a magnetic field are deflected towards
a) North pole
b) South pole
c) Are ionized
d) None of these

3 Magnetism is related to
a) Stationary charges
b) Moving charges
c) Stationary and moving charge
d) Law of motion

4 when charge particle enter perpendicular to magnetic field, the path followed by it is
a) A helix
b) A circle
c) Straight line
d) Ellipse

5 The torque in the coil can be increased by increasing
a) No, of turns
b) Current and magnetic field
c) Area of coil
d) All of above

6 The magnetic flux will be max, for an angle of

a) 0^o

b) 60^o

c) 90^o

d) 180^o

7 The Weber is unit of measure of

a) Conductance

b) Electric current

c) Magnetic flux

d) Electric flux

8 One weber is equal to

a) $N.A^2/m$

b) $N.m^2/A$

c) N.A/m

d) N.m/A

9 An electron moves at 2×10^2m/sec perpendicular to magnetic field of 2T what is the magnitude of magnetic force

a) $1 \times 10^{-6}N$

b) $6.4 \times 10^{-17}N$

c) $3.6 \times 10^{-24}N$

d) 4×10^6N

10 The force on a charge particle moving parallel to magnetic field is

a) Maximum

b) Minimum

c) Zero

d) None of these

11 Ampere's law is applicable to
a) Circular path
b) Rectangular path
c) To any closed path
d) None of these

12 The unit of permiability of free space is
a) T.m/A
b) $T.m^2/A$
c) $T.m/A^2$
d) None of these

CONCEPTUAL QUESTIONS

1. What is the force that a conductor of length L carrying a current I, experiences when placed in a magnetic field B? What is the direction of this force?
2. What is the nature of force between two parallel current carrying wires (in same direction)?
3. What is the magnitude of the force on a charge q moving with a velocity **v** in a magnetic field **B**?
4. In a uniform magnetic field **B**, an electron beam enters with velocity **v**. Write the expression for the force experienced by the electrons.
5. What will be the path of a charged particle moving in a uniform magnetic field at any arbitrary angle with the field?
6. An electron does not suffer any deflection while passing through a region. Are you sure that there is no magnetic field?
7. An electron beam passes through a region of crossed electric and magnetic fields of intensity E and B respectively. For what value of the electron speed the beam will remain un-deflected?
8. Uniform electric and magnetic fields are produced pointing in the same direction. An electron is projected in the direction of the fields. What will be the effect on the kinetic energy of the electron due to the two fields?
9. What is the cyclotron frequency of a charged particle of mass m, charge q moving in a magnetic field B?
10. Can neutrons be accelerated in a cyclotron? Give reason.

11. A current carrying loop, free to turn, is placed in a uniform magnetic field **B**. What will be its orientation relative to **B**, in the equilibrium state?
12. How does a current carrying coil behave like a bar magnet?

Comprehensive Questions

1. Derive an expression for the force acting on a current carrying conductor in a uniform magnetic field.
2. A current carrying loop is placed in a uniform magnetic field. Derive an equation for the torque acting on it?
3. Does a moving charge experiences a force in magnetic field? Explain.
4. How *e/m* ratio for electron is determined using magnetic field?
5. Define and explain magnetic flux?
6. State Ampere's law and use it to derive an expression for the magnetic field of a solenoid?
7. What is galvanometer? How it is converted into an ammeter and a voltmeter?

Numerical Problems

1. At what distance from a long straight wire carrying a current of 10 A is the magnetic field is equal to the earth's magnetic field of 5×10^{-5} T?

   [0.04 m]
2. A long solenoid having 1000 turns uniformly distributed over a length of 0.5 m produces a magnetic field of 2.5×10^{-3} T at the center. Find the current in the solenoid? [1 A]
3. A proton moving at right angles to a magnetic field of 0.1 T experiences a force of 2×10^{-12} N. What is the speed of the proton?

   [1.3×10^{8} m/s]
4. An 8 MeV proton enters perpendicularly into a uniform magnetic field of 2.5 T. Find (a) the force on the electron (b) what will be the radius of the path of proton?

   ((a) 1.6×10^{-11} N (b) 0.17 m)

5. A wire carrying current 10 mA experiences a force of 2 N in a uniform magnetic field. What is the force on it when current rises to 30 mA?

[6 N]

6. What is the time period of an electron projected into a uniform magnetic field of 10 mT and moves in a circle of radius 6 cm?

[3.6 ns]

7. A 0.2 m wire is bent into a circular shape and is placed in uniform magnetic field of 2 T, .If the current in the wire is 20 mA then find the maximum torque acting on the loop?

[1.27×10^{-5} Nm]

8. The full scale deflection for a galvanometer is 10 mA. Its resistance is 100 ohms. How can it be converted into an ammeter of range 100 A?

[$R_S = 0.01\,\Omega$]

9. How a 5 mA, 100 ohms galvanometer is converted into 20 V voltmeter ?

[$R = 3900\ \Omega$]

10. Two parallel wire 10 cm apart carry currents in opposite directions of 8 A. What is the magnetic field halfway between them?

(6.4×10^{-5}T)

11.

UNIT 14

.....Electromagnetic induction......

After studying this chapter students will be able to:

- describe the production of electricity by magnetism.
- explain that induced emf's can be generated in two ways.
  - (i) by relative movement (the generator effect).
  - (ii) by changing a magnetic field (the transformer effect).
- infer the factors affecting the magnitude of the induced emf.
- state Faraday's law of electromagnetic induction.
- account for Lenz's law to predict the direction of an induced current and relate to the principle of conservation of energy.
- apply Faraday's law of electromagnetic induction and Lenz's law to solve problems.
- explain the production of eddy currents and identify their magnetic and heating effects.
- explain the need for laminated iron cores in electric motors, generators and transformers.
- explain what is meant by motional emf. Given a rod or wire moving through a magnetic field in a simple way, compute the potential difference across its ends.
- define mutual inductance (M) and self-inductance (L), and their unit henry.
- describe the main components of an A.C generator and explain how it works.
- describe the main features of an A.C electric motor and the role of each feature.
- explain the production of back emf in electric motors.
- describe the construction of a transformer and explain how it works.
- identify the relationship between the ratio of the number of turns in the primary and secondary coils and the ratio of primary to secondary voltages.

- describe how step-up and step-down transformers can be used to ensure efficient transfer of electricity along cables.

Electricity and magnetism are two aspects of a single electromagnetic force. Moving electric charges produce magnetic forces and moving magnets produce electric forces. The interplay of electric and magnetic forces is the basis for electric motors, generators, and many other modern technologies.

The essential feature of an electric motor is the supply of electrical energy to a coil in a magnetic field causing it to rotate. The generation of electrical power requires relative motion between a magnetic field and a conductor. In a generator, mechanical energy is converted into electrical energy while the opposite occurs in an electric motor. The electric motor is a simple device in principle. It converts electric energy into mechanical energy. In this chapter we will discuss these basic motor principles.

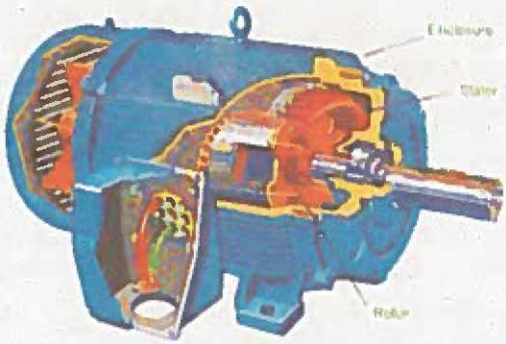


Figure 14.1: electric motor is the heart of domestic appliances such as vacuum cleaners, washing machines, electric trains, lifts, and cars etc.,

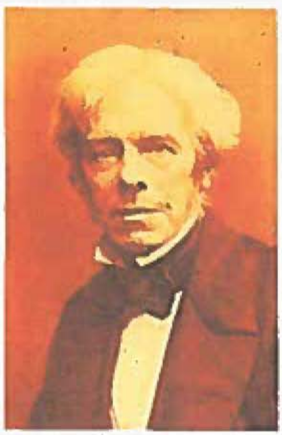


Fig: 14.2: Michael Faraday

In 1831, English scientist Michael Faraday discovered that when the magnetic flux linking a conductor changes, an e.m.f is induced in the conductor.

This phenomenon is known as electromagnetic induction. In this chapter, we will study the various aspects of electromagnetic induction.

14.1 ELECTROMAGNETIC INDUCTION

When the magnetic flux linking a conductor changes, an e.m.f is induced in the conductor this phenomenon is known as electromagnetic induction. The basic requirement for electromagnetic induction is the change in flux linking the conductor (or coil).

Activity :

In order to demonstrate the phenomenon of electromagnetic induction, consider a coil C of wire connected to a galvanometer as shown in Fig14.3(a). Near the coil is a stationary bar magnet. When the magnet is stationary the galvanometer shows no current so there is no source of e.m.f. in the circuit.

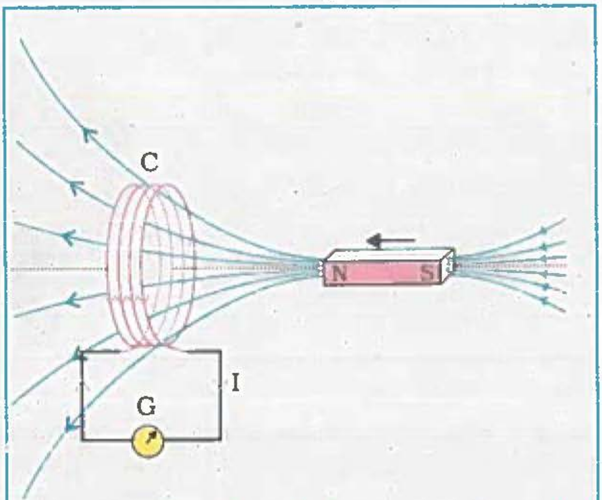


Figure 14.3(a): When the bar magnet is pushed towards the coil, the pointer in the galvanometer G deflects.

Now if we move the magnet towards the coil C, the galvanometer will show deflection in one direction. If we move the magnet away from the coil, the galvanometer again shows deflection but in the opposite direction. In either case, the deflection will persist so long as the magnet is in motion. The production of e.m.f. and hence current in the coil *C* is due to the fact that when the magnet is in motion (towards or away from the coil), the amount of flux linking the coil changes-the basic requirement for inducing e.m.f. in the coil. If the movement of the magnet is stopped, though the flux is linking the coil there is no change in flux so no e.m.f is induced in the coil therefor galvanometer shows zero deflection.

The product of number of turns (N) of the coil and the magnetic flux (Φ) linking the coil is called flux linkages i.e.

$$\text{Flux linkages} = N\,\Phi$$

The phenomenon of electromagnetic induction may be demonstrated in another way as when the current carrying primary coil is moved away or towards

the stationary coil. The current in the primary coil set up a magnetic field that links the stationary coil. Again it is the relative motion of the primary coil which causes induced emf in the coil, and the voltmeter shows deflection.

The e.m.f and current will persist in the coil as long as flux through the coil is changing.

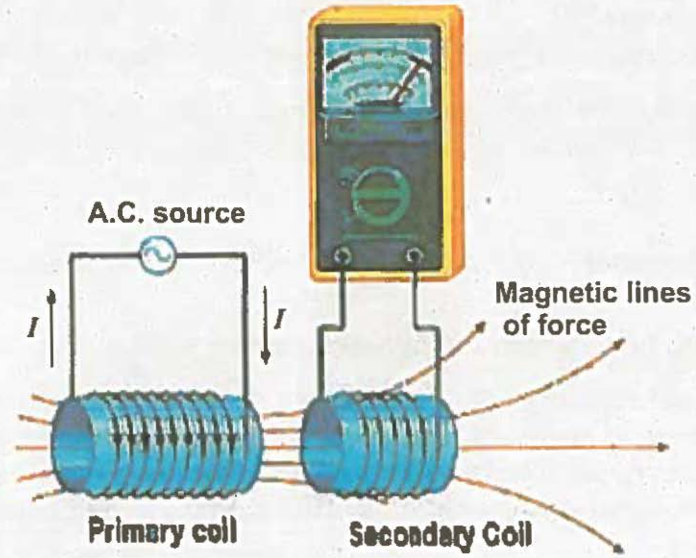


Figure 14.3(b): electromagnetic induction

14.2 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

As the magnitude of e.m.f. induced in a coil is directly proportional to the rate of change of flux linkages. If N is the number of turns of the coil and the magnetic flux linking the coil changes from Φ_1 to Φ_2 in t seconds, then,

Induced e.m.f., $\mathcal{E} \propto$ rate of change of flux linkages

Induced emf , $\mathcal{E} \propto N\dfrac{\Delta\Phi}{\Delta t}$

$$\mathcal{E} = k\,N\frac{\Delta\Phi}{\Delta t}$$

Where k is constant and its value is $k=1$

$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t}$$

Above equation is called Faraday law of induction.

Faraday proposed two laws of electromagnetic induction:

(1) a changing magnetic field induces an electromagnetic force in a conductor;

(2) the electromagnetic force is proportional to the rate of change of the field

The induced emf always opposes the change in flux. The direction of induced emf is given by Lenz's law.

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

The negative sign in the equation represents Lenz's law.

Faradays; law application in Seismometer

An earthquake is a shaking of the earth's surface, known as the crust. The earth's crust is made of huge rock plates, which can shift to cause an earthquake. Most earthquakes happen when two rock plates meet, creating friction. The force is so strong it will send shockwaves through the ground, creating an earthquake.

There are millions of earthquakes every day! Most earthquakes are very small, so no one can feel it. Earthquakes can happen anywhere: land, mountains and oceans. To measure the earthquakes we use a device which is called seismometer.

It's used by seismologists in order to detect earthquakes. One kind of seismometer is called inertial because it is based on Newton's 1st Law. It consists of spring mass system which records the vibrations in the earth's surface and will pick up even the slightest vibration.

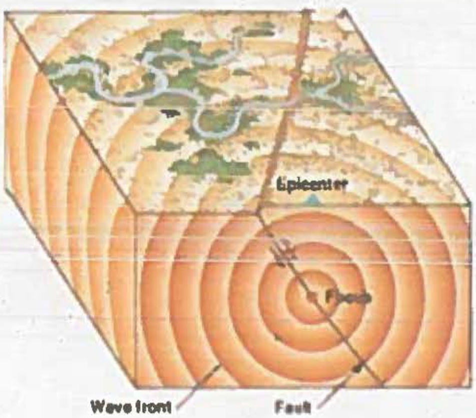


Figure 14.4(a): The focus of an earthquake is the location where rupture begins and energy is released. The place on the surface vertically above the focus is the epicenter. Seismic wave fronts move out in all directions from their source, the focus of an earth.

This is recorded on a sheet of paper under the seismograph needle that writes it. When there are vibrations, the needle sways, causing bigger lines to be drawn. As shown in figure.

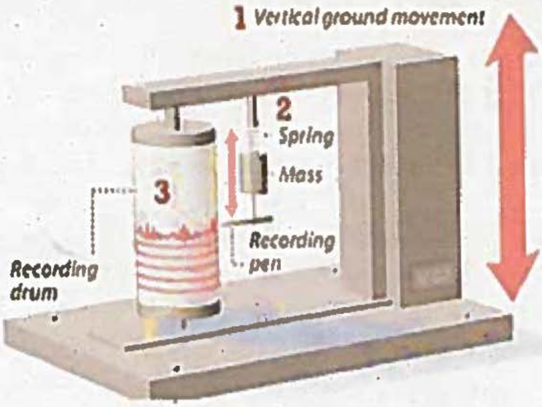


Figure 14.4(b): A vertical-motion seismograph. This seismograph operates on the same principle as a horizontal-motion instrument and records vertical ground movement.

The other kind of seismometer works on the principle of electromagnetic induction. It transforms received vibration energy into an electrical voltage. The relative motion between a magnet and a coil (one of which is attached to the inertial mass and one is attached to the frame) induces an emf in the coil that is proportional to the velocity of the relative motion. The magnitude of emf is also proportional to the strength of the magnet used and the number of turns in the coil.

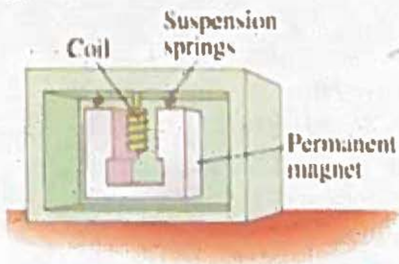


Figure 14.4(c) : electromagnetic seismometer has a coil fixed to the case and moves with the earth. The magnet suspended by spring has inertia and does not move instantaneously with the coil (and case), so there is relative motion between magnet and coil.

In practice either the magnet or the coil can be attached to the inertial mass (in commercial systems the magnet is itself often used as the inertial mass).

14.3 Lenz's Law:

Lenz's law is a convenient alternative method for determining the direction of an induced e.m.f or current. It always gives the same results as the sign rules we have introduced for $\mathcal{E}$ and $\Delta\Phi/\Delta t$ in connection with Faraday's law.

To find the direction of the induced current German scientist Lenz's proposed that:

The induced current will flow in such a direction so as to oppose the cause that produces it.

The negative sign simply reminds us that the induced current opposes the changing magnetic field that caused the induced current. $\mathcal{E} = -\Delta\Phi/\Delta t$.

To demonstrate the lenz's law consider the N-pole of the magnet towards the coil as shown in fig. As the N-pole of the magnet moves towards the coil, the magnetic flux linking the coil produces electromagnetic induction. According to Lenz's law, the direction of the induced current will be such so as to oppose the cause that produces it. In the present case, the cause of the induced current is the increasing magnetic flux linking the coil. So the induced current will set up magnetic flux that opposes the increase in flux through the coil. This is possible only if the left hand face of the coil become N-pole. Once we know the magnetic polarity of the coil face, the direction of the induced current can be easily determined by applying right-hand rule for the coil.

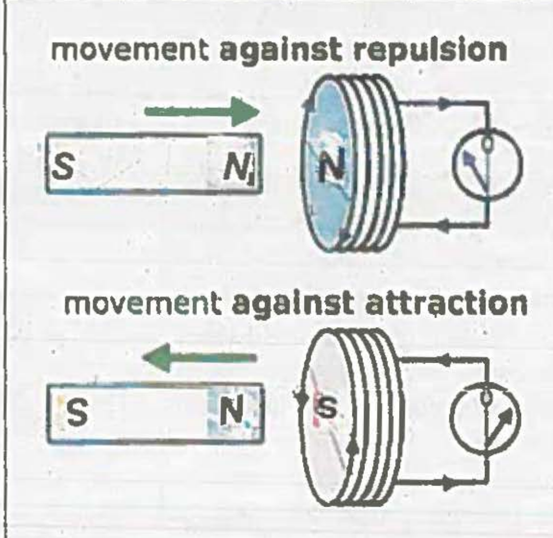


Figure 14.5: magnetic flux linking the coil

Lenz's law and conservation of energy

Lenz's law is a consequence of the law of conservation of energy. In the above case, for example, when the N-pole of the magnet is approaching the coil, the induced current will flow in the coil in such a direction that the left-hand face of

the coil becomes *N*-pole. The result is that the motion of the magnet is opposed. The mechanical energy spent in overcoming this opposition is converted into electrical energy which appears in the coil. Thus Lenz's law is consistent with the law of conservation of energy.

Fleming's Right-Hand Rule and direction:

To find the direction of the induced e.m.f. and hence current, Fleming's right-hand rule may also be used. When the conductor moves at right angles to a stationary magnetic field then the direction of induced current is from right to left as shown in fig. If the motion of the conductor is downward, then the direction of induced current will be from left to right.

It may be stated as under:

Stretch out the **forefinger, middle finger** *and* **thumb** *of your right hand so that they are at right angles to one another. If the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced current.*

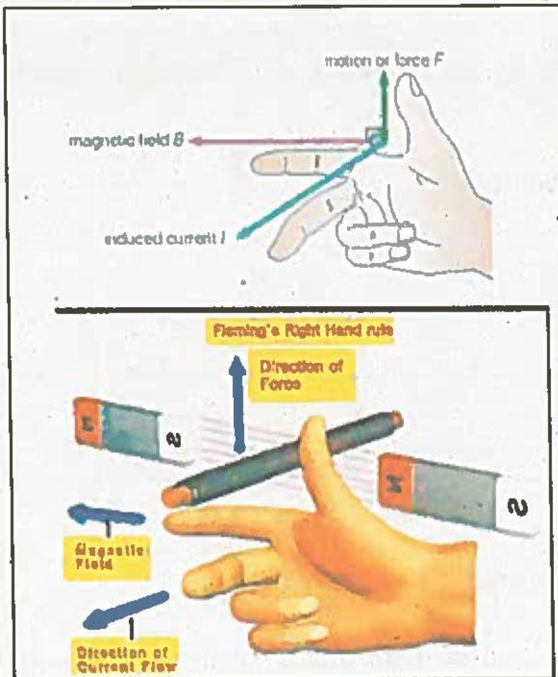


Figure 14.6 : Fleming's right hand rule shows the relative directions of Current (second finger), Magnetic Field (first finger) and the direction of motion or force (thumb).

Example: 14.1.

A coil of 100 turns is linked by a flux of 20 m Wb. If this flux is reversed in a time of 2 ms, calculate the average e.m.f. induced in the coil.

Solution.

Change in flux, $\Delta\Phi = 20 - (-20) = 40 \text{ mWb} = 40 \times 10^{-3} \text{ Wb}$

Time taken for the change, $\Delta t = 2\ ms = 2 \times 10^{-3}\ s$

$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t} = 100\times\frac{40\times10^{-3}}{2\times10^{-3}}$$

$$= 2000\ V$$

Example: 14.2.

At what rate would it be necessary for a single conductor to cut the flux in order that a current of 1.2 mA flows through it when 10 Ω resistor is connected across its ends?

Solution

$$\mathcal{E} = N\frac{\Delta\Phi}{\Delta t}$$

Here $\mathcal{E} = IR = 1.2\times10^{-3}\times10 = 1.2\times10^{-2}\ V;$

$$\frac{\Delta\Phi}{\Delta t} = ?$$

$$\frac{\Delta\Phi}{\Delta t} = \frac{\mathcal{E}}{N} = \frac{1.2\times10^{-2}}{1}$$

$$= 1.2\times10^{-2}\ Wb/s$$

Example: 14.3.

A coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned through 180^0 in 1/10 s. Calculate the average induced e.m.f.

Solution

$\Phi = NBA\cos\theta$

When the plane of the coil is perpendicular to the field, $\theta = 0^0$. When the coil is turned though 180^0, $\theta = 180^0$.

Therefore, initial flux linked with the coil is $\Phi_1 = NBA\cos\theta = NBA$

Flux linked with coil when turned through 180^0 = -NBA

Change in flux linking the coil is

$\Delta\Phi = \Phi_2 - \Phi_1 = (-NBA) - (NBA) = -2\,NBA$

$\therefore$ Average induced e.m.f., $\mathcal{E}=\dfrac{\Delta\Phi}{\Delta t}=\dfrac{2NBA}{\Delta t}$

Here $N=1000$; $B=0.4$ G$=0.4\times10^{-4}$ T

$;A=500\times100^{-4}m^2;$

$\Delta t=0.1$s

$$\mathcal{E}=\frac{2\times1000\times(0.4\times10^{-4})\times500\times10^{-4}}{0.1}$$

$$=0.04\text{V}$$

14.4 INDUCED E.M.F

By Faraday's law *whenever a conductor is placed in a varying magnetic field, EMF is induced in the conductor and this e.m.f. is called induced e.m.f.*

But as the varying magnetic field can be brought about in two ways, therefore induced e.m.f. is of two types

1. Dynamically induced e.m.f.

When the conductor is moved in a stationary magnetic field in such a way that the flux linking it changes in magnitude. Then the e.m.f. induced in this way is called dynamically induced e.m.f. (as in a d.c. generator). It is so called because e.m.f. is induced in the conductor which is in motion.

2. Statically induced e.m.f.

When the conductor is stationary and the magnetic field is moving or changing. Then the e.m.f. induced in this way is called statically induced e.m.f. (as in a transformer). It is so called because the e.m.f. is induced in a conductor which is stationary.

14.4.1 Motional e.m.f.

In order to study the dynamically induced e.m.f. in details, consider a conductor NM moving in a magnetic field. Fig: 14.7 shows a conductor of length l in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the figure, directed into the page.

14.5 Generating Electricity

Alternating current generator

The ability to produce an electromotive force by changing the magnetic field inside a coil is used to generate electricity. The generation of electricity is the important application of electromagnetic induction.

Generator is a device which converts Mechanical energy into electrical energy.

It consists of coil CDEF, two slip rings and two carbon brushes. When the coil CDEF rotates between the poles of a magnet and an electromotive force is induced. The current flows through the external circuit by slip rings A_1 and A_2 which are made of copper.

A_1 is connected to the side CD of the coil and A_2 to the side EF as shown in fig14.12: A_1 is always in contact with the brush B_1 and A_2 with the brush B_2. When the side CD moves upwards Fleming's Right-hand Rules shows that the direction of the current is from C to D and E to F. Thus the current enters the circuit at B_1 and leaves at B_2. Half a revolution later FE will be in the position previously occupied by CD and the current direction is reversed, i.e. it is from F to E and D to C. The current now enters the circuit at B_2 and leaves at B_1. Thus the direction of the induced electromotive force and the current changes every half revolution.

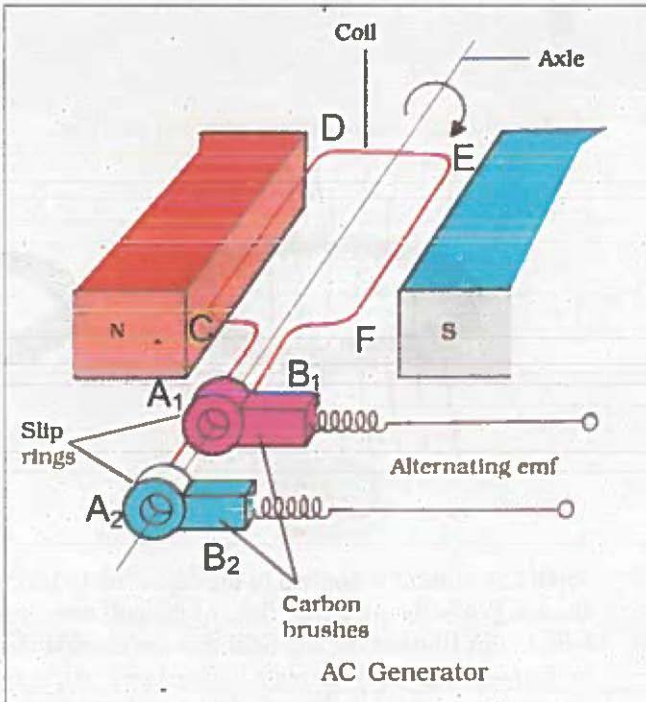


Figure 14.12: AC generator

These vary not only in direction; however, they also vary in magnitude. The graph of electromotive force against time is shown in Fig14.13: The time taken for one revolution T is called the period and frequency f, is the number

of revolutions per second. Hence

$$f = \frac{1}{T}$$

The domestic electricity supply has a frequency of 50 hertz, which means that the generator makes fifty revolutions each second. The graph of current against time has the same shape as that of electromotive force against time, but the magnitude of the current depends upon the resistance of the external circuit. Fig14.13 shows the positions of the coil that correspond to various points on the graph.

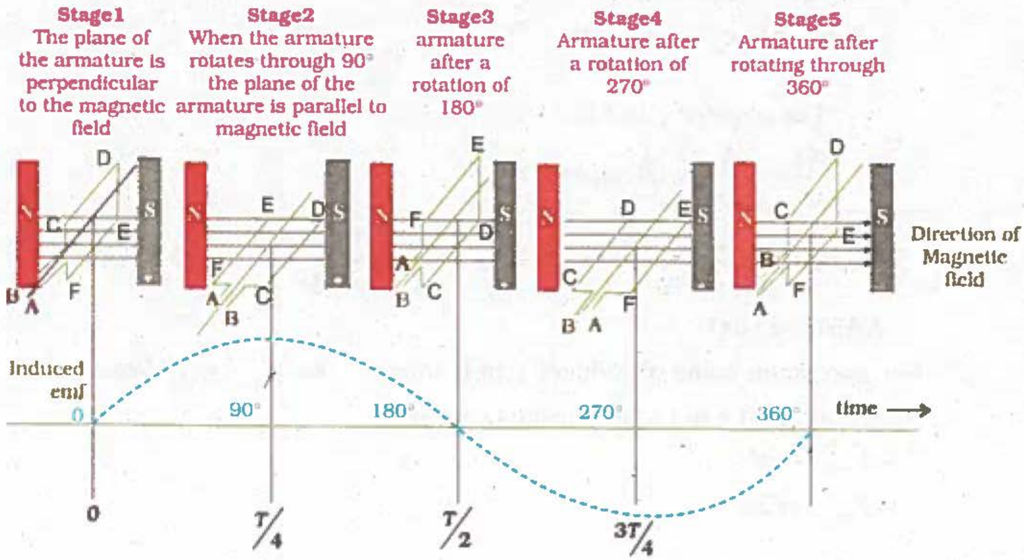


Figure 14.13 An alternating emf is generated by a loop of wire rotating in a magnetic field.

The coil has the maximum number of lines of force passing through it when it is in vertical position, and hence the maximum number of flux linkages. Therefore the induced electromotive force is a maximum when the coil is horizontal and a minimum when the coil is vertical. The induced electromotive force, is determined by the rate at which the number of flux linkages changes. From Fig14.13: it is clear that there is very little change in the number of flux

linkages when the coil is near the vertical position and consequently no, or very little induced electromotive force. Another way of looking at the problem is to consider the sides CD and EF of the coil which are responsible for the induced electromotive force. When the coil is horizontal sides CD and EF are moving perpendicular to the magnetic field and so the induced electromotive force is a maximum; when the coil is perpendicular to the field sides CD and EF are moving parallel to the magnetic field and so the induced electromotive force is zero.
The magnitude of induced e.m.f can be determined by finding the rate of change of flux through the coil. Suppose the coil is rotating with angular velocity ω and at any instant t, θ be the angle which the coil makes with the field B. If N is the number of turns in the coil then the flux through the coil of area A is

$$\Phi = B.A\,N = NAB\cos\theta$$

$$= NAB\cos\omega t \qquad \because \theta = \omega t$$

The induced e.m.f is

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta}{\Delta t}(NAB\cos\omega t)$$

$$\mathcal{E} = -NAB\lim_{\Delta t\to 0}\frac{\Delta(\cos\omega t)}{\Delta t} \qquad \text{As } \lim_{\Delta t\to 0}\frac{\Delta(\cos\omega t)}{\Delta t} = -(\omega\sin\omega t)$$

$$= NAB(\omega\sin\omega t)$$

For maximum value of induced e.m.f. $\sin\omega t=1$, so $\mathcal{E}_{max} = NAB\omega$

Hence induced e.m.f at any instant of time is:

$$\mathcal{E} = \mathcal{E}_{max}\sin\omega t$$

$$\mathcal{E} = \mathcal{E}_{max}\sin(2\pi f t)$$

14.6 AC Motor

An AC motor has two basic electrical parts: a "stator" and a "rotor" as shown in Fig14.14: The stator is the stationary part of the motor while rotor is the rotating part of the motor. The stator consists of a group of individual electro-magnets arranged in such a way that they form a hollow cylinder, with one pole of each magnet facing toward the center of the group. It also consists of a group of electro-magnets arranged around a cylinder, with the poles facing toward the stator poles. The rotor, obviously, is located inside the stator and is mounted on the motor's shaft. The objective of these motor components is to make the rotor

rotate which in turn will rotate the motor shaft. The rotor consists of coils wound on a laminated iron armature mounted on an axle. The rotor coils are not connected to the external power supply, and an induction motor has neither commutator nor brushes. An induction motor is so named because eddy currents are induced in the rotor coils by the rotating magnetic field of the stator.

The eddy currents produce magnetic fields which interact with the rotating field of the stator to exert a torque on the rotor in the direction of rotation of the stator field.

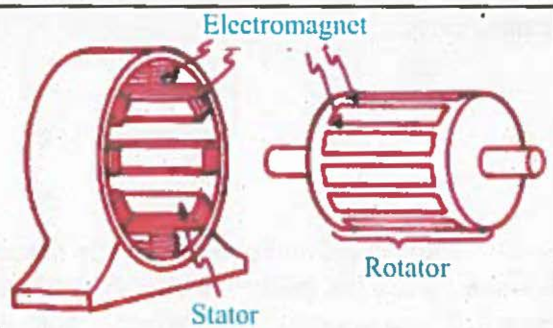


Figure 14.14 : Basic electrical components of an AC motor.

In order to understand the principle on which it works. Let at time t_1 the S-pole of the rotor is attracted by the two N-poles of the stator and the N-pole of the rotor is attracted by the two south poles of the stator. At time t_2, when the polarity of the stator poles is changed then it forces the rotor to rotate 60 degrees to line up with the stator poles as shown in fig 14.15.

At time t_3, the polarity of the stator poles is changed so that the rotor is further rotate 60 degrees to line up with the stator poles. Similarly at time t_4 it further rotates 60 degrees.

As each change is made, the poles of the rotor are attracted by the opposite poles on the stator. Thus, as the magnetic field of the stator rotates, the rotor is also forced to rotate with it.

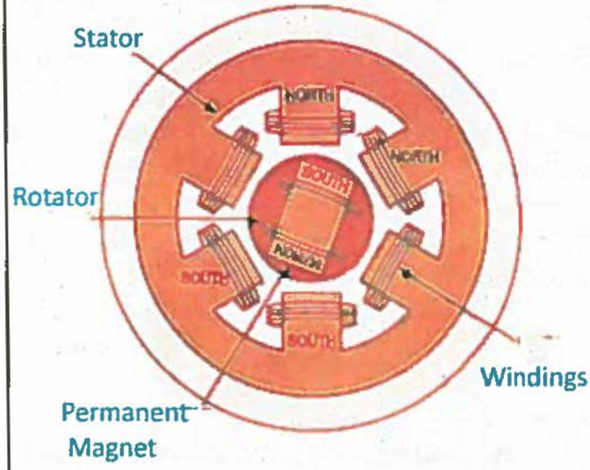


Figure 14.15 : the S-pole of the rotor is attracted by the two N-poles of the stator and the N-pole of the rotor is attracted by the two south poles of the stator.

14.6.1 Back emf

When the coil of electric motor rotates in a magnetic field by applying it with a battery of potential difference V an induced emf $\mathcal{E}$ is produced. This induced emf is produced in such a direction so as to oppose the emf V of battery which is known as back emf.

The applied voltage is equal to back emf $\mathcal{E}$ plus the voltage drop across the resistance R.

$$V = \mathcal{E} + IR$$

As the motor speed increases the eddy currents and the resulting back EMF also increase. When the motor reaches its maximum operating speed back emf will be generated at a constant rate. When a load is applied, the speed of the motor is reduced, which reduces the back emf and hence increases current in the motor. If the load stops the motor from moving then the current may be high enough to burn out the motor coil windings.

Generally, the load slows the armature down and so the current increases as the back emf is decreased. This produces an increase in current and torque to cope with the increased load. To protect the motor at low speeds a resistor in series is switched in to the circuit to protect the coils from burning out. It is switched out when the current drops to a set level and switches back in when this level is exceeded again. At the higher speeds the back emf reduces the current and so the motor continues to operate safely.

14.7 Transformers

A transformer is a device which is used to transform electrical power from one voltage and current level to another. The transformer depends on the use of alternating current. There are three main parts to a transformer: two coils of wire (called the primary and secondary) and a laminated iron core connecting them:

The primary coil is connected to the input electrical supply, which has to be AC to work properly. An AC current through the coil creates a changing magnetic field which is concentrated through the iron core.

The transformer is based on two principles: first, that an electric current can produce a magnetic field (electromagnetism), and, second that a changing magnetic field within a coil of wire induces a voltage across the ends of the coil (electromagnetic induction). Changing the current in the primary coil changes the magnetic flux that is developed. The changing magnetic flux induces a voltage in the secondary coil.

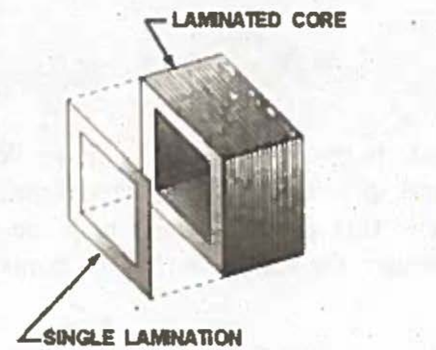


Figure 14.16: laminated iron core

14.7.1 Induction law

Current passing through the primary coil creates a magnetic field. The primary and secondary coils are wrapped around a core of high magnetic permeability, such as iron, so that most of the magnetic flux passes through both the primary and secondary coils. So the flux through the primary and secondary coil is same. The changing magnetic flux through the primary coil gives rise to an induced emf V_s in the secondary coil.

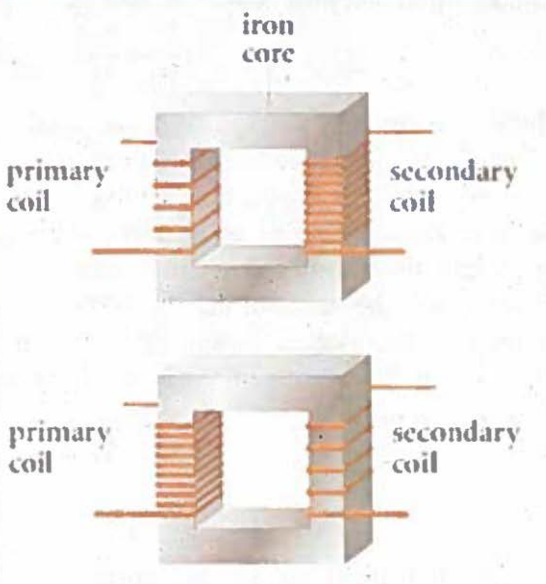


Figure 14.17: step up and step down transformer

The voltage induced across the secondary coil is given by Faraday's law of induction, which states that:

$$V_s = N_s \frac{d\Phi}{dt}$$

where V_s is the instantaneous voltage, N_s is the number of turns in the secondary coil and Φ is the magnetic flux through one turn of the coil. Since the same magnetic flux passes through both the primary and secondary coils in an ideal transformer, the voltage across the primary is

$$V_p = N_p \frac{d\Phi}{dt}$$

Taking the ratio of the two equations for V_s and V_p gives the basic equation for stepping up or stepping down the voltage

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Where V_p = primary voltage, V_s = secondary voltage, N_p = primary turns. If $N_s > N_p$ then $V_s > V_p$ and the transformer is called a step-up transformer. When $N_s < N_p$ it follows that $V_s < V_p$ and it is called a step-down transformer. The amount of energy cannot be stepped up or down. At best, as with the induction coil, as such energy can be obtained from the secondary as is put into the primary, i.e. the efficiency of the transformer is 100%. This theoretical efficiency cannot be obtained in practice, although the transformer is a very good machine with efficiency in the region of 90%. As for induction coil (assuming a theoretical efficiency of 100%),

$$V_s I_s = V_p I_p$$

$$\frac{V_s}{V_p} = \frac{I_s}{I_p}$$

It follows that if the voltage is stepped up, the current is stepped down and vice versa. If a high voltage is required a step-up transformer is used, whereas if a high current is required a step-down transformer is used.

14.7.2 Energy losses in transformers:

(i) Flux Leakage: There is always some flux leakage; that is, not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(ii) Resistance of the windings: The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire (I^2R). In high current, low voltage windings, these are minimised by using thick wire.

(iii) Eddy currents: A transformer has an iron core to concentrate the magnetic field to achieve the maximum possible inductive coupling between the primary and secondary coils. As the changing flux intersects the core, eddy currents are induced in the iron. Heating occurs because of the rather high resistance of the iron to the eddy currents.

Transformer cores are made of laminated iron, that is, many thin sheets of iron pressed together but separated by thin insulating layers. This limits the circulation of any eddy currents to the thickness of one lamina, rather than the whole core, thus reducing the overall heating effect.

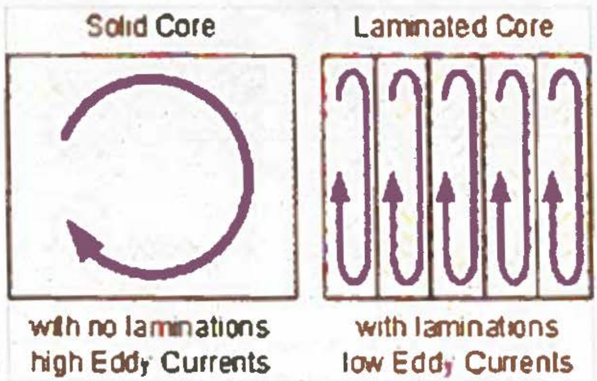


Figure 14.18: Eddy currents in an ordinary iron core, and a laminated iron core.

(iv) Hysteresis: The magnetisation of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

The transformer allowed electricity to be efficiently transmitted over long distances. This made it possible to supply electricity to homes and businesses located far from the electric generating plant. The electricity first goes to a transformer at the power plant that boosts the voltage up to 400,000 volts.

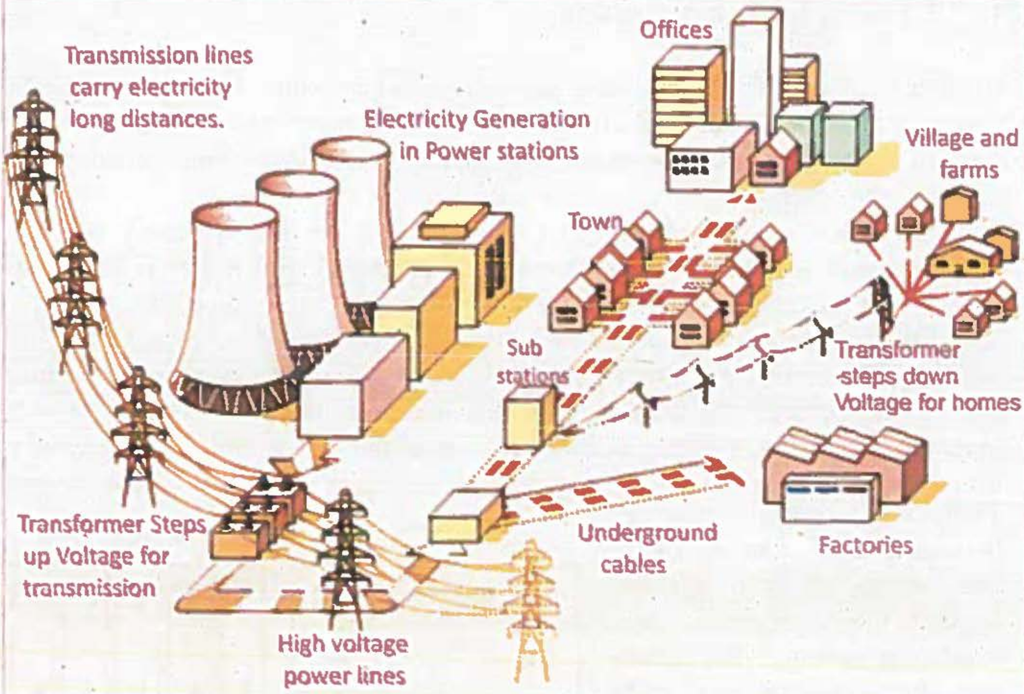


Fig 14.19: Illustrations of electricity transmission from power station to homes

When electricity travels long distances it is better to have it at higher voltages. Another way of saying this is that electricity can be transferred more efficiently at high voltages. The long thick cables of transmission lines are made of copper or aluminum because they have a low resistance. Some of the electrical energy is lost because it is changed into heat energy. High voltage transmission lines carry electricity long distances to a substation. The power lines go into substations near businesses, factories and homes. Here transformers change the very high voltage electricity back into lower voltage electricity.

From these substations, electricity in different power levels is used to run factories, streetcars and mass transit etc., In your neighborhood, another small transformer mounted on pole or in a utility box converts the power to even lower levels to be used in your house. The voltage is eventually reduced to 220 volts for larger appliances, like stoves and clothes dryers, and 110 volts for lights, TVs and other smaller appliances.

Key points

- When the magnetic flux linking a conductor changes, an e.m.f is induced in the conductor this phenomenon is known as electromagnetic induction.
- The basic requirement for electromagnetic induction is the change in flux linking the conductor (or coil).
- The e.m.f and hence the current in this conductor (or coil) will persist so long as this change is taking place.
- If the change of magnetic flux is due to a variation in the current flowing in the same circuit, the phenomenon is known as self-induction; if it is due to a change of current flowing in another circuit it is known as mutual induction.
- When the coil of electric motor rotates in a magnetic field by applying it with a battery of potential difference V an induced emf $\mathcal{E}$ is produced. This induced emf is produced in such a direction so as to oppose the emf V of battery which is known as back emf.
- Lenz's proposed that the induced current will flow in such a direction so as to oppose the cause that produces it.
  When the conductor is moved in a stationary magnetic field in such a way that the flux linking it changes in magnitude. The e.m.f. induced is called dynamically induced e.m.f.
- When the conductor is stationary and the magnetic field is moving or changing. The e.m.f. induced is called statically induced e.m.f.
- The ability to produce an electromotive force by changing the magnetic field inside a coil is used to generate electricity. *Generator is a device which converts Mechanical energy into electrical energy.*
- A transformer is a device which is used to transform electrical power from one voltage and current level to another.
- The transformer is based on two principles: first, that an electric current can produce a magnetic field, and, second that a changing magnetic field within a coil of wire induces a voltage across the ends of the coil.
- One of the best ways to overcome difficulties of heating in transformers is to reduce the size of the eddy currents.

Exercise ?

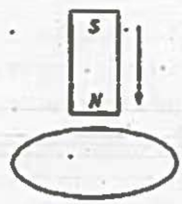
Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. For inducing emf in a coil the basic requirement is that
   (a) Flux should link the coil
   (b) change in flux should link the coil
   (c) coil should form a closed loop
   (d) both (b) and (c) are true

2. The device in which induced emf is statically induced emf is
   (a) transformer (b) ac generator
   (c) alternator (d) dynamo.

3. The north pole of a magnet is brought near a metallic ring as shown in the fig. The direction of induced current in the ring will be



   (a) anticlockwise
   (b) clockwise
   (c) first anti-clockwise and then clockwise
   (d) first clockwise and then anti-clockwise

4. What is the coefficient of mutual inductance, when the magnetic flux changes by 2×10^{-2} Wb, and change in current is 0.01 A?
   (a) 2 H (b) 3H
   (c) 1/2 H (d) zero

5. The induced emf in a coil is proportional to
   (a) magnetic flux through the coil
   (b) rate of change of magnetic flux through the coil
   (c) area of the coil
   (d) product of magnetic flux and area of the coil

6. In a coil current change from 2 to 4 A in .05 s. If the average induced emf is 8V then coefficient of self-inductance is
   (a) 0.2 henry (b) 0.1 henry
   (c) 0.8 henry (d) 0.04 henry

7. Which of the following quantities remain constant in step up transformer?
   (a) current (b) voltage (c) power (d) heat

8. Step-up transformer has a transformation ratio of 3:2. What is the voltage in secondary, if voltage in primary is 30 V?
   (a) 45 V (b) 15 V
   (c) 90 V (d) 300 V

9. Eddy current is produced when
   (a) a metal is kept in varying magnetic field
   (b) a metal is kept in steady magnetic field
   (c) a circular coil is placed in a steady magnetic field
   (d) a current is passed through a circular coil.

Conceptual question's

19

1. Make list of similarities and differences between the motor effect and electromagnetic induction in a moving wire (the dynamo effect).
2. For a simple motor, why must the back e.m.f. always be smaller than the applied potential difference?

20

3. What factors limit the size of the back e.m.f.?
4. Why does back e.m.f. tend to decrease as the rate of doing work increases?

21

5. Explain from $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$ why it possible to say that $\mathcal{E} \propto \frac{\Delta I}{\Delta t}$

6. Show that the relationship $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$ is dimensionally correct.

22

7. Give the formulae for the flux linkage in terms of angular orientation.

8. Explain the eddy current in terms of Lenz's law. Also by drawing a suitable diagram show the direction of eddy current and the polarity produced in the sheet as a result of magnetic field.

9. How electromagnetic brake works? Explain.

10. A bar magnet is dropped inside a long vertical tube. If the tube is made of metal, the magnet quickly approaches a terminal speed, but if the tube is made of cardboard, the magnet falls with constant acceleration. Explain why the magnet falls differently in the metal tube than it does in the cardboard tube.

11. The transformer suffers from Eddy current loss (a) Explain how eddy currents arise. (b) State the features of transformer designed to minimize eddy currents.

12. Analyse information to explain how induction is used in cook tops in electric ranges?

13. (a) Explain what is meant by the term back e.m.f. in any electric motor operation.

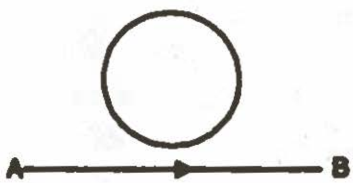
(b) Explain why it is an advantage for the armature to rotate in a radial magnetic field rather than a uniform one?

17.

14. If the armatures rotating freely then explain, in terms of electromagnetic principles (a) why the current in the armatures progressively decreases as 18 the angular velocity of the armature increases (b) why a maximum angular velocity is eventually reached?

15. Transformer cores can be made from a variety of materials. What are the 19 main features that you would require of material to make a good transformer core? Suggest how well each of the following materials would perform; iron, solid soft iron, laminated soft iron, aluminum.

16. 20. Current is increasing in magnitude from A to B as shown in fig:. What is the direction of induced current, if any, in the loop?



Comprehensive question's

1. Describe electromagnetic induction with simple experiments. Explain the factors effecting magnitude and direction of induced emf.
2. State faradays law of electromagnetism. Also explain statically and dynamically induced emf with experiments.
3. Describe lenses law. Show that this law is a manifestation of conservation of energy.
4. Explain the phenomenon of mutual induction. Define the coefficient of mutual induction and units.

5. Explain the phenomenon of self-induction. Define the coefficient of self-induction and units.
6. Explain motional emf. Show that motional induced emf is $\mathcal{E} = Blv$.
7. What is dc generator? Explain how alternating emf is generated by a loop of wire rotating in a magnetic field.
8. What is AC motor? Explain the construction and working of AC motor.
9. What is transformer? Give its principle, mathematical relationship. Also explain why laminated iron core is used instead of solid one.
10. Explain the back emf in an electric motor.
11. Explain eddy current with suitable example. How eddy current can be minimized.

Numerical problems

1. Two identical coils A and B of 500 turns each has on parallel planes such that 70% of flux produced by one coil links with the other. A current of 6 A flowing in coil A produces a flux of 0.06 m Wb in it. If the current in coil A changes from 10A to -10 A in .03s, calculate (a) the mutual inductance and (b) the e.m.f. induced in coil B.

   (a. 3.5×10^{-3}mH, b. 2.33V)

2. A wheel with 12 metal spokes each 0.6m long is rotated with a speed of 180 r.p.m in a plane normal to earth's magnetic field at a place. If the magnitude of the field is 0.6 G, what is the magnitude of induced e.m.f. between the axle and rim of the wheel? (2.035×10^{-4} V)

3. A circuit has 1000 turns enclosing a magnetic circuit 20cm^2 in section with 4A current, the flux density is 1 Wbm^{-2} and with 9A current, it is 1.4Wbm^{-2}. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9A to 4A in .05s.

   (.16H, 16V)

4. A coil of resistance 100Ω is placed in a magnetic field of 1mWb. The coil has 100 turns and a galvanometer of 400Ω resistance is connected in series

with it: find the average emf and the current if the coil is moved in $1/10^{th}$ s from the given field to a field of 0.2mWb.

(1.6 mA)

5. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2} (a) what is the instantaneous value of the emf induced in the wire?
(b) What is the direction of the emf ?
(c) Which end of the wire is at the higher electrical potential?

(1.5×10^{-3} V)

6. Current in a circuit falls from 5.0 A to 0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

(4 H)

7. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

(7.54×10^{-6} V)

8. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

(2.4×10^{-4} V, 2 s, & 0.6×10^{-4} V, 8 s)

9. A 90-mm length of wire moves with an upward velocity of 35 ms^{-1} between the poles of a magnet. The magnetic field is 80 mT directed to the right. If the resistance in the wire is 5.00 m Ω , what are the magnitude and direction of the induced current?

(50.4 A)

10. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

(30 Wb)

11. The back emf in a motor is 120V when the motor is turning at 1680 rev/min. What is the back emf when the motor turns at 3360 rev/min?

(240 V)

UNIT 15

.........A.C Circuit.......

After studying this chapter the students will be able to

- describe the terms time period, frequency, instantaneous peak value and root mean square value of an alternating current and voltage.
- represent a sinusoidally alternating current or voltage by an equation of the form $x = x_o \sin \omega t$.
- describe the phase of A.C and how phase lags and leads in A.C Circuits.
- identify inductors as important components of A.C circuits termed as chokes (devices which present a high resistance to alternating current).
- explain the flow of A.C through resistors, capacitors and inductors.
- apply the knowledge to calculate the reactances of capacitors and inductors.
- describe impedance as vector summation of resistances and reactances.
- construct phasor diagrams and carry out calculations on circuits including resistive and reactive components in series.
- solve the problems using the formulae of A.C Power.
- explain resonance in an A.C circuit and carry out calculations using the resonant frequency formulae.
- describe that maximum power is transferred when the impedances of source and load match to each other.
- describe the qualitative treatment of Maxwell's equations and production of electromagnetic waves.
- become familiar with electromagnetic spectrum (ranging from radiowaves to γ-rays).

- identify that light is a part of a continuous spectrum of electromagnetic waves all of which travel in vacuum with same speed.
- describe that the information can be transmitted by radiowaves.
- identify that the microwaves of a certain frequency cause heating when absorbed by water and cause burns when absorbed by body tissues.
- describe that ultra violet radiation can be produced by special lamps and that prolonged exposure to the Sun may cause skin cancer from ultra violet radiation.

The electricity produced by most generators is in the form of alternating current. In general AC generators, motors and other electrical equipment's are simpler, cheaper and more reliable than their DC counterparts.

Investigation of the behaviour of resistance, inductance and capacitance in AC circuits prepares us to look into the many diverse uses of these circuit elements and AC sources. A theory describing relation between accelerating charges, circuits and electric and magnetic fields was given by James Clerk Maxwell in 1864 called electromagnetic theory. The theory predicts that accelerating electric charges radiate electromagnetic waves, which propagate at the speed of light. When the acceleration is in the form of a continuous oscillation, the frequency of the electromagnetic waves is equal to the frequency of oscillation of the charges.

Maxwell formulated four equations that are regarded as the basis of all electrical and magnetic phenomena.

The consequences of Maxwell's equations are very far reaching. Maxwell predicted the existence of electromagnetic waves and that light is a form of electromagnetic radiation. Thus Maxwell unified the subjects of optics and electromagnetism.

For your information

Metal detectors are used at air ports and other sensitive areas for security purposes. Metal objects cause changes in an electromagnetic field when they pass through the door way. A circuit detects the changes and sets off an alarm.

15.1 Alternating Voltage and Current

The supply of current for electrical devices may come from a direct current source (DC), or an alternating current source (AC). In direct current electricity, electrons flow continuously in one direction from the source of power through a conductor to a load and back to the source of power. The voltage in direct current remains constant. DC power sources include batteries and DC generators. In alternating current an AC generator is used to make electrons flow first in one direction then in another. A source which produces potential difference of changing polarity with time is called as alternating source. *A voltage which changes its polarity at regular interval of time is called an alternating voltage.*

When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction; the direction of current at any instant depends upon the polarity of the voltages. Fig. shows an alternating voltage source connected to a resistor *R*. In Fig the upper terminal of alternating voltage source is positive and lower terminal negative so that current flows in the circuit as shown in Fig.15.1 (i).

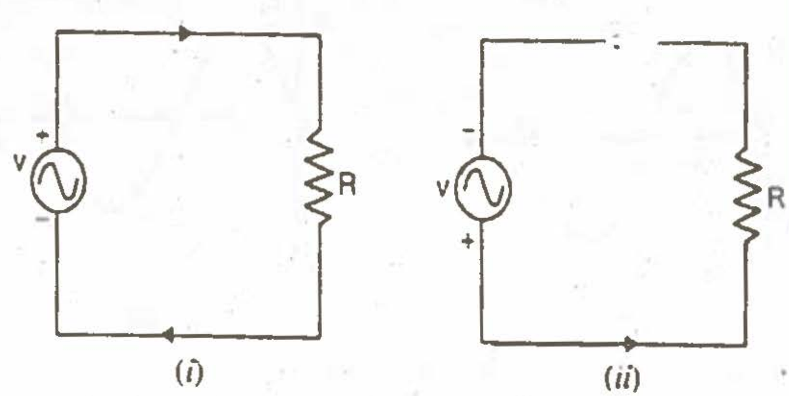


Figure 15.1: AC circuit

After some time, the polarities of the voltage source are reversed, so that current now flows in the opposite direction. This is called alternating current because the current flows in alternate directions in the circuit.

15.2 Sinusoidal Alternating Voltage and Current

We have studied in chapter 14 that sinusoidal alternating voltage can be produced by rotating a coil with a constant angular velocity (say ω rad/s) in a uniform magnetic field. AC voltage switches polarity over time. When, graphed, over time, the "wave" traced by this voltage of alternating polarity from an alternator takes on a distinct shape, known as a sine wave. The sinusoidal alternating voltage can be expressed by the equation:

$$V = V_m \sin\omega t \qquad \text{...(15.1)}$$

Where V is Instantaneous value of alternating voltage, V_m is Max. value of alternating voltage and ω is angular velocity of the coil. Fig.15.2 (i)shows, the waveform of sinusoidal voltage whereas Fig.15.2 (ii) shows the waveform of sinusoidal current. Fig15.2: shows that sinusoidal voltage or current not only changes direction at regular intervals but the magnitude is also changing continuously. The change from one polarity to the other is a smooth one, the voltage level changing most rapidly at the zero ("crossover") point and most slowly at its peak.

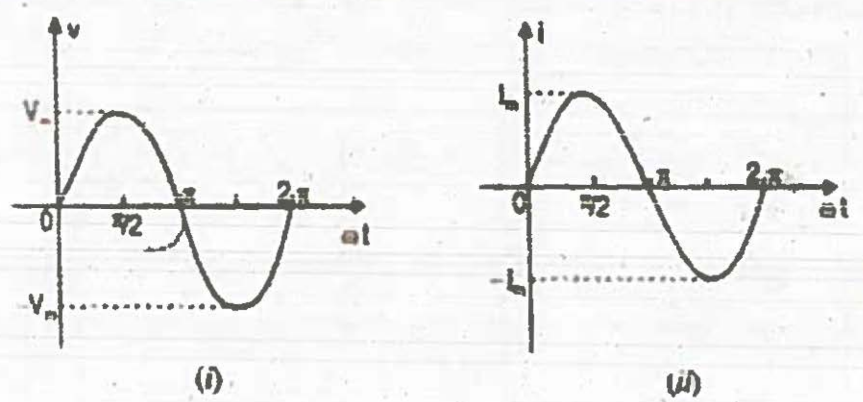


Figure 15.2: alternating current and voltage

15.3 A.C. Terminologies

Alternating voltage or current changes continuously in magnitude and alternates in direction at regular intervals of time.

It rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again (as shown in Fig15.3). The important A.C. terminology is defined below:

1. **Instantaneous value.** The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltage and current are represented by V and I respectively. As an example, the instantaneous values of voltage at 0°, 90° and 270° are 0, + V_m, - V_m respectively as shown in Fig 15.3.

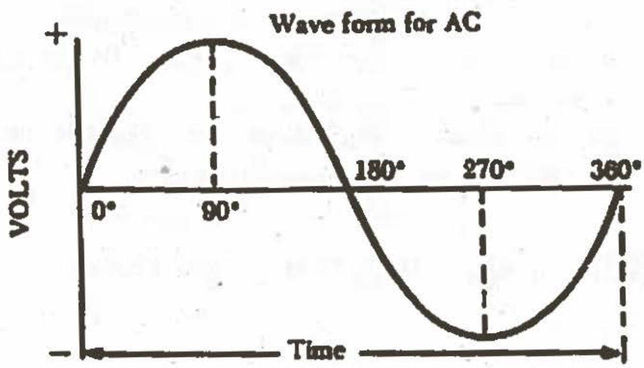


Figure 15.3: AC waveform

2. **Cycle.** One complete set of positive and negative values of an alternating quantity is known as a cycle. Fig. 15.3 shows one cycle of an alternating voltage. A cycle can also be defined in terms of angular measure. One cycle corresponds to 360° electrical or 2π radians.
3. **Time period.** The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.
4. **Frequency.** The number of cycle that occurs in one second is called the frequency (f) of the alternating quantity. It is measured in cycle /s (C/s) or Hertz (Hz). One hertz is equal to 1C/s.

The frequency of power system in Pakistan is 50 C/s or 50 Hz. It means that alternating voltage or current completes 50 cycles in one second. The 50 Hz frequency is the most popular because it gives the best results when used for operating both lights and machinery.

For your Information

Importance of Sine Waveform

Alternating voltages and currents can be produced in variety of waveforms (e.g. square waves, triangular waves, rectangular waves etc), but the engineers still choose to adopt sine waveform. It has following advantages:

1. The sine waveform produces the least disturbance in the electrical circuit and is the smoothest and efficient waveform. For example, when current in a capacitor, in an inductor or in a transformer is sinusoidal, the voltage across the element is also sinusoidal. This is not true of any other waveform.
2. The mathematical computations, connected with alternating current work, are much simpler with this waveform.

15.4 Values of Alternating Voltage and Current

The average value of an AC waveform is not the same value as that for a DC waveforms average value. This is because the AC waveform is constantly changing with time. It can be expressed as, Peak value, Average value or mean value, and R.M.S. value or effective value.

Peak Values

The maximum value reached by an AC waveform is called its peak value. The peak value of a sine wave occurs twice each cycle, once at the positive maximum value and once at the negative maximum value. The peak value of a waveform is sometimes also called its amplitude, but the term "peak value" is more descriptive. The knowledge of peak value is important in case of testing materials. However, peak value is not used to specify the magnitude of alternating voltage or current. The peak or maximum value of an alternating voltage or current is represented by V_m or I_m.

Average Value

The average value of a waveform is the average of all its values over a period of time. Finding an average value over time means adding all the values that occur in a specifying time interval and dividing the sum by that time. In performing such a computation, we regard the area above the time axis as positive area and area below the time axis as negative area. The algebraic signs of the areas must be taken into account when computing the total (net) area.

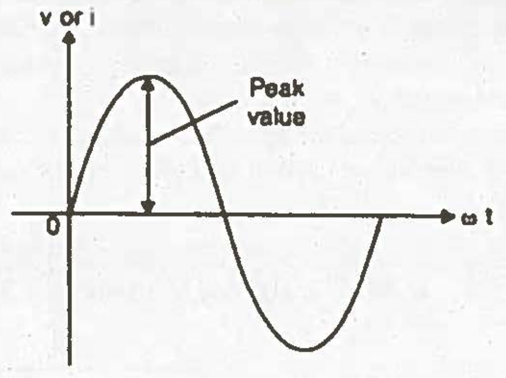


Figure 15.4: peak value

$$\text{Average} = \frac{\text{Total (net)area under curve for time } T}{\text{Time } T}$$

R.M.S. or Effective Value

In order to specify a sinusoidal voltage or current we do not use average value, because its value over one cycle is zero and cannot be used for power calculation. Therefore, we use another suitable method to measure the effectiveness of an alternating current. The equivalent average value for an alternating current system that provides the same power to the load as a DC equivalent circuit is called the "effective value".

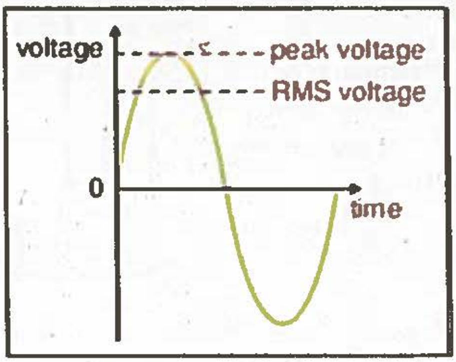


Figure 15.5(a): peak and RMS value of voltage

This effective power in an alternating current system is therefore equal to: (I^2R Average). As power is proportional to current squared, the effective current, I will be equal to $\sqrt{I^2}$ Ave.

Therefore, the effective current in an AC system is called the Root Mean Squared or R.M.S. value.

The effective or r.m.s. value of an alternating current is that steady current (d.c.)which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.

For example if the effective or r.m.s. value of an alternating current is 7A, then the alternating current will produce the same heating effect as that produced by 7A direct current.

15.5 R.M.S. Value of Sinusoidal Current

Although peak, average and peak to peak values may be important in some engineering applications, but it is the r.m.s. or effective value which is used to express the magnitude of an alternating voltage or current.

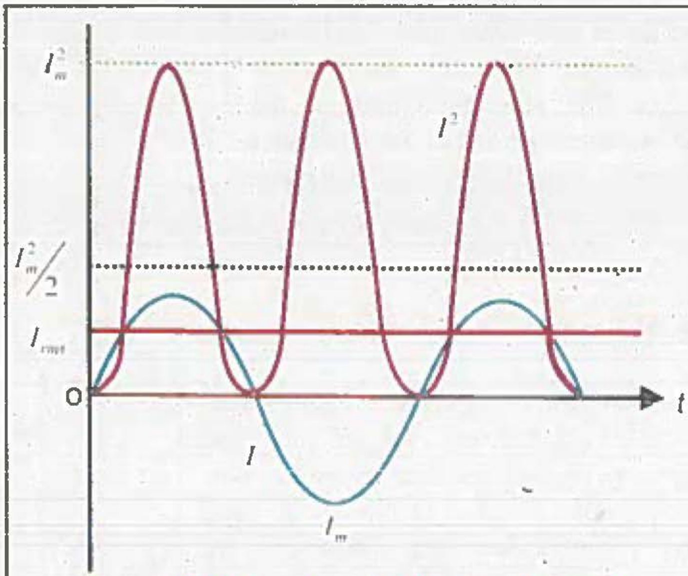


Figure 15.5(b): peak and RMS value of current

The equation of the alternating current varying sinusoidally is given by:

$$I = I_m \sin \omega t \qquad ...(15.2)$$

If this current is passed through a resistance R, then power delivered at any instant is

$$P = I^2 R = (I_m \sin \omega t)^2 \text{R}$$

$$= I_m^2 \text{R} \sin^2 \omega t \qquad ...(15.3)$$

Because the current is squared, power is always positive. Since the value of $\sin^2 \omega t$ varies between 0 and 1, its average value is 1/2

$$\therefore \text{Average power delivered, } P = \frac{1}{2} I_m^2 \text{R} \qquad ...(15.4)$$

If $I_{r.m.s}$ is the r.m.s. (or effective) value of alternating current, then by definition,

Power delivered, $P = I^2_{r.m.s}R$ ...(15.5)

From Eqs. (15.4) and (15.5), we have,

$$I^2_{r.m.s}R = \frac{1}{2}I^2_m R$$

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.7071\,I_m$$

$$I_{r.m.s} = 0.707\ I_m \quad ...(15.6)$$

An alternating current can also be represented as a cosine function of time. $I = I_m\cos\omega t$. Similarly, alternating voltage can be represented as $V = V_m\cos\omega t$.

15.6 Phase of A.C.

In electrical engineering, we are more concerned with relative phases or phase difference between different alternating quantities rather than with their absolute values. The word "phasor" is short for "phase vector." It is a way to represent a sine or cosine function graphically. Consider an alternating voltage wave of time period T second as shown in Fig.15.6.

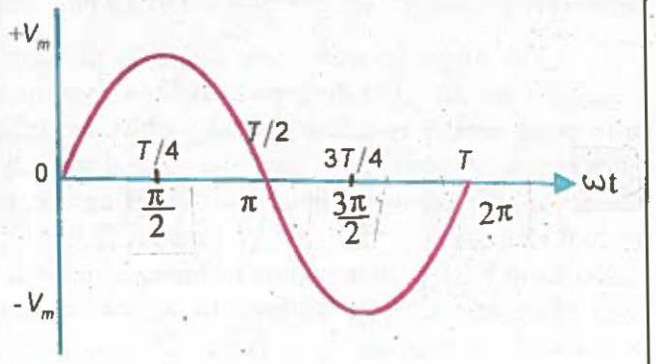


Figure 15.6: alternating voltage wave form

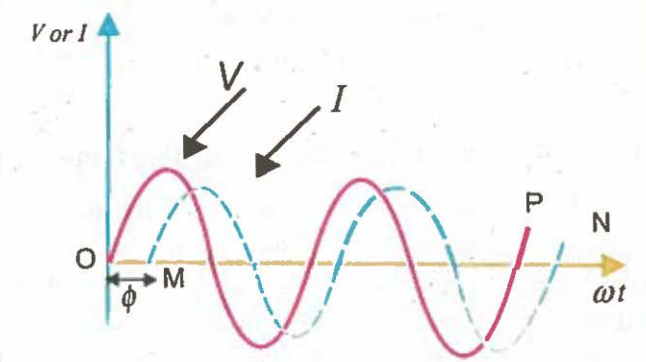


Figure 15.7: phase difference of current and voltage

The maximum positive value ($+V_m$) occurs at $T/4$ second or $\pi/2$ radians. Therefore phase of maximum positive value is $T/4$ second or $\pi/2$ radians.

Similarly, the phase of negative peak ($-V_m$) is $3T/4$ second or $3\pi/2$ radians.

Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase Difference

: In most of practical circuits, alternating voltage and current have different phases. Thus voltage may be passing through its zero point while the current has passed or it is yet to pass through its zero point in the same direction. We say that voltage and current have a phase difference.

Hence *when two alternating quantities of the same frequency have different zero point, they are said to have a phase difference.*

The angle between zero points is the angle of phase difference ϕ. It is generally measured in degrees or radians. The quantity which passes through its zero point earlier is said to leading while the other is said to be lagging. Since both alternating quantities have the same frequency, the phase difference between them remains the same. Phasor of waves can be added as vectors to produce the sum of two sine functions. Consider an A.C. circuit in which current I lags behind the voltage V by ϕ so the phase difference between voltage and current is ϕ.
This phase relationship is shown by waves in Figure. Thus in Fig15.7, voltage V is passing through its zero point 'O' and is rising in the positive direction. Similarly, current I passes through its zero point 'M' as shown in fig and is rising in the positive direction. Therefore, phase difference between voltage and current is OM (= ϕ). Similarly, difference at other points P and N is PN (= ϕ). The equations of voltage and current are:

$$V = V_m \sin\omega t \qquad \text{(i)}$$

$$I = I_m \sin(\omega t - \phi) \qquad \text{(ii)}$$

15.6.1 Alternating Quantities Representation

The sinusoidal alternating voltage or current is represented by a line of definite length rotating in counter clock wise direction at a constant angular velocity (ω). Such a rotating line is called a phasor. The length of the phasor is taken equal to the maximum value (on a suitable scale) of the alternating quantity, the angle with axis of reference (i.e., X-axis) indicates the phase of the alternating quantity (current in this case) and angular velocity equal to the angular velocity of the alternating quantity. A phasor diagram permits addition and subtraction of alternation voltages or current with a fair degree of case.

In AC circuits, currents and voltages are all sinusoidal functions. The general mathematical form of such a function is: $I = I_m \sin \omega t$. Let line OA represents the maximum value I_m on the scale. Imagine the line OA (or Phasor, as it is called) to be rotating in anticlockwise direction at an angular velocity ω rad/s about the point O. Measuring the time from the instant when OA is horizontal, let OA rotate through an angle θ in the anticlockwise direction. The projection of OA on the Y-axis is OB.

$$OB = OA \sin\theta$$
$$I = I_m \sin \omega t$$

where I, is the value of current at that instant.

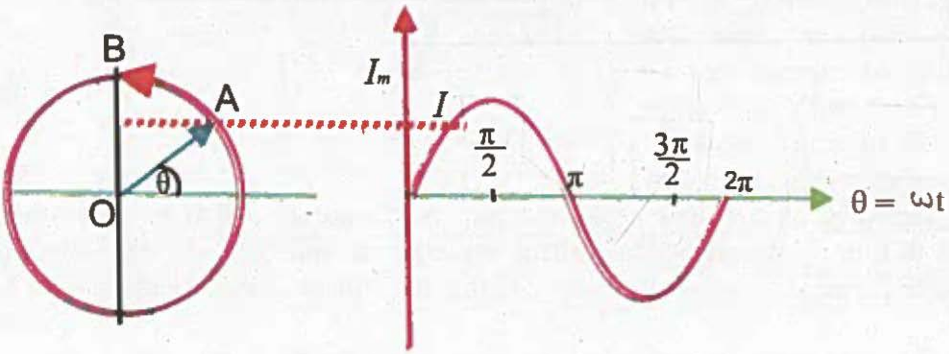


Figure 15.8:Phasor Representation

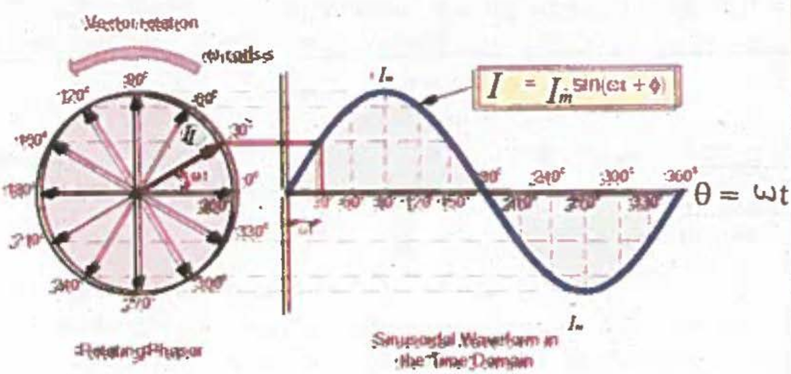


Figure 15.9: rotating vectors at different phase angles

Hence the projection of the phasor OA and the y-axis at any instant gives the value of current at that instant. Thus when $\theta = 90^\circ$, the projection on y-axis is OA ($= I_m$) itself. That the value of current at this instant (i.e. at θ or $\omega t = 90^\circ$) is I_m can be readily established if we put $\theta = 90^\circ$ in the current equation. If we plot the projections of the phasor on the Y-axis versus its angular position point by point, a sinusoidal alternating current wave is generated as shown in Fig15.8. Thus the phase represents the sine wave for every instant of time. Things start to get complicated when we need to relate two or more AC voltages or currents that are out of step with each other. By "out of step," we mean that the two waveforms are not synchronized: that their peaks and zero points do not match up at the same points in time. If the sinusoidal voltage wave V and sinusoidal current wave I of the same frequency are out of phase such that the voltage is leading the current by ϕ^0.

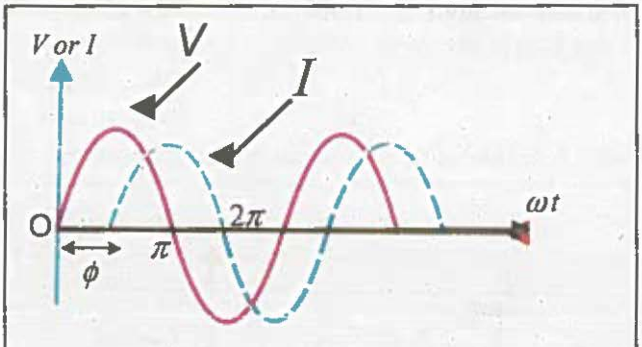


Figure 15.10: Phasor Diagram

Then the alternating quantities can be represented on the same phasor diagram because the phasors V_m and I_m rotate at the same angular velocity ω and hence phase difference ϕ between them remains the same at all times as shown in fig15.10: When each phasor completes one revolution, it generates the corresponding cycle. The equations of the two waves can be represented as:

$$V = V_m \sin \omega t$$

$$I = I_m \sin (\omega t - \phi) \qquad \text{...(15.7)}$$

Since the two phasors have the same angular velocity (ω) and there is no relative motion between them, they can be displayed in a stationary diagram.

Instantaneous Power

The instantaneous power supplied to a circuit is simply the product of the instantaneous voltage and instantaneous current. The instantaneous power is always expressed in watts, irrespective of the type of circuit used. The instantaneous power may be positive or negative. A positive value means that power flows from the source to the load. Consequently, a negative value means that power flows from the load to the source.

15.7 A.C. Through Resistance

Consider a circuit containing a pure resistance of R connected across an alternating voltage source as shown in Fig15.11 (a), then free electrons flow in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit. The applied voltage and current pass through their zero values at the same instant and attain their positive and negative peaks at the same instant such that current is in phase with the applied voltage as shown in fig 15.11(b).The alternating voltage is be given by

$V = V_m \sin \omega t$ (i)

where, V_m is the peak value of the alternating voltage.

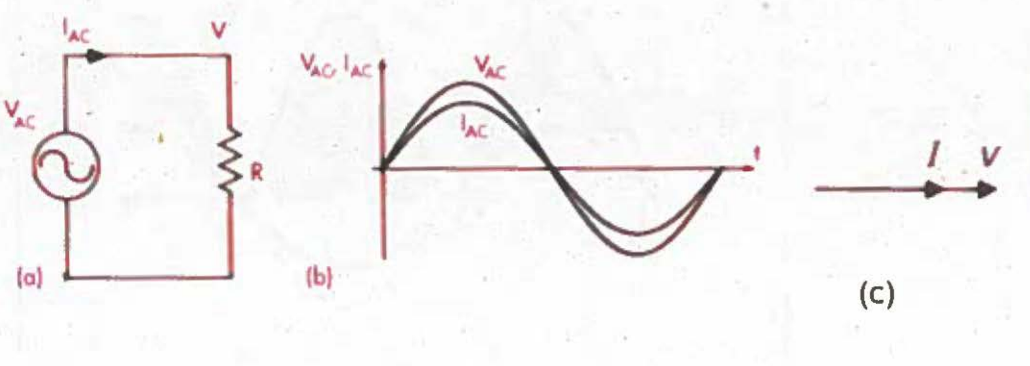


Figure 15.11: AC circuit and waveform

As a result of this voltage, an alternating current I will flow in the circuit. The applied voltage has to overcome the drop in the resistance only i.e., $V = IR$

Or $$I = \frac{V}{R} = \frac{V_m}{R} \sin \omega t$$

∴ $$I_m = \frac{V_m}{R}$$

$$I = I_m \sin \omega t \qquad \text{(ii)}$$

The value of I will be maximum (i.e. I_m) when $\sin \omega t = 1$. Eqs. (i) and (ii) shows that the applied voltage and the circuit current are in phase with each other. This is also indicated by the phasor diagram shown in Fig15.11(c).

In terms of r.m.s. value,

$$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \times R \qquad ...(15.8)$$

Or

$$V_{rms} = I_{rms} R \qquad ...(15.9)$$

15.7.1 Power Loss in an Resistor

The power curve for a pure resistive circuit is obtained from the product of the corresponding instantaneous values of voltage and current. Fig15.12 shows that power is always positive except at points L,M and N at which it drops to zero for a moment.

This means that the voltage source is constantly delivering power to the circuit which is consumed by the circuit.

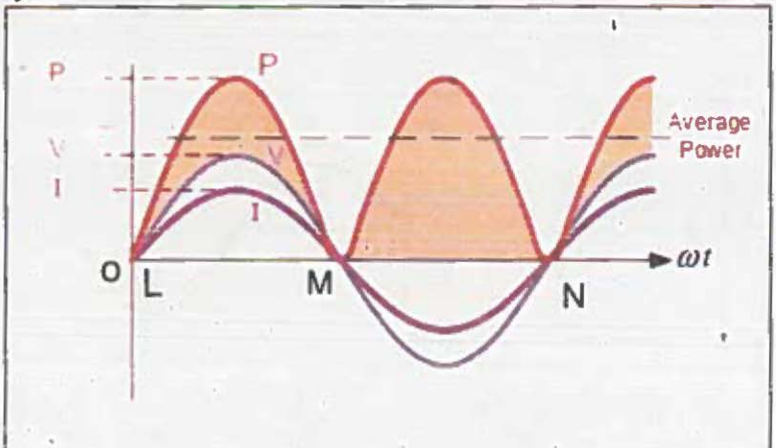


Figure 15.12: power curve

The average power dissipated in resistor R over one complete cycle of the applied is:

$$P = \langle VI \rangle = \langle V_m \sin \omega t \times I_m \sin \omega t \rangle \qquad ...(15.10)$$

$$= V_m I_m \langle \sin^2 \omega t \rangle$$

$$= \frac{V_m I_m}{2} \qquad \because \langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms} \qquad ...(15.11)$$

Example 15.1

An A.C. circuit consists of a pure resistance of 20Ω and is connected across A.C. supply of 220V, 50Hz. Calculate (a) current (b) power consumed and (c) equation for voltage and current.

Solution.

Resistance $R = 20\ \Omega$
Voltage $V = 220$ V
Frequency $f = 50$ Hz

Maximum value of an alternating voltage is $V_m = \sqrt{2}\,V = \sqrt{2} \times 220 = 311.1$V

(a) Current, $I = V/R = 220/20 = 11$A

(b) Average power P dissipated in the resistor is

$$P = VI = 220 \times 11 = 2420 \text{ w}$$

Maximum value of an alternating current $I_m = \sqrt{2}\,I = \sqrt{2} \times 11 = 15.55$ A

$$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad s}^{-1}$$

(c) Equations for voltage and current is

$V = V_m \sin(\omega t)$ & $I = I_m \sin(\omega t)$

putting values

$V = 311.1\sin(314t)$ & $I = 15.55\sin(314t)$

15.8 A.C. Through Pure Inductance

An inductor is a two-terminal electrical component which resists changes in electric current passing through it. It consists of a conductor such as a wire, usually wound into a coil. Consider an alternating voltage applied to a pure inductance of L as shown in Fig15.13 when a sinusoidal current I flow in time t then a back e.m.f. ($= L\,\Delta I/\Delta t$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. As there is no drop in potential, so the applied voltage has to overcome the back. emf

∴ Applied alternating voltage = Back e.m.f.

So the energy which is required in building up current in inductance L, is returned back during the decay of the current.

Let the equation for alternating current is :

$$I = I_m \sin \omega t \qquad \ldots(i)$$

The changing current sets up a back e.m.f in the coil.
The magnitude of back e.m.f is

$$\mathcal{E} = L\frac{\Delta I}{\Delta t}$$

To maintain a constant current the applied e.m.f must be constantly applied.
The magnitude of applied voltage is

$$V = L\frac{\Delta I}{\Delta t} = L\frac{\Delta(I_m \sin \omega t)}{\Delta t}$$

$$= LI_m \frac{\Delta(\sin \omega t)}{\Delta t} \qquad \text{(ii)}$$

Using the result of simple calculus:

$$\frac{\Delta(\sin \omega t)}{\Delta t} = \omega \cos \omega t \qquad \text{(iii)}$$

putting the values of Eq (iii) in (ii) we get

$$V = \omega L I_m \cos \omega t$$

$$\therefore (\omega L I_m = V_m)$$

$$or \quad V = V_m \cos \omega t$$

$$V = V_m \sin(\omega t + \frac{\pi}{2}) \qquad \text{(iv)}$$

From Eqs (i) and (iv) it is clear that current lags behind the voltage by $\pi/2$ radians or 90°. Hence in a pure inductance, current lags behind the voltage by 90°.
Fig. 15.13(b) also shows that current lags the voltage in an inductive coil. Inductance opposes the change in current and serves to delay the increase or decrease of current in the circuit.

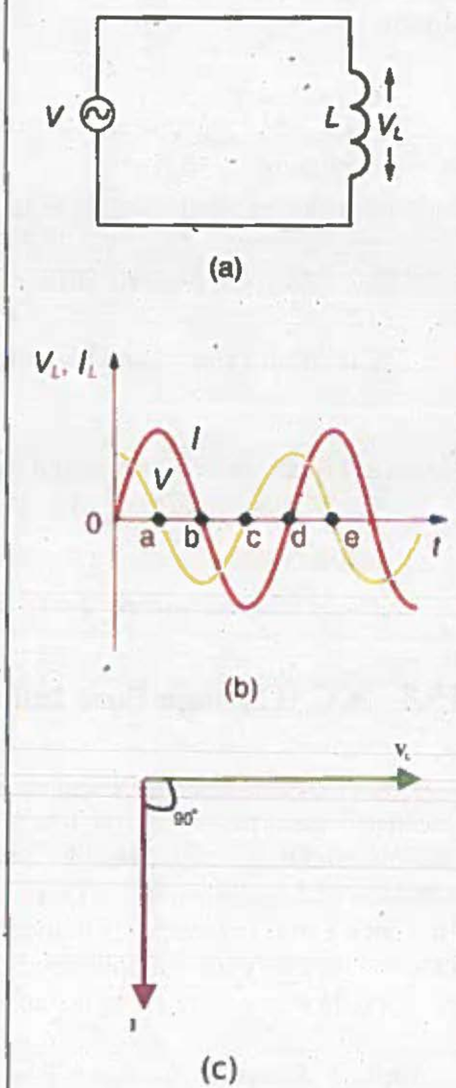


Figure 15.13:

This causes the current to lag behind the applied voltage which is indicated by the phasor diagram shown in Fig.15.13(c).
Inductance opposes the flow of current in the circuit.
So *the opposition offered by an inductor to the flow of A.C. is called inductive reactive reactance* X_L.
Therefore in analogy to ohm law we can write:

$$V_m = I_m X_L$$

Since inductive reactance is ratio of voltage to current. So

$$\text{or } \frac{V_m}{I_m} = X_L$$

$$X_L = \frac{V_m}{I_m}$$

$$X_L = \frac{I_m \omega L}{I_m} = \omega L$$

$$or\ X_L = \omega L \qquad ...(15.12)$$

$$X_L = 2\pi f L \qquad ...(15.13)$$

The reactance of coil depends upon frequency of A.C. In case of D.C. inductive reactance X_L is zero.

15.8.1 Power loss in an inductor

Fig15.14: shows the power curve for a pure inductive circuit. During the first 90° of the cycle, the voltage is positive and the current is negative, therefore, the

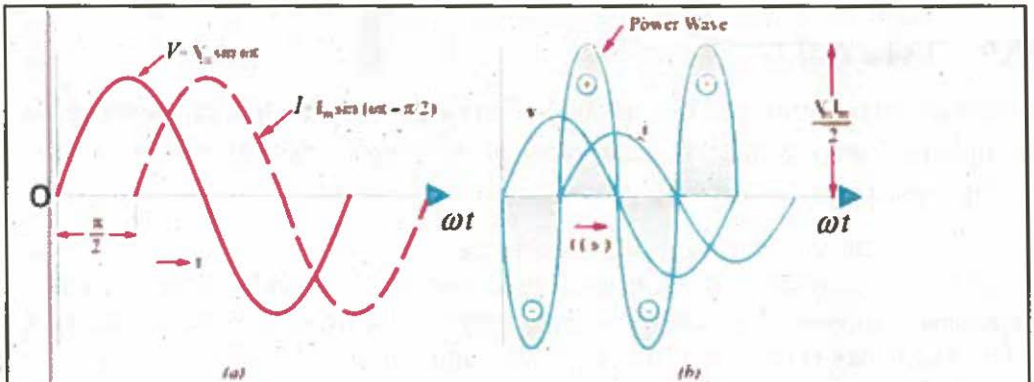


Figure 15.14: The shaded portion of the graph indicates that on adding portion I, II, III and IV, vector p is equal to zero. Average power over a complete cycle of AC through an ideal inductor is zero.

power supplied is negative. This means the power is flowing from the coil to the source. During the next 90° of the cycle, both voltage and current are positive and the power supplied is positive. Therefore, power flows from the source to the coil. Similarly, for the next 90° of the cycle, power flows from the coil to the source and during the last 90° of the cycle, power flow from the source to the coil. The power curve over one cycle shows that positive power is equal to the negative power. Hence the resultant power over one cycle is zero i.e. a pure inductance consumes no power. The electric power merely flows from the source to the coil and back again.

In any circuit, electric power consumed at any instant is the product of voltage and current at that instant. The average power loss in an inductive circuit is,

$$P = \langle VI \rangle = \langle V_m \cos \omega t \times I_m \sin \omega t \rangle$$

$$= V_m I_m \langle \sin \omega t \rangle \langle \cos \omega t \rangle$$

$$= 0 \qquad \because \langle \sin \omega t \rangle \langle \cos \omega t \rangle = 0$$

For your Information

The purpose of passing current through a circuit is to transfer power from the source to the circuit. *The power which is actually consumed in the circuit is called the true power or active power.*

We know that current and voltage are in phase in a resistance whereas they are 90° out of phase in L or C. Therefore, we come to the conclusion that current in phase with voltage produces true or active power whereas current 90° out of phase with voltage contributes to reactive power *i.e.*

True Power = Voltage × Current in phase with voltage

15.9 Choke coil

A Choke is an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current. In a DC circuit, a resistor is used to restrict the current.

If I is the current and R is the resistance, the power loss in the form of heat is I^2R. In AC circuit, inductor is used. Its impedance is X_L and is large at high frequencies. In general, a choke is used to prevent electric signals along undesired paths. The choke is used as a filter in power supply to prevent ripple. It also prevents unwanted signals to enter other parts of the circuits, e.g. radio frequency choke (RFC) prevents radio frequency signals from entering audio frequency circuits. Thus, undesired signals and noise can be attenuated.

Example: 15.2

A pure inductive coil allows a current of 20A to flow from a 220 V, 50Hz supply. Find (1) inductive reactance (2) inductance of the coil (3) power absorbed. Write down the equation for voltage and current.

Solution:

Current $I = 20$ A, Voltage $V = 220$ V
Frequency $f = 50$ Hz

1. Circuit current, $I = V/X_L$

$\therefore$ Inductive reactance, $X_L = V/I = 220/20 = 11\ \Omega$

2. Now, $X_L = 2\pi f L$

$$\therefore L = \frac{X_L}{2\pi f} = \frac{11}{2\pi \times 50} = 0.035\text{H}$$

3. Power absorbed = Zero

$V_m = 220 \times \sqrt{2} = 311.1\text{V}$; $I_m = 20 \times \sqrt{2} = 28.28$ A ;
$\omega = 2\pi \times 50 = 314$ rad s^{-1}

Since in a pure inductive circuit, current lags behind the voltage by $\pi/2$ radians, the equations are:

$V = 311.1 \sin(314t)\text{V}$ $I = 28.28 \sin(314t - \pi/2)$ A

Example:15·3

The current through an 60 mH inductor is 0.2 sin ($377t - 25^\circ$)A. Write the mathematical expression for the voltage across it.

Solution:

Inductance $L = 60$ m H ,

current $I = 0.2 \sin(377t - 25^\circ)$A

Mathematical expression for the voltage $V = ?$

Inductive reactance, $X_L = 2\pi fL = 377 \times 60 \times 10^{-3} = 22.62\,\Omega$

maximum value of an alternating $V_m = I_m X_L = 0.2 \times 22.62 = 4.5\,\text{V}$

Since the voltage leads the current by 90^0 therefore 90^0 is added to the phase angle of voltage.

$V = V_m \sin(377t - 25^0 + 90^0)$ $V = 4.5 \sin(377t + 65^0)\,\text{V}$

15.10 A.C. Through Capacitance

Consider an alternating voltage applied to a capacitor of capacitance C as shown in Fig. When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electron move to and fro around the circuit, connecting the plates, thus constituting alternating current. The basic relation between the charge q on the capacitor and voltage V across its plates i.e. $q=CV$ holds at every instant. Let the applied alternating voltage is

$$V = V_m \sin \omega t \qquad \text{(i)}$$

Then, at any instant I be the current and q be the charge on the plates.

Charge on capacitor,

$$q = CV = C\, V_m \sin \omega t$$

$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta(CV_m \sin(\omega t))}{\Delta t}$$

by using maths formulae

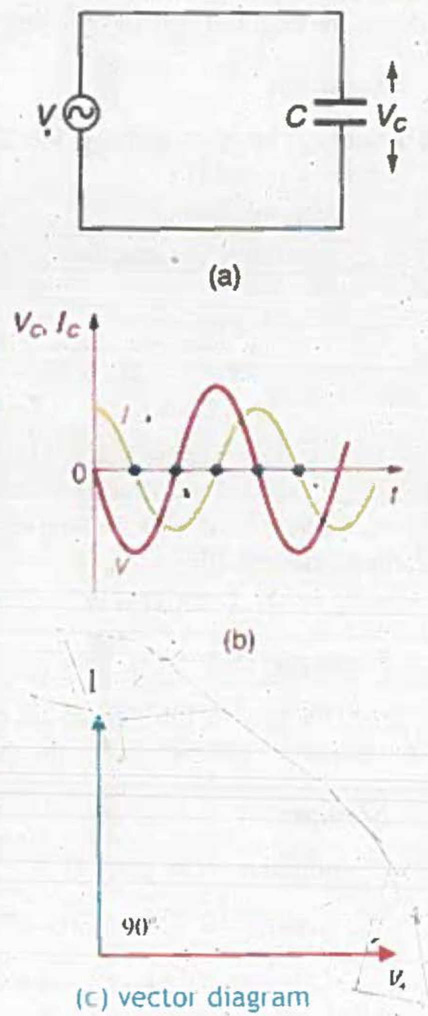
$$\therefore \Delta \sin(\omega t) = \omega \cos(\omega t)$$

$$= CV_m \omega \cos(\omega t)$$

$$= CV_m \omega \sin(\omega t + \frac{\pi}{2})$$

$$\therefore\ CV_m \omega = I_m \qquad \ldots(15.14)$$

$$I = I_m \sin(\omega t + \frac{\pi}{2}) \qquad \ldots(ii)$$



(c) vector diagram

Figure 15.16:Capacitor in AC circuit

Eqs. (i) and (ii) shows that current leads the voltage by $\pi/2$ radians or 90°. Hence in a pure capacitance, current leads the voltage by 90°. Capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across the capacitor.

This causes the voltage to lag behind the current. This is also illustrated in the phasor diagram shown in Fig:15.16(b): Like inductance which opposes the flow of A.C., capacitance also opposes the flow of AC current in the circuit. From the above:

$$I_m = \omega C\, V_m \qquad ...(15.15)$$

Or $$\frac{V_m}{I_m} = \frac{1}{C\omega}$$

Just like ohm law the ratio of V/I is the measure of opposition offered by a resistor to the flow of current. In case of capacitor this opposition is capacitive reactance which opposes the flow of current.

$$\frac{V_m}{I_m} = \frac{V_C}{I} = \frac{1}{C\omega} \qquad ...(15.16)$$

Clearly, the opposition offered by capacitance to current flow is $I / \omega C$. This quantity $I / \omega C$ is called the capacitive reactance X_c of the capacitor. It has the same dimensions as resistance and is, therefore, measured in Ω.

$$I = V_c / X_c$$

Where capacitive reactance is

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad ...(15.17)$$

The capacitive reactance depends upon frequency of A.C. In case of D.C., X_c has infinite value.

15.10.1 Power loss in a capacitive circuit

In pure capacitive circuit the current leads the voltage by 90^0 in phase therefore; the power curve for capacitor and inductor is same. The average power loss in capacitive circuit is,

$$P = \langle VI \rangle = \langle V_m \sin \omega t \times I_m \cos \omega t \rangle$$

$$= V_m I_m < \sin \omega t >< \cos \omega t >$$

$$= 0 \qquad \because < \sin \omega t >< \cos \omega t > = 0$$

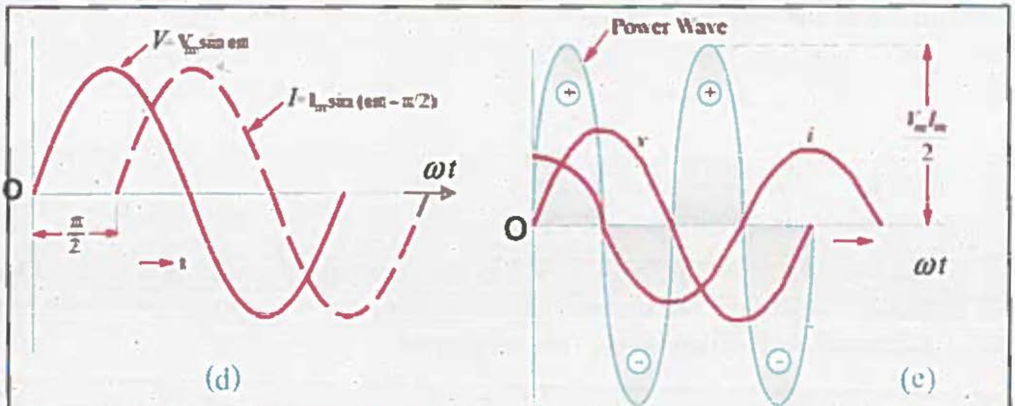


Figure 15.16:phase diagram for current and voltage and power curve for pure capacitive circuit

This fact is also illustrated in the wave diagram shown in Fig:15.16: Which shows that in one cycle the positive power is equal to negative power so power absorbed by capacitor in one cycle is zero.

Example 15.4

A318μF capacitor is connected across a 220V, 50Hz system. Determine (a) the capacitive reactance (b) RMS value of current and (c) equations for voltage and current.

Solution:

Capacitance, C= 318μF= 318×10^{-6} F

Voltage V=220 V

Frequency f = 50 Hz

(a) Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times318\times10^{-6}} = 10.26\Omega$

(b) RMS value of current, $I = V / Xc = 220/10.26 = 21.44A$

(c) maximum value of an alternating voltage & current is

$V_m = 220\times\sqrt{2} = 311.1V,$

$I_m = I\times\sqrt{2} = \sqrt{2}\times21.44 = 30.32A,$

Frequency $\omega = 2\pi\times50 = 314Hz$

∴Equations for voltage and current are:

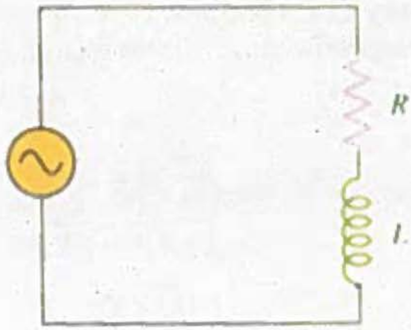
$$V = V_m \sin(\omega t)$$

& $$I = I_m \sin(\omega t + \pi/2)$$

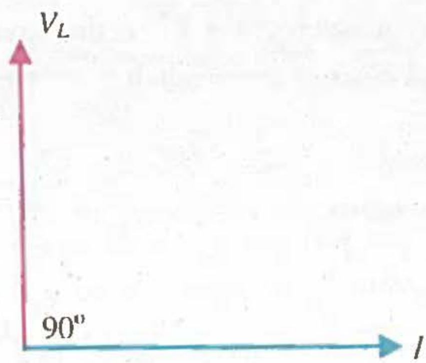
putting values

$$V = 311.1 \sin 314t,$$

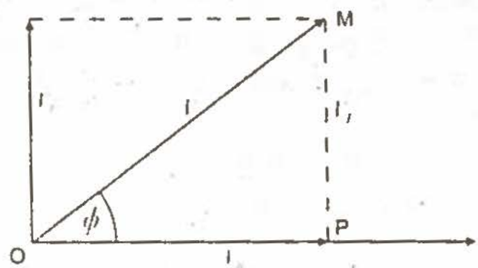
$$I = 30.32 \sin(314t + \pi/2),$$



(a) : RL series circuit



(b) phasor diagram



(c):phasor diagram of RL series circuit

Figure 15.17

15.11 R.L Series A.C. Circuit

In an AC circuit that contains inductance, L and resistance, R the voltage, V will be the phasor sum of the two component voltages, V_R and V_L. This means that the current flowing through the coil will still lag the voltage, but by an amount less than 90° depending upon the values of V_R and V_L. The new phase angle between the voltage and the current is known as the phase angle ϕ of the circuit. V is the rms value of the applied voltage, I is the r.m.s. value of the circuit current and $V_L = I X_L$. Fig15.17 (a): shows a pure resistance R connected in series with a coil of pure inductance L.

Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig.15.17(c).

The voltage drop V_R (= I R) is in phase with current and is represented in magnitude and direction by the phasor OP.
The voltage drop V_L (= I X_L) leads the current by 90^o and is represented in magnitude and direction by the phasor *PM*.
The applied voltage V is the phasor sum of these two drops *i.e.*

$$V^2=V_R^2+V_L^2 \qquad ...(15.18)$$

or

$$V=\sqrt{V_R^2+V_L^2}$$

$$=\sqrt{(I\,R)^2+(I\,X_L)^2}$$

$$=I\sqrt{R^2+X_L^2} \qquad ...(15.19)$$

$$or \quad I=\frac{V}{\sqrt{R^2+X_L^2}}$$

The quantity $\sqrt{R^2+X_L^2}$ is the opposition offered to current flow and is called impedance of the circuit. It is represented by Z and is measured in ohms (Ω)

$$I = V/Z$$

where $$Z=\sqrt{R^2+X_L^2} \qquad ...(15.20)$$

The phasor diagram shows that circuit current I lags behind the applied voltage V by ϕ^{O}. This fact is also illustrated in the wave diagram shown in Fig.15.17(b) The value of phase angle ϕ can be determined from the phasor diagram.

$$\tan\phi=\frac{V_L}{V_R}=\frac{IX_L}{IR}=\frac{X_L}{R} \qquad ...(15.21)$$

Since X_L and R are known, ϕ can be calculated. If the applied voltage is $V=V_m\sin\omega t$, then equation for the circuit current will be:

$$I=I_m\sin(\omega t-\phi)$$

where $I_m = V_m/Z$

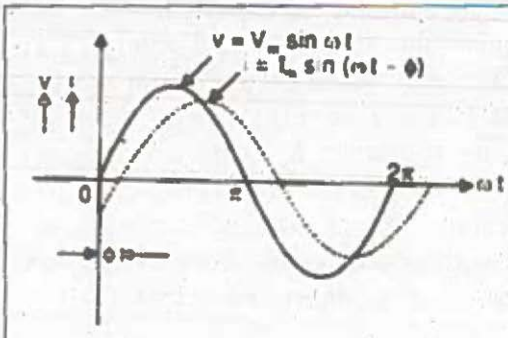


Figure 15.17 (d) :wave diagram

Fig15.17(d): shows that in an inductive circuit current lags behind the applied

voltage. The angle (i.e. ϕ) of lagging is greater than 0° but less than 90°. It is determined by the ratio of inductive reactance to resistance ($\tan\phi = X_L / R$) in the circuit.

The greater the value of this ratio, the greater will be the phase angle ϕ .

15.11.1 Power in RL circuit

$$\therefore \quad Average\ Power, P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$$

or $$P = V\,I\cos\phi \qquad ...(15.22)$$

Where V and I are the r.m.s. values of voltage and current. The term $\cos\phi$ is called power factor of the circuit and its value is given by (from phasor diagram):

Power factor, $$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \qquad ...(15.23)$$

Or Power factor = $\cos\phi$ cosin of angle between V and I.

Or $$P = VI\cos\phi = (IZ)\,I\,(R/Z) = I^2 R \quad [\because \cos\phi = R/Z \text{ and } V = IZ]$$

In a resistor, the current and voltage are in phase i.e. $\phi = 0^\circ$.

Therefore, power factor of a pure resistive circuit is $\cos 0^\circ = 1$. Similarly, phase difference between voltage and current in a pure inductance or capacitance is 90°. Hence power factor of pure L or C is zero.

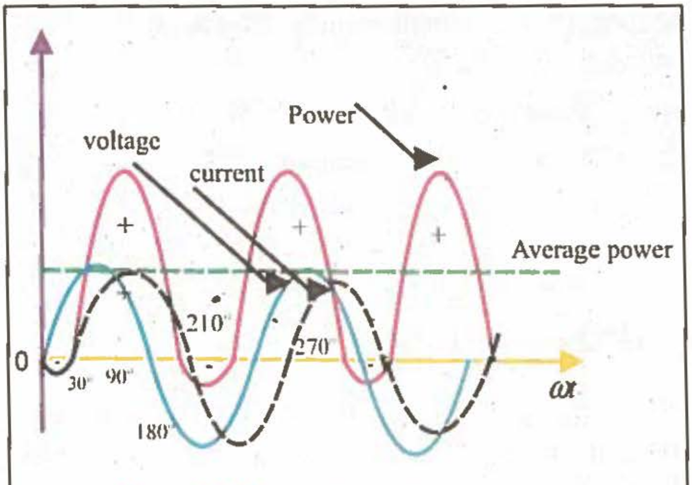


Figure 15.18 : power curve

This is the reason that power consumed by pure L or C is zero. For a circuit having R, L and C in varying proportions, the value of power factor will lie between 0 and 1.

Fig:15.18 shows that power is negative between 0^0 and 30^0 and between 180^0 and 210^0. The negative area means that the inductance of the circuit returns the power

to the source. Conversely power is positive between 30^0 and 180^0 and so on. But as the area of positive curve is greater than negative area of curve. So net power over one cycle is positive. This shows that power is consumed in R-L series circuit.

15.12 R-L Series Impedance Triangle

In a DC circuit, the ratio of voltage to current is called resistance. However, in an AC circuit this ratio is known as Impedance, Z. Impedance is the total resistance to current flow in an "AC circuit" containing both resistance and inductive reactance. In R.L. series circuit.

$$Z = \sqrt{R^2 + X_L^2} \quad \text{where } X_L = 2\pi f L$$

The magnitude of impedance in R.L series circuit depends upon the values of R.L and the supply frequency f. The R-L series circuit is shown in Fig15.17(a). The phasor diagram is a triangle whose sides represent R, X_L and Z. This triangle is called an "Impedance Triangle".

Impendence triangle is a useful concept in A.C. circuits as it enables us to calculate:

1. Power angle ϕ i.e. $\cos\phi = R/Z$
2. Phase angle ϕ i.e. $\tan\phi = X_L/R$

Therefore, it is always useful to draw the impedance triangle while analyzing an a.c. circuit.

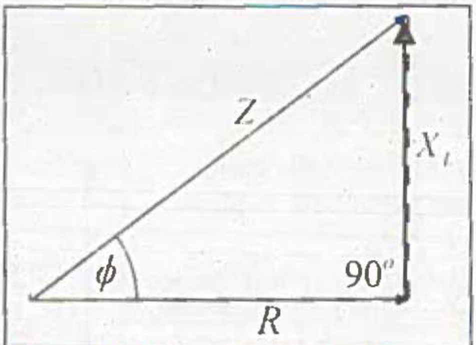


Figure 15.19: impedance triangle

15.12.1 Q-factor

The quality factor or Q-factor of a component is its energy storing ability. The Q-factor of a circuit is a ratio of energy stored in the circuit to the energy dissipated in one cycle.

The ratio of the inductive reactance (X_L) of a coil to its resistance (R) at a given frequency is known as Q-factor of the coil at that frequency i.e.,

$$Q - \text{factor} = \frac{X_L}{R} = \frac{\omega L}{R} \qquad ...(15.24)$$

Also, $$Q-\text{factor}=2\pi\times\frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

The Q-factor is used to describe the quality or effectiveness of a coil. A coil is usually designed to have high value of L compared to its resistance R. The greater the value of Q-factor of a coil, the greater is its inductance (L) as compared to its resistance (R).

Example:15.5

A coil having a resistance of 7Ω and an inductance of 31.8 mH is connected to 220V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor and (d) power consumed.

Solution.

Inductance, $L = 31.8$ m H= 31.8×10^{-3}H
Voltage V=220 V
coil resistance R=7Ω
Frequency f = 50 Hz

(a) Inductive reactance, $X_L = 2\pi f L = 2\pi\times50\times31.8\times10^{-3} = 10\Omega$

Coil impedance, $Z=\sqrt{R^2+X_L^2}=\sqrt{7^2+10^2}=12.2\Omega$

Circuit current, $I = V/Z$

$=220/12.2=18.03\text{A}$

(b) $\tan\phi = X_L/R = 10/7$

∴ Phase angle, $\phi=\tan^{-1}(10/7)=55^0$ lag

(c) Power factor $=\cos\phi=\cos55^0=0.573$ lag

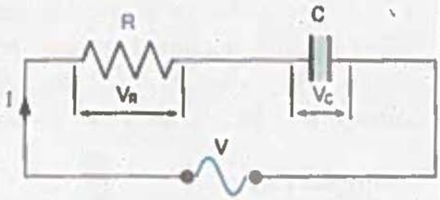
(d) Power consumed, $P = VI\cos\phi$

$=220\times18.03\times0.573=2272.8$ W

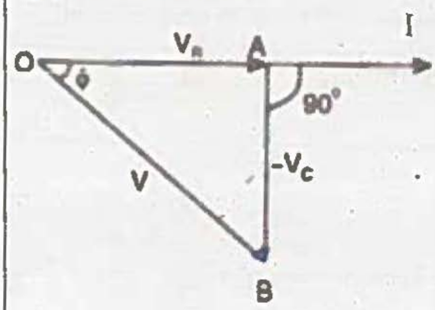
15.13 R.C Series A.C. Circuit

Consider AC circuit that contains both capacitance, C and resistance, R which are connected in series with each other as shown in fig. 15.20(a). The voltage, V across the combination is equal to the phasor sum of two component voltages, V_R and V_C. Where $V_R = IR$ and, $V_C = IX_C$.

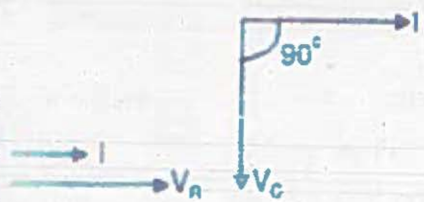
In order to draw a vector diagram for a capacitance a reference must be found. In a series AC circuit the current is common and can therefore be used as the reference source because the same current flows through the resistance and capacitance. The individual vector diagrams for a pure resistance and a pure capacitance is shown in fig15.20.



(a) RC series circuit



(b) Impedance triangle



(C) phasor diagram

Figure 15.20:

The voltage drop V_R (= IR) is in phase with current and is represented by the phasor OA. The voltage drop V_C (= IX_C) lags behind the current by 90^0 and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drop i.e.

$$V^2 = V_R^2 + V_C^2 \qquad (i)$$

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} \qquad \ldots(15.25)$$

The quantity $\sqrt{R^2 + X_C^2}$ offer opposition to current flow and is called impedance of the circuit. $I = V/Z$

where

$$Z = \sqrt{R^2 + X_C^2} \qquad \ldots(15.26)$$

The phasor diagram shows that circuit current I leads the applied voltage $\dot{V}$ by ϕ. this fact is also illustrated in the wave diagram and impedance triangle (as shown in Fig15.21(b)) of the circuit.

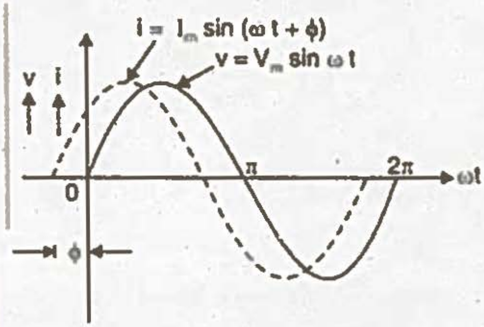
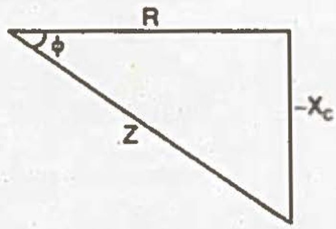


Figure 15.21(a): phasor diagram



(b) impedance triangle

The value of the phase can be determined as under:

$$\tan\phi = -\frac{V_C}{V_R}$$

$$= -\frac{IX_C}{IR} = -\frac{X_C}{R} \qquad \ldots(15.27)$$

15.14 Power in R.C. circuit

The equation for voltage and current are:

$$V = V_m \sin\omega t \quad ; \quad I_m \sin(\omega t + \phi)$$

$\therefore$ Average power, $\langle P\rangle = \langle V\rangle\langle I\rangle$

$$= VI\cos\phi$$

Example 15.6

A 100V, 50Hz a.c. supply is applied to a capacitor of capacitance 79.5 μ F connected in series with a non-inductive resistance of 30 Ω. Find (1) impedance (2) current (3) phase angle and (4) equation for the instantaneous value of current.

Solution:

Voltage V=100 V
resistance R=30Ω
Frequency f = 50 Hz

, capacitance C = 79.5 μ F

(1) Capacitive reactance, $X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 79.5} = 40\Omega$

Circuit impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50\Omega$

(2) Circuit current, $I = V/Z = 100/50 = 2$ A

(3) $\tan\phi = Xc/R = 40/30 = 1.33$

∴Phase angle, $\phi = \tan^{-1} 1.33 = 53^0 lead$

$$I_m = I \times \sqrt{2} \Rightarrow I_m = 2 \times \sqrt{2} = 2.828\,\text{A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314\,\text{rad s}^{-1}$$

(4) equation for current is $I = 2.828 \sin(314t + 53^0)$

15.15 R-L-C Series A.C Circuit

Many AC circuits are very useful for us, which include resistance, inductive reactance and capacitive reactance. In this section, we will look at some implications of connecting a resistor (*R*), an inductor (*L*), and a capacitor (*C*) together in what is called a series RLC circuit. The simplest and most important AC circuit we can analyze is the series LRC circuit, illustrated in Fig15.22 (a).

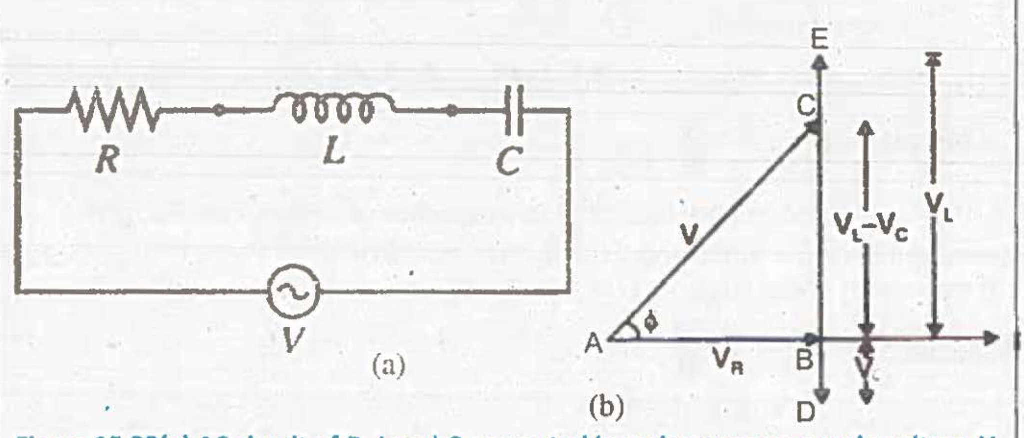


Figure 15.22(a) AC circuit of *R, L* and *C* connected in series across a supply voltage *V* (*r.m.s.*). The resulting circuit current is *I* (*r.m.s.*). (b) phasor sum

The analysis of this circuit is quite easy since all the circuit elements share the same current. We can draw a phasor diagram for the current and voltages across the inductor, capacitor, and resistor. The P.D. across R, is $V_R = IR$ in this case V_R is in phase with I. The P.D. across L, is $V_L = IX_L$ in this case V_L leads I by 90^0. The P.D. across C, is $V_C = IX_C$ in this case where V_C lags I by 90^0. V_L and V_C are thus 180^0 out of phase. In phasor diagram (Fig15.22 (b)), AB represents V_R. BE represents V_L and BD represents V_C. It may be seen that V_L is in phase opposition to V_C.

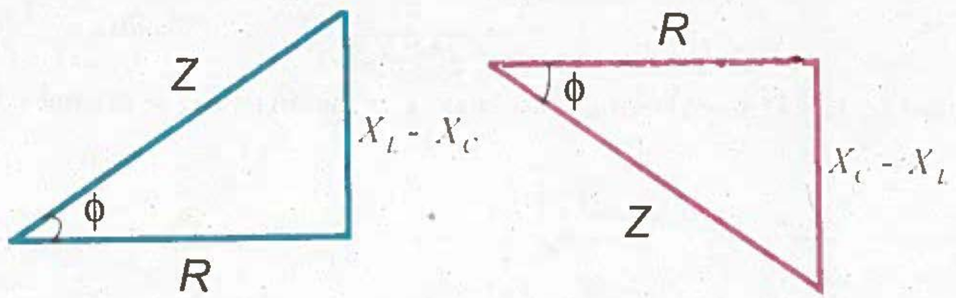


Figure 15.22 (c), (d) : impedance triangle

It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. If $V_L > V_C$ then the net voltage drop across L-C combination is $V_L - V_C$ and their resultant is in the direction V_L represented by BC. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and represented by AC.

$$V^2 = V_R^2 + (V_L - V_C)^2 \quad \text{...(15.28)}$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$= IZ$$

Where $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$ ...(15.29)

The quantity $(X_L - X_C)$ is called the reactance of the circuit, denoted by X

$$X^2 = (X_L - X_C)^2$$

Finally we can write

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X^2}$ is the opposition offered to current flow and is called impedance of the circuit.

$$\text{Circuit power factor, } \cos\phi = \frac{R}{Z}$$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad (15.30)$$

Since X_L, X_C and R, are known, phase angle ϕ of the circuit can be determined.

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$= \frac{X_L - X_C}{R}$$

$$= \frac{X}{R} \qquad (15.31)$$

So if the current is represented by a cosine function, $I = I_m \cos \omega t$

The source voltage leads the current by an angle and its equation is

$$I = I_m \cos(\omega t + \phi)$$

Power consumed, $P = V I \cos\phi$

We have seen that the impedance of a R-L-C series circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(i) When $X_L - X_C$ is positive (*i.e.* $X_L > X_C$), phase angle ϕ is positive and the circuit will be inductive. In other words, in such a case, the circuit current I will lag behind the applied voltage V by ϕ .

(ii) When $X_L - X_C$ is negative (*i.e.* $X_C > X_L$), phase angle ϕ is negative and the circuit is capacitive. That is to say the circuit current I leads the applied voltage V by ϕ ; the value of ϕ being given by Eq.(15.31) above.

(iii) When $X_L - X_C = 0$ (i.e. $X_L = X_C$), the circuit is purely resistive. In other words, circuit current I and applied voltage V will be in phase i.e. $\phi = 0^o$ the circuit will then have unity power factor.

If the equation for the applied voltage is $V = V_m \sin \omega t$, then equation for the circuit current will be

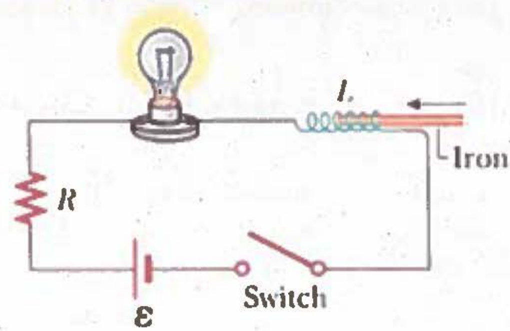
$$I = I_m \operatorname{Sin}(\omega t \pm \phi)$$

where $I_m = V_m / Z$

The value of ϕ will be positive or negative depending upon which reactant (X_L or X_C) predominates.

Fig.15.22(*c*) shows the impedance triangle of the circuit for the case when $X_L > X_C$ whereas impendence triangle in Fig.(*d*) is for the case when $X_C > X_L$.

Quiz?



The switch in the circuit shown in Figure is closed and the light bulb glows steadily. The inductor is a simple air-core solenoid. As an iron rod is being inserted into the interior of the solenoid, the brightness of the light bulb (a) increases, (b) decreases, or (c) remains the same.

Example15.7

A 220V, 50Hz A.C. supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 μ F capacitor. Calculate

(a) impedance (b) current (c) phase angle between current and voltage (d) power factor and (e) power consumed.

Solution

$$X_L = 2\pi f L = 2\pi \times 50 \times .06 = 18.85\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\,\Omega$$

(a) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(2.5)^2 + (18.85 - 468)^2} = 449.2\Omega$$

(b) Circuit Current, $I = V/Z = 220/449.2 = 0.4897\,A$

(c) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{18.85 - 468}{2.5} = -179.66$

$\therefore$ Phase angle, $\phi = \tan^{-1}(-179.66) = -89.7^\circ$

The negative sign with ϕ shows that current is leading the voltage

(d) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{2.5}{449.2} = 0.00557$

(e) Power consumed, $P = VI\cos\phi = 220 \times 0.4897 \times 0.00557 = 0.60007$w

15.16 Resonance in A.C Circuits

In the impedance equation along with the equations for the inductive and capacitive reactance, we see that impedance has a rather complicated dependence on the frequency of the oscillator.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}$$

When the frequency is very small, the capacitive reactance is large and $X_C = Z$. When the frequency is very large, the inductive reactance is large and $X_L = Z$. Z is a minimum when $X_L = X_C$, and Z is a minimum, the current in the circuit is a maximum. When this happens, the resistance provides the only

impedance in the circuit, $Z=R$. This condition is called resonance and is electrical analog to resonance in harmonic oscillators such as a swinging pendulum or a mass on the end of a spring.

Resonance means to be in step with. When applied voltage and circuit current in an A.C. circuit are in step with (i.e. phase angle is zero or power factor is unity), the circuit is said to be in electrical resonance. If this condition exists in a series a.c. circuit, it is called series resonance. The frequency at which resonance occurs is called resonant frequency (f_r).

An A.C. circuit containing reactive elements (L and C) is said to be in resonance when the circuit power factor is unity.

15.16.1 Resonance in R-L-C Series Circuits

R-L-C series circuit is said to be in resonance when the circuit power factor is unity i.e. $X_L = X_C$. The frequency f_r at which it occurs is called resonant frequency. The resonance (i.e. $X_L = X_C$) in an R-L-C series circuit can be achieved by changing the supply frequency because X_L and X_C are frequency dependent. At a certain frequency f_r, X_L becomes equal to X_C and resonance takes place.

At series resonance, $$X_L = X_C \qquad \text{...(15.32)}$$

Or $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$\therefore$ Resonant frequency, $$f_r = \frac{1}{2\pi\sqrt{LC}} \qquad \text{...(15.33)}$$

From Eq. (15.33), it is clear that on increasing either the inductance or the capacitance causes the resonant frequency to decrease. For a given value of inductance and capacitance, there is only one resonant frequency.

There are an infinite number of inductor and capacitor combinations for any specified resonant frequency.

Resonance Curve: The curve between current and frequency is known as resonance curve of a typical *R-L-C* series circuit. Current reaches its maximum value at the resonant frequency (f_r), falling off rapidly on either side at that point.

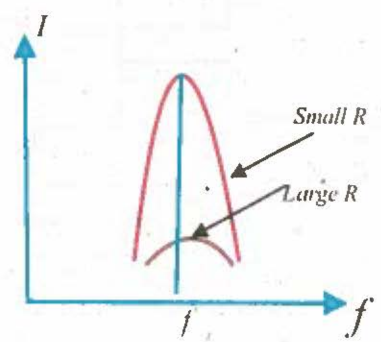


Figure 15.23: resonance curve

It is because if the frequency is below f_r, $X_C>X_L$ and the net reactance is no longer zero. If the frequency is above f_r, then $X_L> X_C$ and the net reactance is again not zero. In both cases, the circuit impedance will be more than the impedance (Z_r= R) at resonance. The result is that the magnitude of circuit current decreases rapidly as the frequency changes from the resonant frequency.

The Q-factor of a series circuit circle indicates how many times the P.D. across L or C is greater than the applied voltage at resonance. For example, when R-L-C series circuit is connected to a 220V source having a Q-factor of the coil as 20, then voltage across the coil or capacitor will be

$$V_C = V_L= QV_R = 20 \times 220 = 4400 \text{ V at resonance.}$$

15.17 Principle of metal detectors

A coil and capacitor are electrical components, which together can produce oscillations of current. An L-C circuit behaves just like an oscillating mass-spring system. In this case energy oscillates between a capacitor and an inductor. The circuit is called an electrical oscillator. Two such oscillators A and B are used for the operation of common type of metal detector (Fig 15.24). In the absence of any nearby metal object, the inductances L_A and L_B are the same and hence the resonance frequency of the two circuits is also same.

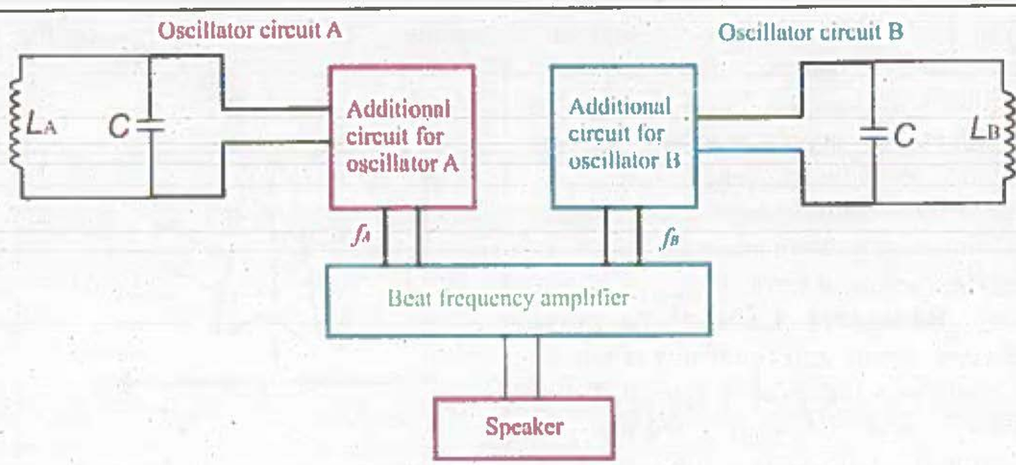


Figure 15.24 : metal detectors

When inductor B, called the search coil comes near a metal object the inductance L_B decreases and corresponding oscillator frequency increases and thus a beat note is heard in the attached speaker. Such detectors are extensively used not only for various security checks but also to locate buried metal objects.

15.18 Maximum power transfer

The maximum power-transfer theorem says that to transfer the maximum amount of power from a source to a load, the load impedance should match the source impedance.

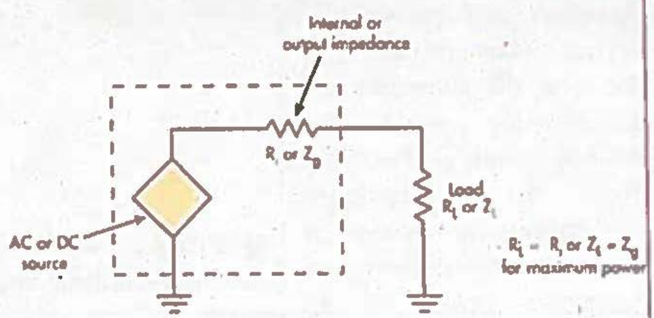


Figure 15.25(a): Maximum power is transferred from a source to a load when the load resistance equals the internal resistance of the source.

In the basic circuit, a source may be AC or DC, and its internal resistance (R_i) or generator output impedance (Z_g) drives a load resistance (R_L) or impedance (Z_L) (Fig15.25(a):

$$R_L = R_i$$

Or

$$Z_L = Z_g \qquad \ldots(15.34)$$

A plot of load power versus load resistance reveals that matching load and source impedances will achieve maximum power (fig15.25(b)).

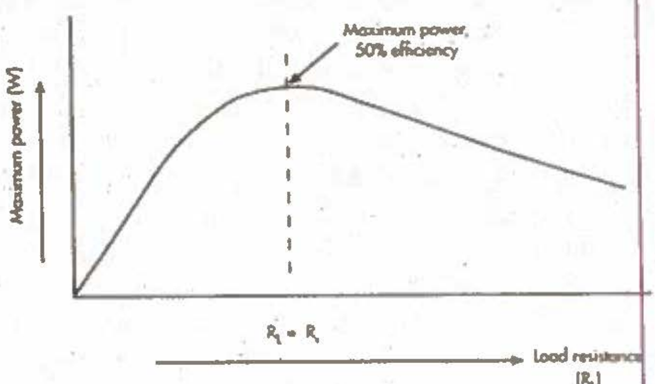


Fig 15.25(b). Varying the load resistance on a source shows that maximum power to the load is achieved by matching load and source impedances. At this time, efficiency is 50%.

A key factor of this theorem is that when the load matches the source, the amount of power delivered to the load is the same as the power dissipated in the source. Therefore, transfer of maximum power is only 50% efficient.

The source must be able to dissipate this power. To deliver maximum power to the load, the generator has to develop twice the desired output power. One of the important applications of maximum power is the delivery of maximum power to an antenna (fig15.25(c)).

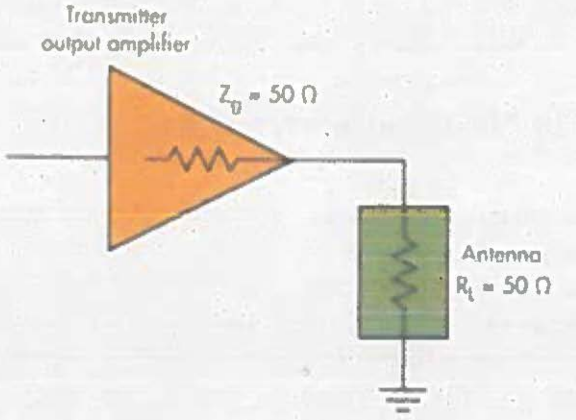


Fig 15.25(c) . Antenna impedance must equal the transmitter output impedance to receive maximum power.

15.19 MAXWELLS EQUATIONS

In the early days of 19th century, two different units of electric charge were used, one for electrostatics and the other for magnetic phenomena involving currents. These two units of charge had different physical dimensions. Their ratio has units of velocity and measurements showed that the ratio had a numerical value that was precisely equal to the speed of light. It was regarded as an extraordinary coincidence that had no explanation. Maxwell in a search for an explanation of the coincidence found that all the basic principles of electromagnetism can be formulated in terms of four fundamental equations, now called Maxwell's equations. These equations exists as experimental laws in the form of Gauss' law, Faraday's law and Ampere's law. These equations predict the existence of electromagnetic waves and that such waves are radiated by accelerating charges. For simplicity, we present Maxwell's equations as applied to free space. We know that changing magnetic flux density B through a certain region of space produces an induced emf in the region.

So an induced current will flow in a closed loop of wire in the region as shown in Fig(15.26(a)

According to Faraday's law the induced emf or the induced potential difference V is given by

$$\mathcal{E} = \frac{\Delta\Phi}{\Delta t}$$

$$= \frac{\Delta}{\Delta t}(B.A) \qquad ...(15.34)$$

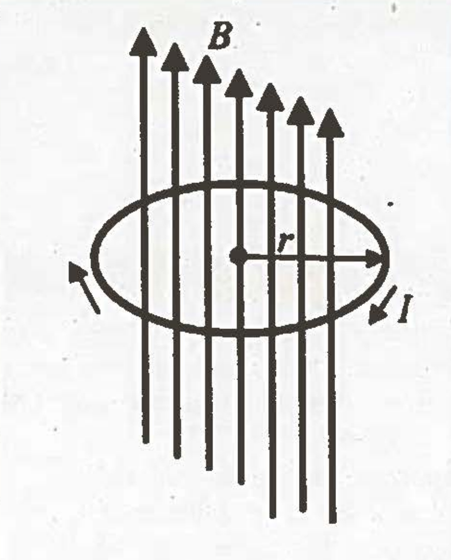


Figure 15.26 :changing magnetic field

As potential difference is due to an electric field, it means an electric field will be generated at each point of the loop. By symmetry E is circular in direction and constant in magnitude E at each point of the loop. If a unit positive charge is circulated once round the circular loop of radius r, the work done will be

$$W = 2\pi r F_e = 2\pi r(qE)$$

$$\mathcal{E} = \frac{W}{q} = 2\pi r E \qquad ...(15.35)$$

By definition W will equal to the emf or V in the loop:

$$\mathcal{E} = \frac{\Delta\Phi}{\Delta t} = 2\pi r E \qquad ...(15.36)$$

$$E = \frac{1}{2\pi r}\frac{\Delta\Phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t} \qquad ...(15.37)$$

This equation shows that a changing magnetic flux gives rise to an electric field. Experiments have shown that the electric field produced by changing magnetic field is present even if the conducting loop is absent.

(b): Analogues to changing magnetic flux it is found that a changing electric flux gives rise to a magnetic field. In order to arrive at this statement, let a capacitor be connected to a battery as arranged in Fig. (15.27). Current starts growing in the circuit but very quickly decreases to zero when the capacitor is fully charged. An electric field is established between the plates of the capacitor.

The charge Q on an air filled capacitor of capacitance C, plate area A, plate separation d and potential difference V is given by

$$Q = CV = \frac{\varepsilon_o A}{d} V = \varepsilon_o AE \qquad ...(15.38)$$

$$\therefore \frac{V}{d} = E$$

In Fig (15.27) the current through the dielectric of the capacitor is due to changing electric field.

Suppose the capacitor is now connected to an alternating emf source. It is observed that the current flows continuously in the circuit. It does not stop. Why is to so different than the case of the battery! The reason is that outside the capacitor the current in the wires is due to the conduction electrons but what is the entity that drives that current in the dielectric between the plates of the capacitor. We note that in the present case the E-Field between the plates of the capacitor changes with time. There exists $\Delta E/\Delta t$. It was first conceived by Maxwell that the change in the electric field is the cause of current in the capacitor. From Eq. (15.38'), we have

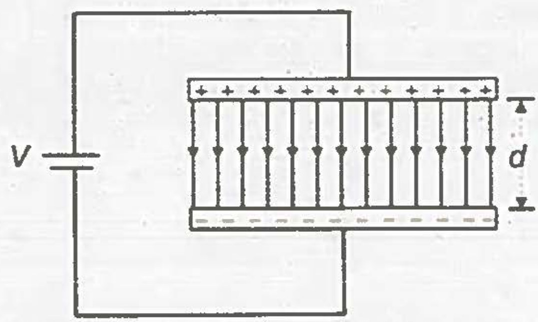


Figure 15.27 Uniform electric field between a pair of charged plate.

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta(\varepsilon_o AE)}{\Delta t} = \frac{\varepsilon_o \Delta(AE)}{\Delta t} \qquad ...(15.39)$$

$$I = \frac{\varepsilon_o \Delta(\Phi)}{\Delta t} \qquad ...(15.40)$$

Where I is the current and Φ is the electric flux through the area A. Eq. (15.40) shows that a changing electric flux is equivalent to a current. This type of current, which is due to changing electric flux is called displacement current.

We must extend our concept about current. A current can arise due to the flow of charges and also due to changing electric flux. The former is called conduction

current and the later is called displacement current. According to Ampere's law, each type of current produces magnetic field around itself. We have thus shown that a changing E –field creates a B-field.

(c). A changing E-field creates a B-field, which in turn creates an E-field, an electromagnetic disturbance or waves are generated. The fundamental requirement for generation of electromagnetic waves is an electric charge with changing velocity (acceleration) since it will create changing electric flux. The velocity of an oscillating charge as it moves to and fro along a wire is always changing.

(d). Light is a type of electromagnetic waves. Maxwell predicted theoretically that the velocity of electromagnetic waves in free space is given by

$$c=\frac{1}{\sqrt{\varepsilon_o\mu_o}} \qquad ...(15.41)$$

Where $\varepsilon_o = 8.85\times10^{-12}C^2\ N^{-1}m^{-2}$ is the permitivity of free space and

$\mu_o = 4\pi\times10^{-7}Wb\ A^{-1}m^{-1}$ is the permeability of free space.

Put these values in the above relation we find that the velocity of lights is given by

$$c=\frac{1}{\sqrt{8.85\times10^{-12}\times4\pi\times10^{-7}}}=3\times10^{8}m\,s^{-1}$$

Faraday's law shows that a changing magnetic field gives rise to an electric field. Ampere-Maxwell law shows that a changing electric field gives rise to a magnetic field. It follows that when either electric or magnetic field varies with time, the other field is induced in space. The net effect is that an electromagnetic disturbance is generated due to changing electric and magnetic fields. The disturbance propagates in the form of an electromagnetic wave.

15.20 ELECTROMAGNETIC WAVE

Electromagnetic radiations such as infrared ,ultravoilet,etc are different from each other due to their properties. But they have some features in common such as electric field and magnetic field. Therefore it can be described in terms of electric and magnetic fields, and they all travel through vacuum with the same speed (the speed of light).

Fundamentally these radiations are different only in wavelength or frequency. The names given to them in Fig. 15.28 shows various regions of the spectrum along

with given names. There are no gaps in the spectrum, nor their sharp boundaries between the various categories. (Certain regions of the spectrum are assigned by law for commercial or other uses, such as TV. AM. or FM broadcasting).

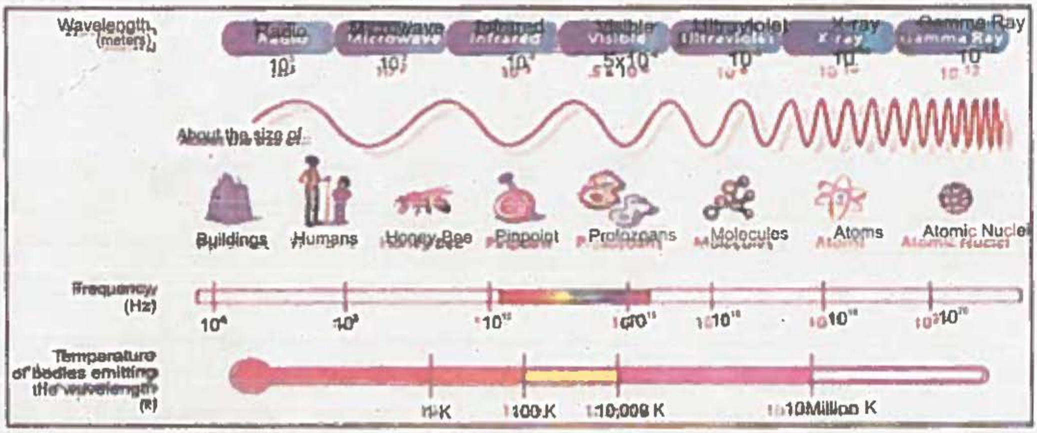


Figure 15.28(a) : electromagnetic spectrum

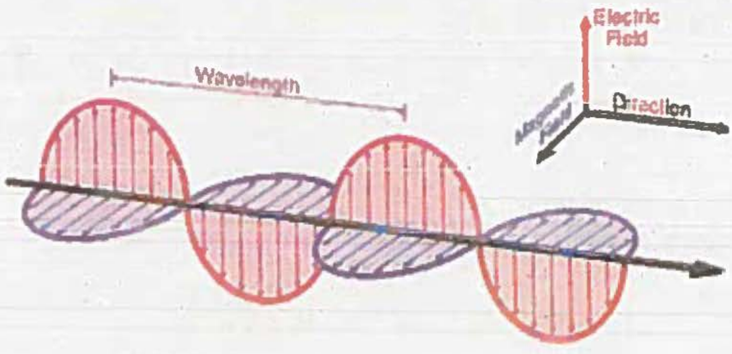


Figure 15.28(b):EM waves

Let us consider some of these types of electromagnetic radiation in more details.

1. **Light.** The visible region of the spectrum, most familiar to us, is the electromagnetic radiation emitted by the Sun. The wavelength of the visible region ranges from about 400 nm (violet) to about 700nm (red).

Light is often emitted when the outer (or valence) electrons in atoms change their state of motion: for this reason, such transitions in the state of the electron are called optical transitions. The color of the light tells us something about the atoms or the object from which it was emitted. The study of the light emitted from the Sun and from distant stars gives infomation about their composition.

2. Infrared. Infrared radiation, which has wavelength stonger than the visible (from 0.7 μm to about 1 mm), is commonly emitted by atoms or molecules when they change their rotational or vibrational motion. Infrared radiation is an important means of heat transfer and is sometimes called heat radiation. The warmth you feel when you place your hand near a glowing light bulb is primarily a result of the infrared radiation emitted from the bulb.
All objects emit electromagnetic radiation (called "thermal radiation;") because of their temperature. Objects of temperatures ranges from , 3 K to 3000 K emit their most intense thermal radiation in the infrared region of the spectrum.

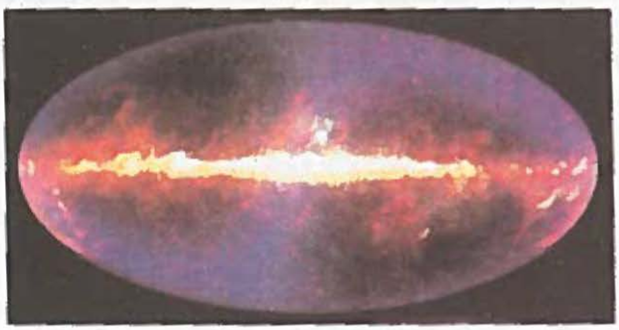


Figure 15.29: infrared image of Milky Way galaxy

Figure 15.30: Microwaves relay station

A **remote control** is a component of an electronics device, most commonly a television set, DVD player and home theater systems originally used for operating the device wirelessly from a short line-of-sight distance. The main technology used in home remote controls is infrared light. The signal between a

remote control handset and the device it controls consists of pulses of infrared light, which is invisible to the human eye. Infrared radiation is also used for cooking the surface of food (the interior is then heated by convection and conduction).

3. Microwaves. Microwaves can be regarded as short radio waves, with typical wavelengths in the range 1 mm to 1 m. They are commonly produced by electromagnetic oscillators in electric circuits, as in the case of microwave ovens. Microwaves are often used to transmit telephone conversations; Fig. 15.31 shows a Microwaves station that serves to relay telephone calls. Microwaves also reach us from extraterrestrial sources.

Figure 15.31: remote

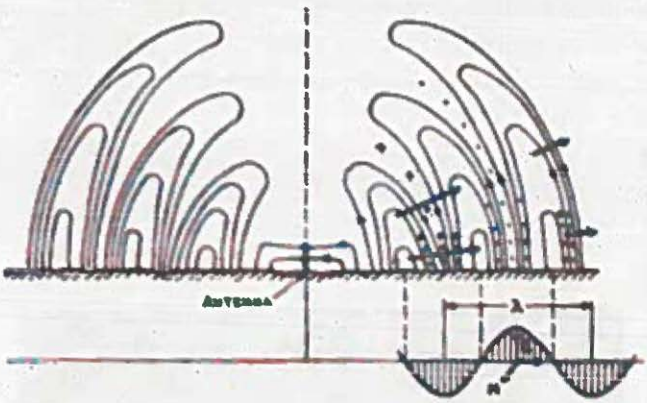


Figure 15.32: radio waves propagation

Neutral hydrogen atoms, which populate the regions between the stars in our galaxy, are common extraterrestrial source of Microwaves emitting radiation with a wavelength of 21 cm.

4. Radio waves. Radio waves have wavelengths longer than 1 m. They are produced from terrestrial sources through electrons oscillating in wires of electric circuits. By carefully choosing the geometry of these circuits, as in an antenna,we can control the distribution in space of the emitted radiation (if the antenna acts as a transmitter) or the sensitivity of the detector (if the antenna acts as a receiver. Traveling outward at the speed of light, the expanding of TV signals transmitted on Earth.

Radio waves reach us from extraterrestrial sources, the sun being a major source that often interferes with radio or TV reception on Earth. Mapping the radio emissions from exraterrestrial sources, known as radio astronomy, has provided infomation about the universe that is often not obtainable using optical telescopes.

5. Ultraviolet. The radiations of wavelengths shorter than the visible begin with the ultraviolet (1 nm to 400nm), which can be produced in atomic transitions of the outer- electrons as well as in radiation from thermal sources such as the Sun. Because our atmosphere absorbs strongly at ultraviolet wavelengths, little of this radiation from the Sun reaches the ground. However, the principal agent of this absorption is atmospheric ozone, which has been depleted in recent years as a result of chemical reactions with fluorocarbons released from aerosol sprays, refrigeration equipment, and other sources. Brief exposure to ultraviolet radiation causes common sun burn but long-term exposure can lead to more serious effects, including skin cancer.

Ultraviolet Lamp,(UV Light).A lamp producing Ultraviolet (UV) radiation is emitted through clear, pre-filtered, particle free water. This UV light is extremely effective in killing and eliminating bacteria, yeast's, viruses, molds and other harmful organisms known to man. Used in industry and hospitals to treat water. Many times used as a post disinfecting method for residential water treatment.

Figure 15.33: ultraviolet lamp

6. X-rays. X rays (typical wavelengths 0.01 nm to 10 nm) can be produced with discrete wavelengths in individual transitions among the inner (most tightly bound) electrons of an atom, and they can also be produced when charged particles (such as electrons) are decelerated. X rays can easily penetrate soft tissue but are stopped by bone and other solid matter: for this reason they have found wide use in medical diagnosis.

7. Gamma rays. Gamma rays are electromagnetic radiations with the shortest wavelengths (less than 10 pm).

They are the most penetrating of electromagnetic radiations, and exposure to intense gamma radiation can have a harmful effect on the human body. These radiations can be emitted in transitions of an atomic nucleus from one state to another and can also occur in the decays of certain elementary particles: for example, a neutral pion can decay into two gamma rays according to

$\pi^{\circ} \longrightarrow \gamma+\gamma$

15.21 Electrocardiogram (E.C.G)

The electrocardiogram or ECG is worldwide used for diagnosing heart conditions. An electrocardiogram is a recording of the small electric waves being generated during heart activity.

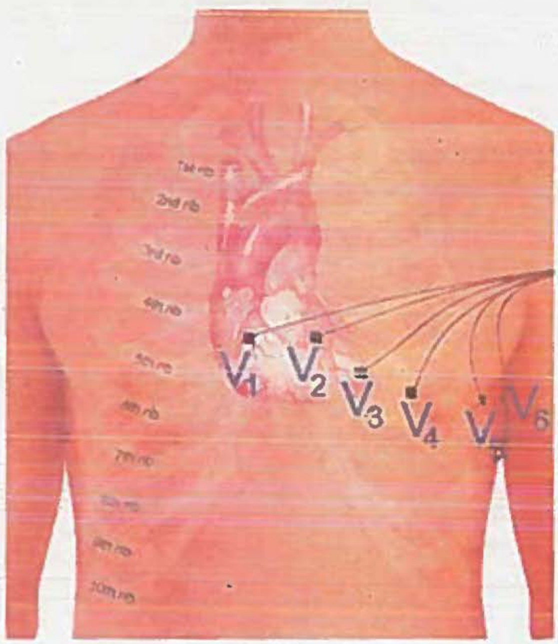


Figure 15.34(a) : Sensitive electrodes are placed on certain parts of the body.

The electric activity starts at the top of the heart and spreads down. A normal heart beat is initiated by a small pulse of electric current. This tiny electric "shock" spreads rapidly in the heart and makes the heart muscle contract.

If the whole heart muscle contracted at the same time, there would be no pumping effect.

Therefore the electric activity starts at the top of the heart and spreads down, and then up again, causing the heart muscle to contract in an optimal way for pumping blood. The electric waves in the heart are recorded in millivolts by the electrocardiograph.

Our heart produces time-varying voltages as it beats. These heart voltages produce small voltage differences between points on your skin that can be measured and used to diagnose the condition of your heart. The waves are registered by electrodes placed on certain parts of the body. Which are then printed on paper in form of a curve as shown in fig:15.34(b).

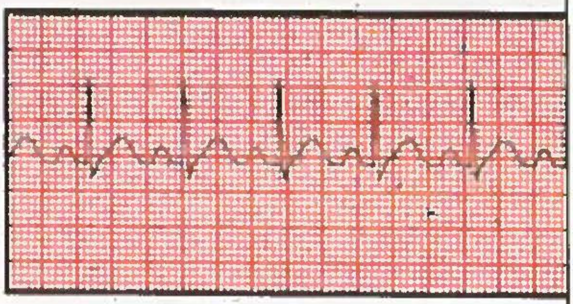


Fig 15.34(b):

Some of the characteristics of ECG curve are shown in fig15.34(b): When the curve falls below the base line it shows a negative deflection and when it rises above the base line it shows a positive deflection.

A negative deflection indicates that the recorded wave has traveled away from the electrode and a positive deflection means it has traveled towards it.

The tiny rise and fall in the voltage between two electrodes placed either side of the heart which is displayed as a wavy line either on a screen or on paper.

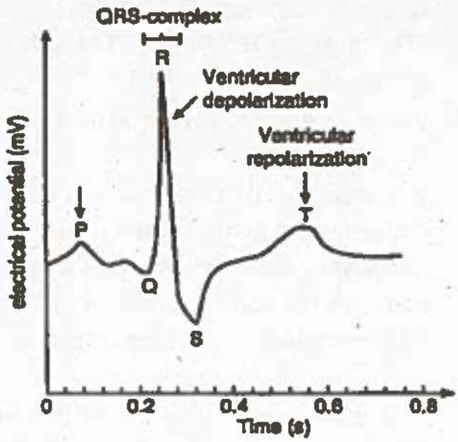


Fig 15.34(c) : An ECG curve reflects the perspective of the electrode recording it.

A typical plot of voltage difference between two points on the human body vs time is shown in fig 15.34(b). The P deflection corresponds to the contraction of the atria at the start of the heartbeat. The QRS group corresponds to the contraction of the ventricles. The T deflection corresponds to a re-polarization or recovery of the heart cells in preparation for the next beat.

Key points

- *A voltage which changes its polarity at regular interval of time is called an alternating voltage.*
- The sinusoidal alternating voltage can be expressed by the equation:
$$V = V_m \sin\omega t$$
- The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa is called its waveform or wave shape.
- One complete set of positive and negative values of an alternating quantity is known as a cycle.
- The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.
- The number of cycle that occurs in one second is called the frequency (f) of the alternating quantity.
- *The average value of a waveform is the average of all its values over a period of time.*
- *The effective or r.m.s. value of an alternating current is that steady current (d.c.)which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.*
- The equation of the alternating current varying sinusoidally is given by:
$$I = I_m \sin \omega t$$
- When two alternating quantities of the same frequency have different zero point, they are said to have a phase difference.
- Sinusoidal alternating voltage or current is represented by a line of definite length rotating in counter clock wise direction at a constant angular velocity (ω). Such a rotating line is called a phasor.
- The phasor representation enables us to quickly obtain the numerical value and at the same time as the events taking place in the circuit.

- The applied voltage and current across resistor are in phase with each other. As they pass through their zero values at the same instant and attain their positive and negative peaks at the same instant.

- When an alternating current flows through a pure inductive coil then the current lags behind the voltage by $\pi/2$ radians or 90°. *The opposition offered by an inductor to the flow of charges is called inductive reactance X_L*
- When an alternating voltage is applied across the plates of a capacitor then the current is leading the voltage by $\pi/2$ radians or 90°.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. The AC system is preferred to DC system because.
   (a) AC voltage can be easily changed in magnitude
   (b) DC motor angular velocity is effected badly
   (c) High voltage AC transmission is less efficient
   (d) Domestic appliances require AC voltage for their operation.
2. A capacitor is perfectly insulator for
   (a) Direct current (b) Alternating current
   (c) Direct as well as alternating current (d) none of these
3. The peak value of alternating current is $5\sqrt{2}$ A. The mean square value of current will be
   (a) 5A, (b) 2.5A, (c) $5\sqrt{2}$ A (d) 5^2 A
4. In choke coil the reactance X_L and resistance R are
   (a) $X_L = R$ (b) $X_L \ll R$
   (c) $X_L \gg R$ (d) $X_L = \infty$
5. In an LRC circuit, the capacitance is made one-fourth, when in resonance. Then what should be change in inductance, so that the circuit remains in resonance?
   (a) 4 times (b) 1/4 times
   (c) 8 times (d) 2 times
6. In AC system we generate sine wave form because
   (a) It can be easily draw
   (b) It produces least disturbance in electrical circuits
   (c) It is nature standard
   (d) Other waves cannot be produced easily.

7. The phase difference between the current and voltage at resonance is
   (a) 0 (b) π (c) $-\pi$ (d) $\pi/2$

8. An alternating voltage is given by 20 sin 157 t. The frequency of alternating voltage is
   (a) 50Hz (b) 25Hz (c) 100Hz (d) 75Hz

9. In LR current which one of the following statements is correct?
   (a) L and R opposes each other (b) R value increases with frequency
   (c) The inductive reactance increases with frequency
   (d) The inductive reactance decreases with frequency.

10. An alternating quantity (voltage or current) is completely known if we know it's
    (*a*) maximum value
    (*b*) frequency and phase.
    (*c*) effective value
    (d) both (a) & (b)

11. For electromagnetic waves, Maxwell generalized
    (a) gausses law for magnetism
    (b) gausses law for electricity
    (c) faradays law
    (d) amperes law

12. An electromagnetic wave goes from air to glass which of the following does not change?
    (a) Radio waves (b) X-rays
    (c) Ultra violet radiation (d) Ultra sound waves

13. The circuit in which current and voltage are in phase, the power factor is
    (a) Zero (b) 1 (c) -1 (d) 2

Conceptual questions

1. (a) Sketch a graph of e.m.f. induced in an inductive coil against rate of change of current. What is the significance of the gradient?

(b) Explain why it is difficult to measure the rate of change of current?

(c) How do graphs of e.m.f. against time and current against time make it possible to measure self- inductance?

2. (a) Current and voltage provided by an AC generator are sometimes negative and sometimes positive. Explain why for, an AC generator connected to a resistor, power can never be negative?

   (b) Explain, using sketch graphs, why the frequency of variation of power in an AC generator is twice as that of the current and voltage.

3. What determines the gradient of a graph of inductive reactance against frequency?

4. How does doubling the frequency affect the reactance of (a) an inductor (b) a capacitor?

5. If the peak value of a sine wave is 1,000 volts, what is the effective (E_{eff}) value?

6. Show that reactance is measured in ohms for both inductors and capacitors.

7. Describe the principle of ECG.

Comprehensive questions

1. Explain with diagrams sinusoidal alternating voltage and sinusoidal alternating current.

2. Define mean, peak and rms value of sinusoidal current and sinusoidal voltage. Obtain mathematical expression for the rms value of current.

3. A sinusoidal alternating voltage of angular frequency ω is connected across a resistor R. Find mathematical expression for instantaneous voltage instantaneous current and the average power dissipated per cycle of the applied voltage.

4. A sinusoidal alternating voltage of angular frequency ω is connected across a capacitor C. Find mathematical expression for instantaneous voltage instantaneous current and the average power dissipated per cycle of the applied voltage.
5. A sinusoidal alternating voltage of angular frequency ω is connected across an inductor of inductance L and resistance R. Find mathematical expression for instantaneous voltage, instantaneous current, average power dissipated per cycle of the applied voltage and draw power curve.
6. Explain the term impedance of an AC circuit. Find expression for the of the RLC series circuit.
7. Describe that maximum power is transferred when the impedances of source and load match to each other.
8. In an RL series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.
9. In an RC series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.
10. What do you mean by the term phasor diagram? Why we use it for a sinusoidal current and voltage?
11. Explain the resonance of a series RLC circuit. Show that resonance occurs at a frequency determined by: $f_r = \frac{1}{2\pi\sqrt{LC}}$
12. Describe that maximum power is transferred when the impedances of source and load match to each other.
13. Describe the statement of four Maxwell's Equations. Use Maxwell's theory to show that $E = \frac{1}{2\pi r}\frac{\Delta\Phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t}$ and $I = \frac{\varepsilon_o \Delta(\Phi)}{\Delta t}$

14. What is the principle of ECG? Sketch the wave curve of heart beats and explain the terms positive deflection and negative deflection.
15. Describe the principle of metal detectors with suitable diagram.
16. What is Ecg and its principle, by sketching its curve identify the terms positive deflection, negative deflection ,P deflection, QRS and T deflection?

Numerical Problems

1. The peak voltage of an ac supply is 300 V. What is the rms voltage?
(212.1V)
2. The rms value of current in an ac circuit is 10 A. What is the peak current?
(14.1 A)
3. The a.c. voltage across a 0.5 μF capacitor is 16 sin $(2\times10^3 t)$V. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.
(1000Ω,16mA)
4. The voltage across a 0.01 μF capacitor is 240 sin $(1.25\times10^4 t-30^\circ)$V. Write the mathematical expression for the current through it.
$(I=.03\sin(1.25\times10^4 t+60^\circ)\text{A})$
5. An inductor with an inductance of 100μH passes a current of 10mA when its terminal voltage is 6.3V. Calculate the frequency of A.C supply.
(10^6 Hz)
6. (a) Calculate the inductive reactance of a 3.00 mH inductor, when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?
((a) 1.13 Ω, 188 Ω (b) 106 A, 0.637 A)
7. For the same *RLC* series circuit having a 40.0 Ω resistor, a 3.00 mH inductor, and a 5.00 μF capacitor: (a) Find the resonant frequency. (b) Calculate I_{rms} at resonance if V_{rms} is 120 V.
((a) 1.30 kHz. , (b) 3.00 A)

8. A coil of pure inductance 318mH is connected in series with a pure resistance of 75 Ω. The voltage across resistor is 150V and the frequency of power supply is 50Hz. Calculate the voltage of power supply and the phase angle.

(250V, 53.06^o lag)

9. A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5μF across a power supply of 50Hz and 100V. Find (a) impedance (b) current (c) phase angle and(c) equation for the instantaneous value of current.

(a. 50 Ω , b. 2A, c. 53^o lead, d. 2.828 sin (314t + 53^o)

10. A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor.(d) power consumed.

(18.85, 55^0 lag, 0.573 lag, 2484.24W)

UNIT 16

......Physics of Solids......

After studying this chapter the students will be able to:

- distinguish between the structure of crystalline, glassy, amorphous and polymeric solids.
- describe that deformation in solids is caused by a force and that in one dimension, the deformation can be tensile or compressive.
- describe the behaviour of springs in terms of load-extension, Hooke's law and the spring constant.
- define and use the terms Young's modulus, bulk modulus and shear modulus.
- demonstrate knowledge of the force-extension graphs for typical ductile, brittle and polymeric materials.
- become familiar of ultimate tensile stress, elastic deformation and plastic deformation of a material.
- describe the idea about energy bands in solids.
- classify insulators, conductors, semiconductors on the basis of energy bands.
- become familiar with the behaviour of superconductors and their potential uses.
- distinguish between dia, para and ferro magnetic materials.
- describe the concepts of magnetic domains in a material.
- explain the Curie point.
- classify hard and soft ferromagnetic substances.
- describe hysteresis loss.
- synthesise from hysteresis loop how magnetic field strength varies with magnetizing current.

Materials have specific uses depending upon their characteristics and properties, such as, hardness, brittleness, ductility, malleability, conductivity etc. What

makes steel hard, lead soft, iron magnetic, and copper electrically conducting? It depends upon the structure – the particular order and bonding of atoms in a material.

This clue has made it possible to design and creates materials with new and unusual properties for use in modern technology.

A solid consists of atoms or clusters of atoms arranged in close proximity. The physical structure of a solid and its properties are closely related to the scheme of arrangement of atoms within the solid.

In crystals the arrangement of atoms is regular and periodic. The concepts of lattice and unit cell help us in understanding the atomic arrangement in crystals.

The crystal structures are analyzed using x-ray diffraction technique invented by Max von Laue and extensively employed by Bragg. The real crystals have imperfections of different kinds. The study of crystal geometry helps us to understand the diverse behavior of solids in their mechanical, metallurgical, electrical, magnetic and optical properties. The imperfections in real crystals can be controlled and suitably altered to improve the selected physical properties of the material.

For your Information

Magnetic-levitation is an application where superconductors perform extremely well. Transport vehicles such as trains can be made to "float" on strong superconducting magnets, virtually eliminating friction between the train and its tracks. It can attain an incredible speed of 361 mph (581 kph). On the other hand conventional electromagnets waste much of the electrical energy as heat.

16.1 CLASSIFICATION OF SOLIDS

On the basis of atomic arrangement solids may be classified into three categories, namely crystals, polycrystalline solids and amorphous solids.

1. Crystals

In a crystalline solid, the particles (ion, molecule or atoms) are arranged in definite geometric pattern in the three dimensional network. This is known as long range order. This arrangement repeats periodically over the entire crystal. Due to this arrangement, they have short range as well as long range order.

Crystals have smooth faces and straight edges. When a crystal is broken, it cleaves along certain preferred directions. X-ray diffraction studies have shown that in crystals the constituent atoms are arranged in a regular periodic pattern in three dimensions. The arrangement of atoms in specific relation to each other is called order.

In crystals the order exists in the immediate neighborhood of a given atoms as well as over large distance corresponding to several layers of atoms. Therefore, crystals possess both short range order and long range order.

Quartz, Sucrose (sugar), diamond and rock salt (NaCl) are examples of solids that occur as large size single crystals.

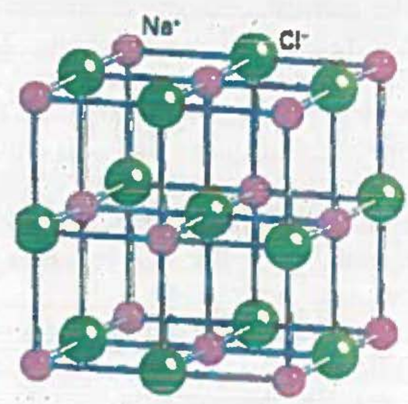


Figure 16.1: Nacl crystal

2. Polycrystalline solids

Polycrystalline is a material made up of *many small single crystals* (also called crystallites or grains).

ethene (or ethylene) C_2H_4 polythene (or polyethylene)

Polycrystalline solids consist of fine grains, having a size of 10^3 to $10^4 A^o$, separated by well define boundaries and oriented in different directions. Each such grain is a single crystal of an irregular shape. Since the grains are oriented

haphazardly, a polycrystalline material is isotropic and exhibits the same properties in all directions. Majority of the natural solids have polycrystalline structure. Metals are examples of polycrystalline solids.

In these solids the ordered regions, vary in size and orientation with respect to one another.

These regions are called as grains (domain) and are separated from one another by grain boundaries. The atomic order can vary from one domain to the next. The grains are usually 100 nm - 100 microns in diameter.

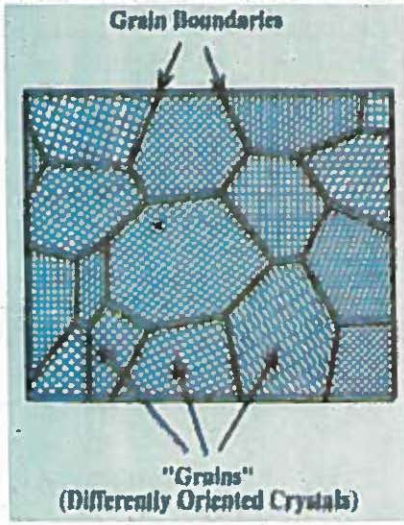


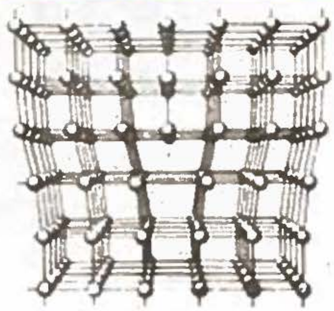
Figure 16.2: crystal grains

3. Amorphous Solids

Amorphous (Non-crystalline) solid is composed of randomly orientated atoms, ions, or molecules that do not form defined patterns or lattice structures.

So in amorphous solids the arrangement of atoms is random. These are the substance whose constituent particles don't possess a regular orderly arrangement. In an amorphous solid, the particles are arranged in a regular manner up to a small region only. This is called short range order. So, in these solids, the particles are not in regular arrangement and possess only short range order and have irregular shape.

For your information



Although metals can be thought of as polycrystalline, the structure is by no means perfect. For example there are dislocations and point defects where atomic bonding is mis-matched.

Some liquids when cooled become more and more viscous and then rigid, retaining random atom characteristic

distribution. This state is called undercooled liquid or amorphous solid. Some of the metals may be prepared in amorphous solid form by rapid cooling from molten state. Glass, rubber and polymers are examples of amorphous substances.

Do you know?

- Amorphous solids glass and plastics are very useful materials and are widely used in construction, house ware, laboratory ware, etc.
- Amorphous silica is one of the best materials for converting sunlight into electricity (photovoltaic).
- Amorphous solid rubber is used in making tyres, shoes soles etc.

The atomic arrangement in two dimensions in cases of the three classes of solids is illustrated in Fig. 16.3

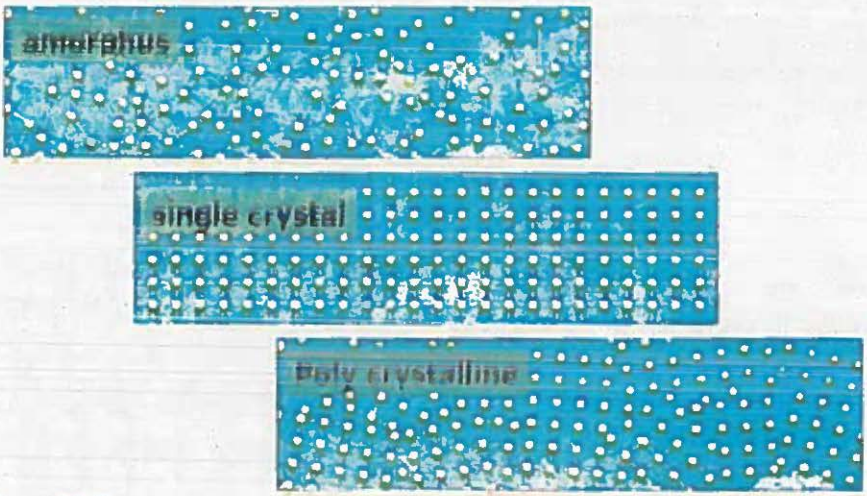


Figure 16.3:

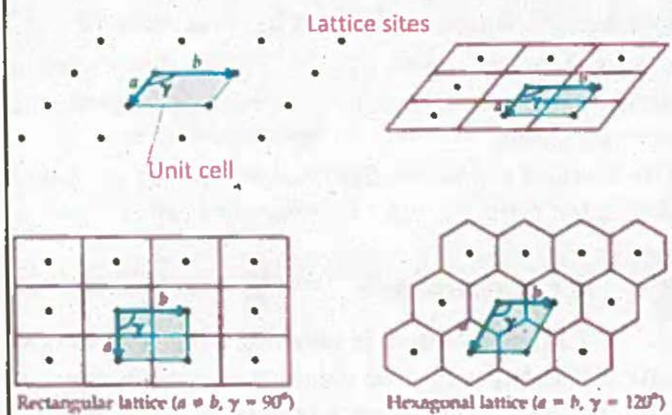
X-ray investigations show that the crystals are composed of atoms (or group of atoms) held in an orderly three-dimensional array. The array may be viewed as produced when the group of atoms, henceforth called a basis or unit, is repeated at regular intervals along all directions in the crystal. The regular and periodic arrangement of the basis is the basic feature of crystals. Geometrical analysis of crystal structure is made by referring to an imaginary array of points in

space. It is conventional to represent the periodicity by replacing the basis with a point.

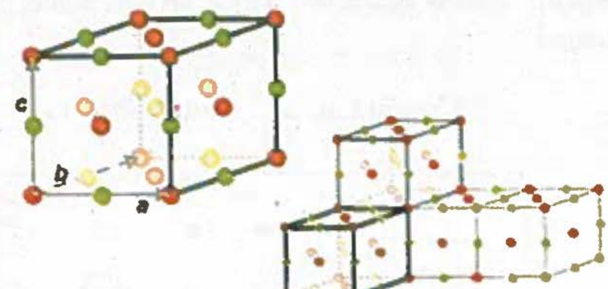
The collection of infinite number of points in a periodic arrangement is called a lattice.

A point is a dimension less and shapeless entity. Therefore, a lattice is merely an imaginary geometrical framework. The space lattice is a skeleton upon which crystal structure is built by placing atoms on or near the lattice points. Therefore, the study of crystal structure becomes simpler when it is represented by a space lattice. The points which are forming a lattice are called lattice sites.

The distance between the consecutive neighbours sites is called lattice constant. When we assign direction to these lattice constant, they become lattice translation vectors.



(a)two dimensional lattice points with each having an identical environment.



(b) three dimensional lattice

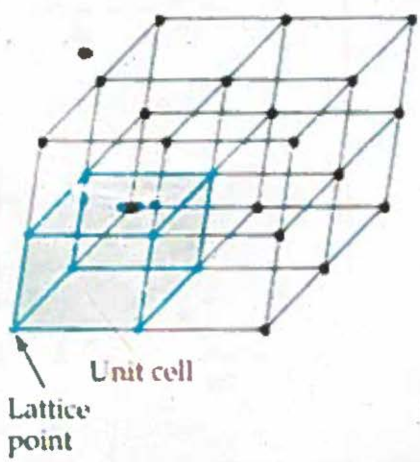


Figure 16.4(c) :unit cell

The two dimensional arrangements of lattice is shown in fig:. which shows that a plane lattice is obtained by translation of $\vec{a}$ and $\vec{b}$ and a space lattice is obtained by translation of $\vec{a}$, $\vec{b}$ and $\vec{c}$ as shown in fig16.4(a): It is possible to divide the crystal into a (large) number of identical unit cells, each containing one or more atoms.

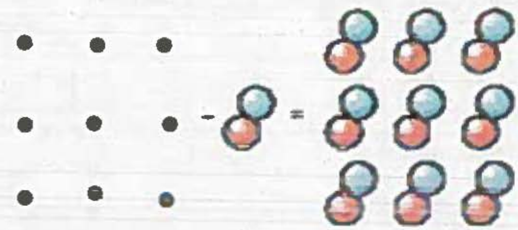
The smallest geometric figure or unit whose periodic repetition in two or three dimension form a crystal is called unit cell.

Crystal structure

Crystal structure is obtained when a basis is added at each point in the lattice. The basis must be identical in composition, arrangement and orientation such that the crystal appears exactly the same at one point as it does at other equivalent points. Fig.16.5(a) shows the basis consisting of a group of two atoms.

When the basis is associated with each lattice site, the crystal structure is obtained.

Crystal Lattice + Basis Structure= Crystal



(a):crystal structure

Figure 16.5 (b): some unit cells

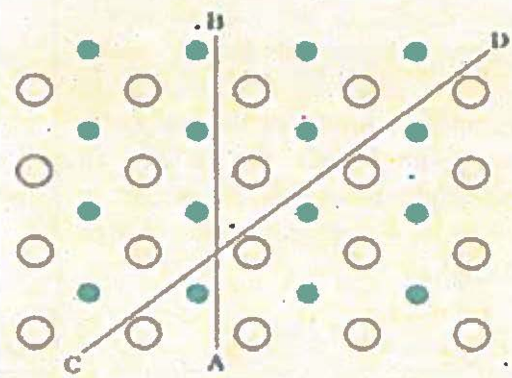
16.2 ELASTIC MODULI

When an external force is applied on the object, it changes its shape or size. Generally this deformation is small and often temporary. If an object regains its original shape when external force is removed it is called elastic. If it remains in deformed state it is called plastic. There are three types of deformations and corresponding three elastic moduli. The deforming force is expressed in terms of stress and the deformation is expressed in terms of strain. For elastic solids, the ratio of stress to strain is constant and it is called elastic modulus. That is

$$\text{Elastic modulus} = \frac{\text{Stress}}{\text{Strain}}$$

This is known as Hooke's law. There are three elastic moduli: Young's modulus (Y), Shear or Rigidity modulus (S) and Bulk modulus (B).

Do you know?



In crystalline solid, the properties like electrical conductance, refractive index, thermal expansion, etc., have different value in different directions as in AB and CD. This type of behavior is called Anisotropy and the substances with this property are called Anisotropic. A crystalline solid gives a clean surface after cleavage with knife rather than an irregular breakage.

Material	Young modulus Y(Pa)	Bulk modulus B(Pa)	Shear modulus S(Pa)
Aluminium	7×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9×10^{10}	6×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Crown glass	6×10^{10}	5×10^{10}	2.5×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}

16.2.1 Young's Modulus

The change in the length due to deforming force is described by Young's modulus. Consider a rod of length L having cross-sectional area A as shown in Fig16.6. If it is clamped at one end and a force is applied perpendicular to the area of cross-section, it changes its length, which is greater than the original length. The internal forces of the rod resist change in length but attain an equilibrium in which the length is $(L+\Delta L)$, where, ΔL is the change in length.

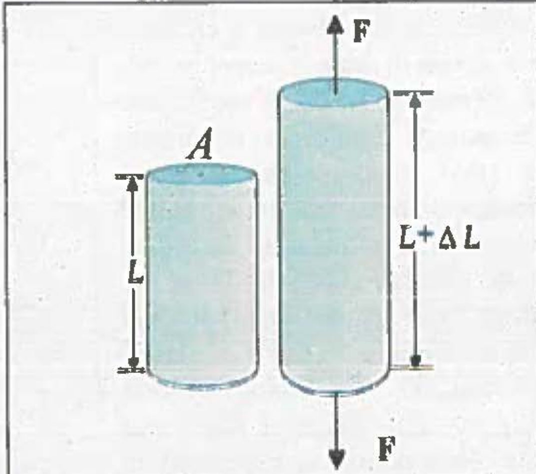


Figure 16.6: deforming forces on a rod

The ratio of the magnitude of external force F to the area of cross-section A is called as tensile stress. Where

$$\text{Tensile stress} = \frac{F}{A} \qquad ...(16.1)$$

The ratio of the change in length to the original length is called as tensile strain. And

$$\text{Tensile strain} = \frac{\Delta L}{L} \qquad ...(16.2)$$

Young's modulus is defined as the ratio of tensile stress to tensile strain.

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F/A}{\Delta L/L} \qquad ...(16.3)$$

Its SI unit is N/m^2.

The Hooke's law is valid within the elastic limit and for small strains. When the metal rod is subjected to increasing deforming force, the relation between stress and strain is as shown in Fig. 16.6.

16.2.2 Shear or Rigidity Modulus

When an object is subjected to a force tangential to one of its faces while the opposite face is held fixed there occurs a deformation (Fig16.7).

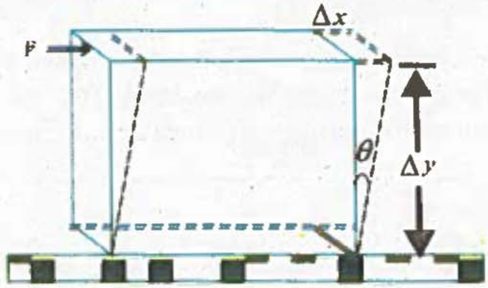


Figure 16.7: tangential deforming force on a box

The shear stress is defined as the ratio of tangential deforming force F to the area A of the face being sheared, i.e.

$$\text{Shear stress} = \frac{F}{A} \qquad \text{...(1)}$$

The shear strain is the ratio of displacement of the sheared face Δx and the fixed face y.

$$\text{Shear strain} = \frac{\text{displacement of Sheared face}}{\text{displacement of fixed face}}$$

$$= \frac{\Delta x}{y} \qquad \text{...(2)}$$

It may be described in terms of angle θ, which is called as angle of shear. Hence

$$\tan\theta = \frac{\Delta x}{y} \qquad \text{...(16.4)}$$

When the angle is small,

$$\tan\theta \approx \theta \qquad \text{...(16.5)}$$

The shear modulus is defined as the ratio of shear stress to the shear strain.

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/y} \qquad \text{...(16.6)}$$

$$= \frac{F/A}{\tan\theta} \approx \frac{F}{A\theta} \qquad \text{...(16.7)}$$

16.2.3 Bulk Modulus

When deforming forces, acting on all the surfaces of a body, are at right angles, the body undergoes deformation. The forces are distributed uniformly on all the surfaces as illustrated in Fig16.8.

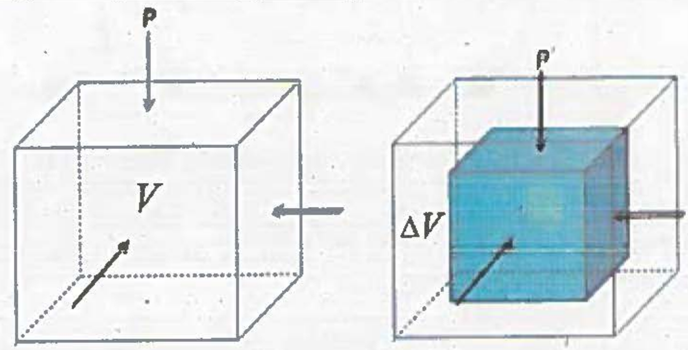


Figure 16.8: change in volume *V* of a box

In this case there is a change in the volume and not in the shape. The volume stress is the ratio of magnitude of the normal force *F* to the area *A*. Therefore

$$\text{Bulk or Volume stress} = \frac{F}{A}$$

In case of fluids it is called as pressure

$$\Delta P = \frac{F}{A}$$

The ratio of change in volume ΔV and original volume V is the volume strain. Where

$$\text{Volume strain} = -\frac{\Delta V}{V} \qquad \text{...(16.8)}$$

The bulk modulus is defined as the ratio of volume stress to volume strain. Therefore,

$$B = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{(-\Delta V/V)}$$

$$= \frac{\Delta P}{(-\Delta V/V)} \qquad \text{...(16.9)}$$

The negative sign is inserted because increase in pressure cause decrease in volume. The value of bulk modulus is always taken as positive.

16.3 HOOKE'S LAW

Stress and strain take different forms in the situations shown in the Fig. (16.9). For small deformations the stress and strain are proportional to each other. This is known as Hooke's law.
Thus,

$$\text{stress} \propto \text{strain}$$

$$\text{stress} = k \times (\text{strain}) \qquad \ldots(16.10)$$

Where, k is the proportionality constant and is known as modulus of elasticity. Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibits this linear relationship

16.3.1 STRESS-STRAIN CURVE

Engineering stress and strain are usually measured using a machine that strains the material at a fixed linear rate and records the stress.
The applied stress is gradually increased in steps and the change in length is noted. These values are then plotted on a graph .A typical graph for a metal is shown in Fig16.9. The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. From the graph, we can see that in the region between O to A, the curve is linear. In this region, Hooke's law is obeyed. The body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body. In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as yield point (also known as elastic limit) and the corresponding stress is known as yield strength (S_y) of the material.
If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The region of the plasticity is represented by the portion of the curve between B and C. Where point C in the fig16.9: shows the ultimate tensile strength (S_u) of the material. If the stress is increased beyond point C, the material is said to be permanently changed, and the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. This type of deformation is called plastic

deformation. At point D the material cannot be stretched further and smallest increase in stress can result in breaking the sample.

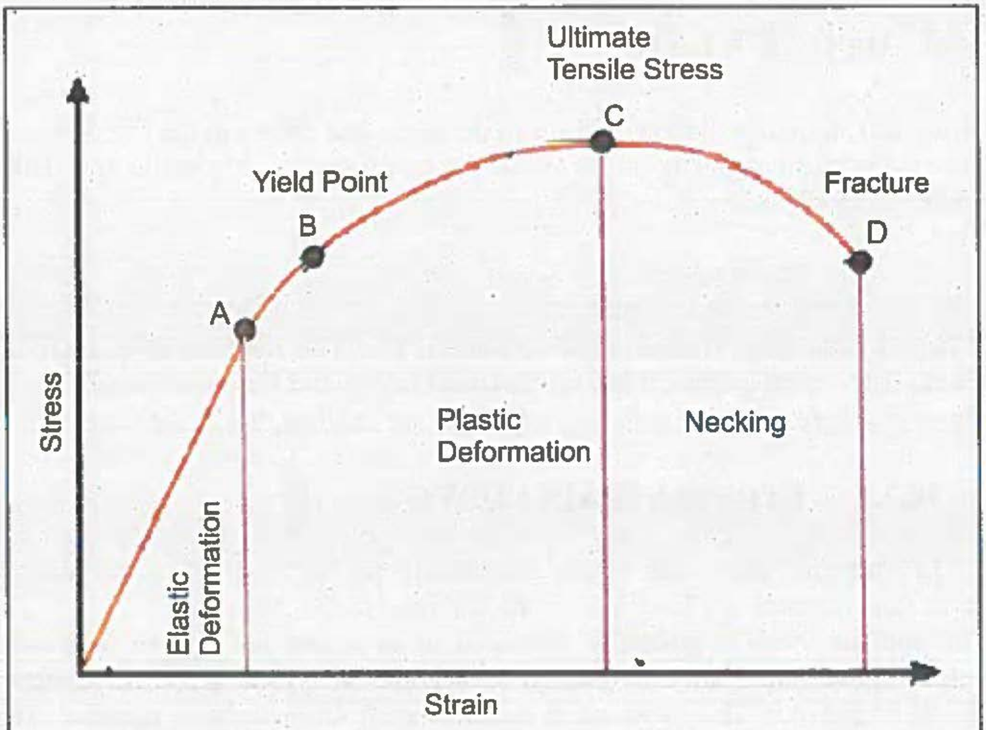


Figure 16.9: stress strain curve

If the ultimate strength and fracture points C and D are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.

Example 16.1

A force of 500N is applied to one end of a cylindrical steel rod of diameter 50cm. What is the tensile stress?

Solution:

$$\text{Since Stress} = \frac{\text{force}}{\text{area}} = \frac{500}{\pi(25\times10^{-2})^2}$$

$$\text{Stress} = 2.5 \times 10^5\ \text{Nm}^{-2}$$

Thus the tensile stress in the rod is

$2.5 \times 10^5\ \text{Nm}^{-2}$

Example 16.2

In an experiment to measure Young's modulus, a load of 500Kg hanging from a steel wire 2.4m long, of cross section $16cm^2$, was found to stretch the wire .30cm from its un-stretched length. What is (a) the stress? (b) the strain (c) the value of Young's modulus for the steel of which the wire is composed?

Solution:

Stretching in wire = x = .30cm = 3×10^{-3}m
Cross section of wire = A = $.16cm^2 = 1.6\times10^{-5}m^2$

(a) $$\text{Stress}=\frac{F}{A}=\frac{5000}{1.6\times10^{-5}m^2}=3.1\times10^{8}\,Nm^{-2}$$

(b) $$Strain=\frac{x}{L}=\frac{3.0\times10^{-3}}{2.4}=1.25\times10^{-3}$$

(c) $$\text{Young's Modulus}\quad y=\frac{\text{Stress}}{\text{Strain}}=\frac{3.1\times10^{8}}{1.25\times10^{-3}}=2.48\times10^{11}Nm^{-2}$$

16.4 Mechanical properties of solids

The mechanical properties of a material are those properties that involve a reaction to an applied load. The mechanical properties of metals determine the range of usefulness of a material and establish the service life that can be expected. Mechanical properties are also used to help classify and identify material. The most common properties considered are strength, ductility, hardness, impact resistance, and fracture toughness.

1. **Strength:** The general ability of a material to withstand an applied force.
2. **Hardness :** Hardness is a measure of how easily a material can be scratched or indented. Hard materials are often also very brittle - this means they have a low resistance to impact. Well known hard materials include diamond and hardened high carbon steels.
3. **Brittleness:** A material that has a tendency to break easily or suddenly without any extension first. Good examples are Cast iron, concrete, high carbon steels, ceramics, and some polymers such as urea formaldehyde (UF).
4. **Toughness:** A material that absorbs impact (sudden forces or shocks such as hammer blows) well is tough - this is the opposite to brittleness. [units for toughness are energy per unit area - J/m^2]

5. **Plasticity**: The materials which deform permanently when small forces are applied show plasticity. Plasticine and clay are good examples.
6. **Elasticity**: The ability of a material to return to its original form after a load has been applied and removed. Good examples include rubber, mild steel and some plastics such as nylon.
7. **Stiffness**: The ability to resist bending.
8. **Ductility**: The ability to be drawn out into a thin wire or threads. It is a measure of how easily a material can be worked. Good examples are gold, copper, titanium, wrought iron, low carbon steels and brass. It also provides an indication of how visible overload damage to a component might become before the component fractures. Ductility is also used a quality control measure to assess the level of impurities and proper processing of a material.

The conventional measures of ductility are the engineering strain at fracture (usually called the elongation) and the reduction of area at fracture. Both of these properties are obtained by fitting the specimen back together after fracture and measuring the change in length and cross-sectional area.

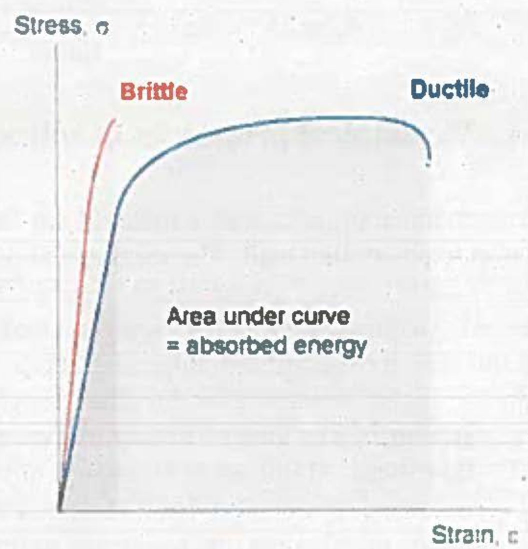


Figure 16.10: Stress-strain curves for brittle and ductile materials. Brittle materials fracture at low strains and absorb little energy. Conversely, ductile materials fail after significant plastic strain (deformation) and absorb more energy.

16.4.1 Strain energy

When a body is loaded without exceeding elastic limit, it changes its dimensions. When the load is removed, it regains its original dimensions. For the period of time it has remained loaded, it stores energy in itself and this energy is called elastic strain energy.

The work done when a wire is stretched results in energy being stored in it, called strain energy.

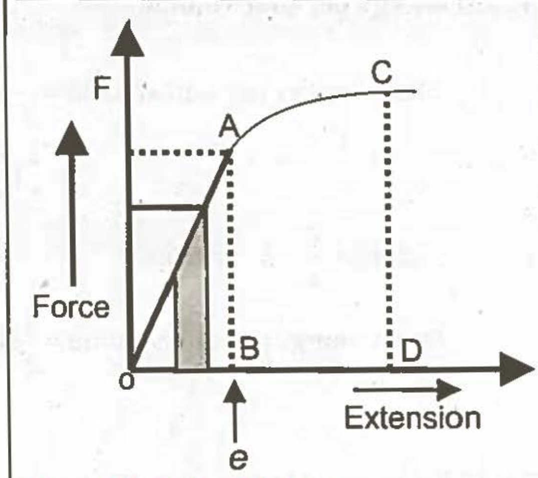


Figure 16.11: stretched wire curve

The above graph of force against extension has the same shape as the corresponding stress against strain graph.

The force varies from 0 at the start to F at the end when the wire is stretched by an amount e.

Therefore:

Work done on the wire during stretching = (average force) × (extension)

$$= \frac{1}{2}Fe \text{ =area of shaded strip} \qquad ...(16.11)$$

The diagram shows that F is not constant during the extension. However, though the strip has to be drawn quite wide for clarity, we can imagine it to be as thin as we wish, and the thinner it is, the smaller the change in F over the correspondingly small extension.

Total work done while producing extension Δe in the string:
=area of triangle OAB

$$\text{Strain energy} = \frac{1}{2}\text{base}\times\text{height} = \frac{1}{2}(OB)(AB) \qquad ...(16.12)$$

$$\text{Strain energy} = \frac{1}{2}F\times e \qquad ...(16.13)$$

Strain energy per unit volume

$$\text{Strain energy per unit volume} = \frac{1}{2}\frac{F \times e}{A\ell}$$

$$= \frac{1}{2}\left(\frac{F}{A} \times \frac{e}{\ell}\right) \quad \text{...(16.14)}$$

$$\therefore \text{stress} = \frac{F}{A} \quad \& \quad \text{strain} = \frac{e}{\ell}$$

$$\text{Strain energy per unit volume} = \frac{1}{2}(\text{stress} \times \text{strain})$$

This is the area under the stress-strain graph.

16.5 ENERGY BAND THEORY

When atoms bond together to become molecules their energy levels merge and split-this results in the splitting of spectral lines in molecular spectra. In a solid this process takes place between large numbers of atoms, and the energy levels divide into bands of closely spaced levels with large energy gaps between the bands. There is a space between these energy bands which cannot be occupied by electrons these are called forbidden energy gap or forbidden energy states. So electron in an atom can only occupy certain discrete energy states which are called permissible energy states

The energy band structure of solids can explain many of their electrical and optical properties. Fig:16.12 a: shows two energy bands, the upper band is called the conduction band in which the electrons are forced to move. In this band electrons move freely and therefore can conduct electric current through solids. It may be either empty or partially filled with electrons.
The lower energy band is called valence band, in which the electrons are tightly bound to their atoms and are not free to move about.

INSULATOR

In an insulating material the outer electrons are all shared between atoms to form bonds, so they are not available as charge carriers (the inner electrons are, of course, tightly bound to individual atoms). These valence electrons have a range of allowed energies which form a "valence band". This valence band is completely filled, so there are no vacant allowed energy levels which would allow the electrons to gain energy from an applied electric field and move from an occupied state in one atom to an empty state in another. There is another allowed energy band above the valence band and if electrons could somehow get into this empty band they could skip from atom to atom through the structure. Thermal vibration might give individual electrons energy boosts, but the energy gap is much greater than the typical size of these thermal excitations so the material is an insulator.

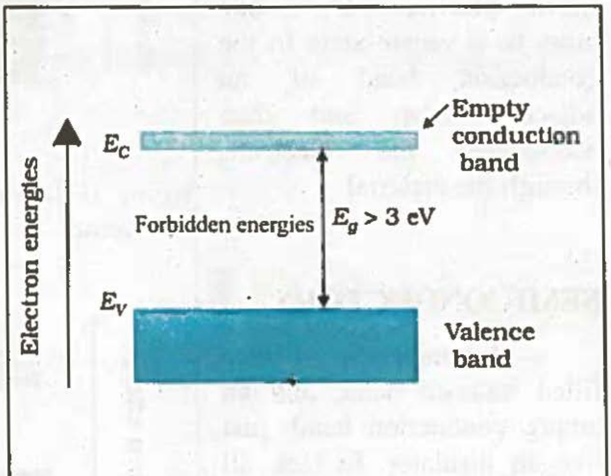


Figure 16.12 (a) : Energy band structure for an insulator

Typical thermal excitation = $kT = 0.025$eV

Thermal excitation is a random process, so electrons will sometimes get energy boosts a lot greater than kT, but the band gap in an insulator is so great that the probability of an electron jumping into the conduction band and becoming a useful charge carrier is virtually zero. In the valence band the electrons are unable to gain energy from an applied electrical field as there are no vacant energy levels for them to jump into. This explains the extremely high resistivity and low conductivity of many insulators.

CONDUCTORS

In a conducting material the energy levels of adjacent atoms have spread to form bands that overlap. The top of the valence band is above the bottom of the conduction band. This means that electrons in the valence band can easily move

to vacant energy levels in the partially filled conduction band. In practice this means that an applied electric field (e.g. created by a cell connected across the conductor) supplies the tiny amount of energy needed to move electrons from one atom to a vacant state in the conduction band of an adjacent atom and then accelerates the electrons through the material.

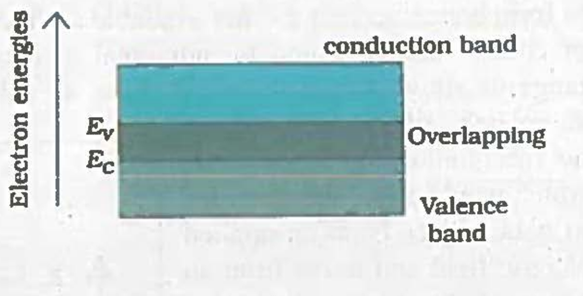


Figure 16.12 (b): Energy band structure for a conductor.

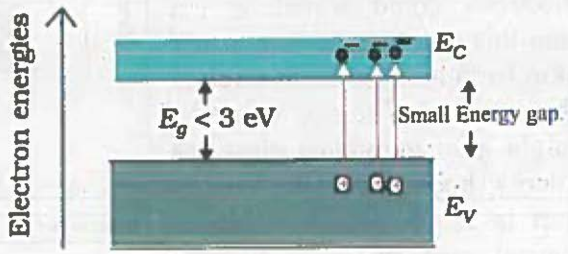


Figure 16.12 (c): Energy band structure for a semiconductor

SEMICONDUCTORS

A semiconductor has a filled valence band, and an empty conduction band, just like an insulator. In fact, all semiconductors are insulators at very low temperatures. The band gap, however, is much smaller than in materials that are insulators at room temperature. As a result, some electron will be thermally excited into the conduction band where they can move freely through the material. When this happens the electrons leave 'holes' in the valence band.

These holes behave just like positive charge carriers and can also move through the material. When a potential difference is connected across a semiconductor, holes and electrons drift in opposite directions and both contribute to the current that flows. The concentration of charge carriers in a typical semiconductor at room temperature is about 10^{21} m^{-3}. The concentration of charge carriers in a metal is about ten million times greater; as their conductivity is in between conductors and insulators so these substances are called semiconductors.

16.6 Superconductors

A superconductor is a material that can conduct electricity or transport electrons from one atom to another with no resistance.

This means no heat, sound or any other form of energy would be released from the material when it has reached "critical temperature" (T_c), or the temperature at which the material becomes superconductive. Unfortunately, most materials must be in an extremely low energy state (very cold) in order to become superconductive. The resistance of a superconductor is zero, since there are no heat losses for currents through them; they are used in magnets needing high currents, such as in MRI machines, and it does not offer resistance to transmission line.

In the past decade, tremendous advances have been made in producing materials that become superconductors at relatively high temperatures. There is hope that room temperature superconductors may someday be manufactured.

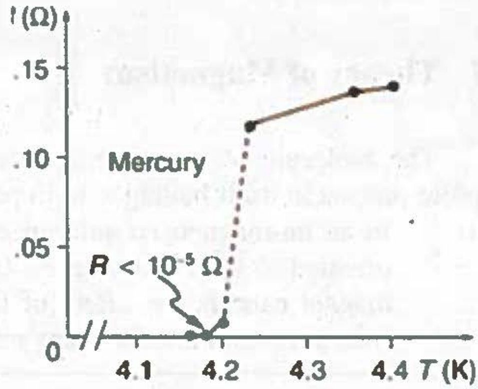


Figure 16.13: A graph of resistivity versus temperature for a superconductor shows a sharp transition to zero at the critical temperature Tc.

High temperature superconductors have verifiable Tc greater than 125 K, well above the easily achieved 77-K temperature of liquid nitrogen.

The temperature at which and below which a material becomes a superconductor is said to be its critical temperature, denoted by T_c.

Certain other elements were also found to become superconductors, but all had T_c less than 10 K, which are expensive to maintain. In 1986, a ceramic compound was found to have T_c of 35 K. It looked as if much higher critical temperatures could be possible, and by early 1988 another ceramic (thallium, calcium, barium, copper, and oxygen) had been found to have T_c = 125 K. The

first commercial use of a high temperature superconductor is in an electronic filter for cellular phones.

High-temperature superconductors are used in experimental apparatus. The search is on for even higher T_c superconductors, many of complex and exotic copper oxide ceramics, sometimes including strontium, mercury, or yttrium as well as barium, calcium, and other elements. Room temperature (about 293 K) would be ideal, but any temperature close to room temperature is relatively cheap to produce and maintain.

16.7 Theory of Magnetism

The molecule of a magnetic substance (where magnetized or not) is a complete magnet in itself having a north pole and a south pole of equal strength.

(i) In an un-magnetised substance, the molecular magnets are randomly oriented as shown in Fig. 16.14 (a). The north pole of one molecular magnet cancels the effect of the south pole of the other so that the substance does not show any net magnetism.

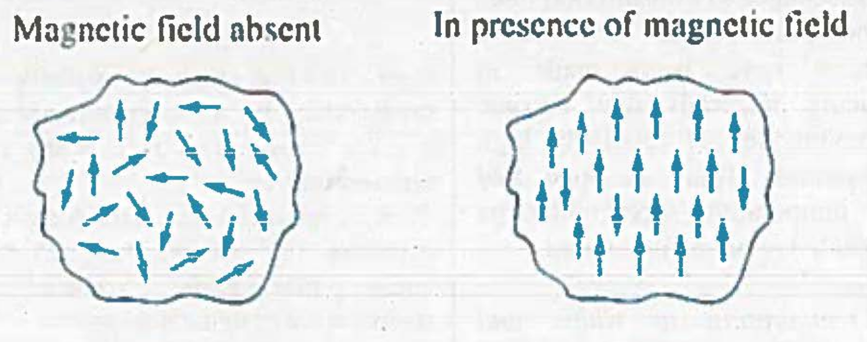


Figure 16.14: (a) (b)

(ii) When a magnetising force is applied to the substance, the molecular magnets are turned and tend to align in the same direction with N-pole of one molecular magnet facing the S-pole of other as shown in Fig.16.14 (b). The result is that magnetic field of the molecular magnets aid each other and two definite N and S-pole are developed near the ends of the specimen; the strength of the two poles being equal. Hence the substance gets magnetised.

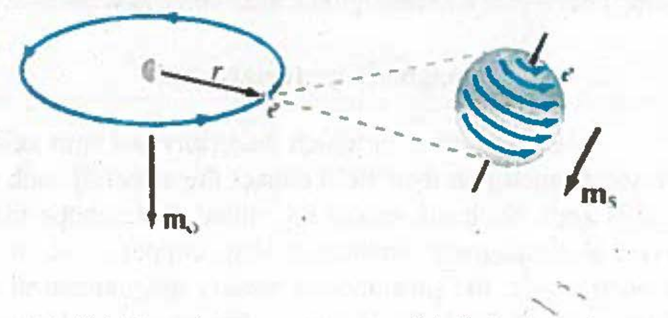
(iii) The extent of magnetisation of the substance depends upon the extent of alignment of molecular magnets. When all the molecular magnets are fully aligned, the substance is said to be saturated with magnetism.

(iv) When a magnetised substance (or a magnet) is heated, the molecular magnets acquire kinetic energy and some of them loss their arrangement. For this reason, a magnet loses some magnetism on heating.

16.8 MODERN VIEW ABOUT MAGNETISM

According to modern view, the magnetic properties of a substance are due to the motions of electrons.(orbital and spin) in the atoms. We know that an atom consists of central nucleus with electrons revolving around the nucleus in different orbits. This motion of electrons is called orbital motion [Fig16.15(a).].

The electrons also rotate around their own axis. This motion of electrons is called spin motion [Fig16.15(b)].



(a) Orbiting electron (b) Spinning electron

Figure 16.15:

Due to these two motions, each atom is equivalent to a current loop i.e. each atom behaves as a magnetic dipole.

1. In the un-magnetised substances, the magnetic dipoles are randomly oriented so that magnetic fields mutually cancel. When the substance is magnetized, the magnetic dipoles are aligned in the same direction. Hence the substance shows net magnetism.
2. Since the revolving and spinning electrons in each atoms cause magnetism, no substance is non-magnetic.
3. It is important to note that spinning motion of electrons in particular is responsible for magnetism of a substance.

16.8.1 CLASSIFICATION OF MAGNETIC MATERIALS

We can classify materials into three categories *viz.* diamagnetic, paramagnetic and ferromagnetic. The behaviour of these three classes of substances is different in an external magnetic field.

1. Paramagnetic material

In these substances the orbital and spin axis of the electrons in an atom are so oriented that their field support each other and the atom behaves like a tiny magnet. These substances are called paramagnetic materials.
When a paramagnetic substance (e.g. aluminium, antimony etc.) is placed in a magnetic field, the substance is weakly magnetized in the direction of the applied field. Therefore, a paramagnetic substance is weakly attracted by a strong magnet.

2. Diamagnetic materials

The substances in which the orbits and spin axis of the electron in an atom are so oriented that their field cancel the effect of each other so that their resultant field is zero. Such substances are called diamagnetic materials.
When a diamagnetic substance (e.g. copper, zinc, bismuth etc.) is placed in a magnetic field, the substance is weakly magnetized in a direction opposite to that of the applied field. Therefore, a diamagnetic substance is weakly repelled by a strong magnet.

3. Ferromagnetic Materials

In these substances the individual atoms act like tiny magnêts. The interaction between these tiny atomic magnets is so strong that it line up parallel to each other even when no external magnetic fields is present. Such substances are called ferromagnetic materials. These tiny magnets are called magnetic domains. The size of these domains is very small of the order of millimeters or less but large enough to contain atoms from 10^{12} to 10^{16}. Each domain acts like a small magnet with its own north and South Pole. The coupling or interaction between the neighboring tiny magnets is reduced by increasing the temperature of a substance. The temperature at which a ferromagnetic material becomes paramagnetic is called its curie temperature.

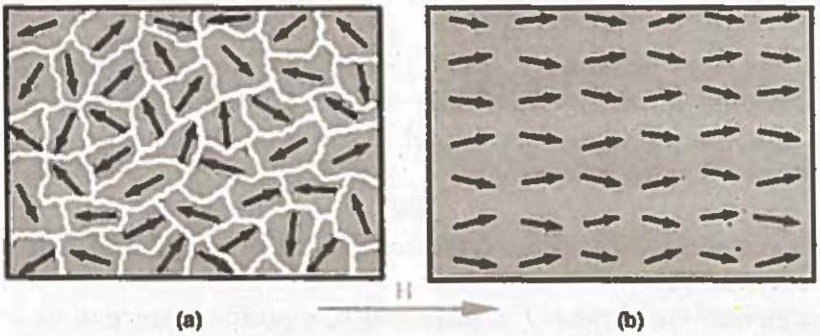


Figure 16.16: (a) a piece of ferromagnetic material which is not magnetised, where the domain poles are not aligned; (b) the domain poles aligned with an external magnetising force (H).

When a ferromagnetic substance (e.g. iron, nickel, cobalt etc.) is placed in a magnetic field, the substance is strongly magnetised in the direction of the applied field. Therefore, ferromagnetic material is strongly attracted by a magnet. The diamagnetism and paramagnetism are weak forms of magnetism. However, ferromagnetic substances exhibit very strong magnetic effects.

16.9 MAGNETIC HYSTERESIS

Hysteresis loop

When an electric current is passed through a coil of wire, the coil acts like a bar magnet with a north pole at one end and a south pole at the other. In other words, it acts like an electromagnet. If a bar of soft iron is placed inside the coil, the strength of the electromagnet is much increased. This is because the piece of soft iron is itself converted into a magnet by the effect of the current in the coil. The Magnetic Flux Density is a measure of the amount of magnetic flux in a unit area perpendicular to the direction of magnetic flow, or the amount of magnetism induced in a substance placed in the magnetic field. Magnetic field strength (H) is the amount of magnetizing force. The intensity of the Magnetic Flux Density, (B), is affected by the intensity of the Magnetic Field, (H) the quantities of the substance and the intervening media between the source of the magnetic field and the substance.

The relationship between magnetic field strength and magnetic flux density is:

$$B = H \times \mu_o$$

When a magnetic material is subjected to a cycle of magnetism (i.e. it is magnetised first in one direction and then in the other), it is found that flux density B in the material lags behind the applied magnetizing force H. This phenomenon is known as hysteresis.

The phenomenon of lagging of flux density (B) behind the magnetizing force (H) in a magnetic material subjected to cycle of magnetization is known as magnetic hysteresis.

If the magnetization current, I is increased in a positive direction to some value the magnetic field strength H increases linearly with I and the flux density B will also increase as shown by the curve from point O to point a as it heads towards saturation. Now if the magnetizing current in the coil is reduced to zero the magnetic field around the core reduces to zero but the magnetic flux does not reach zero due to the residual magnetism present within the core and this is shown on the curve from point a to point b. Fig.16.17

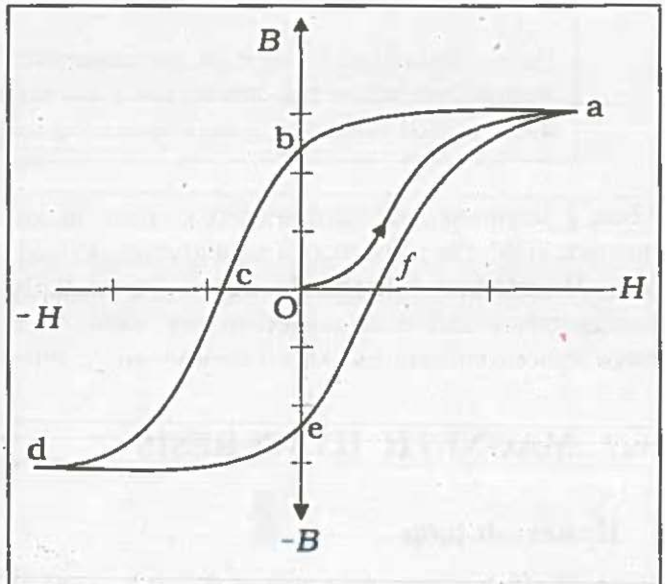


Figure 16.17: hysteresis loop

To reduce the flux density at point b to zero we need to reverse the current flowing through the coil. The magnetizing force which must be applied to null the residual flux density is called a "Coercive Force". This coercive force reverses the magnetic field re-arranging the molecular magnets until the core becomes un-magnetized at point c. An increase in the reverse current causes the core to be magnetized in the opposite direction and increasing this magnetization current will cause the core to reach saturation but in the opposite direction, point d on the curve. If the magnetizing current is reduced again to zero the residual magnetism present in the core will be equal to the previous value but in reverse at point e. Again reversing the magnetizing current flowing through the coil this time into a

positive direction will cause the magnetic flux to reach zero, point f on the curve and as before increasing the magnetization current further in a positive direction will cause the core to reach saturation at point a. Then the B-H curve follows the path of a-b-c-d-e-f-a as the magnetizing current flowing through the coil alternates between a positive and negative value such as the cycle of an AC voltage. This path is called a Magnetic Hysteresis Loop. Thus when a magnetic material is subjected to one cycle of magnetization, B always lags behind H so that the resultant B-H curve forms a closed loop, called hysteresis loop.

From the hysteresis loop, a number of magnetic properties of a material are

1. **Retentivity** - It is a material's ability to retain a certain amount of residual magnetic field when the magnetizing force is removed after achieving saturation.
2. **Residual Magnetism or Residual Flux** - the magnetic flux density that remains in a material when the magnetizing force is zero.
3. **Coercive Force** - The amount of reverse magnetic field which must be applied to a magnetic material to make the magnetic flux return to zero.
4. **Reluctance** - Is the opposition that a ferromagnetic material shows to the establishment of a magnetic field. Reluctance is analogous to the resistance in an electrical circuit.

16.9.1 Magnetic Hysteresis Loops for Soft and Hard Materials

Soft magnetic materials:

Soft ferromagnetic materials such as iron or silicon steel have very narrow magnetic hysteresis loops resulting in very small amounts of residual magnetism making them ideal for use in relays, solenoids and transformers as they can be easily magnetised and de-magnetised.
Since a coercive force must be applied to overcome this residual magnetism, work must be done in closing the hysteresis loop with the energy being used being dissipated as heat in the magnetic material. This heat is known as hysteresis loss, the amount of loss depends on the material's value of coercive force.
By adding silicon, to iron a material with a very small coercive force can be made, such materials typically contain 5% silicon and have very narrow hysteresis loop (figure 16.18b).

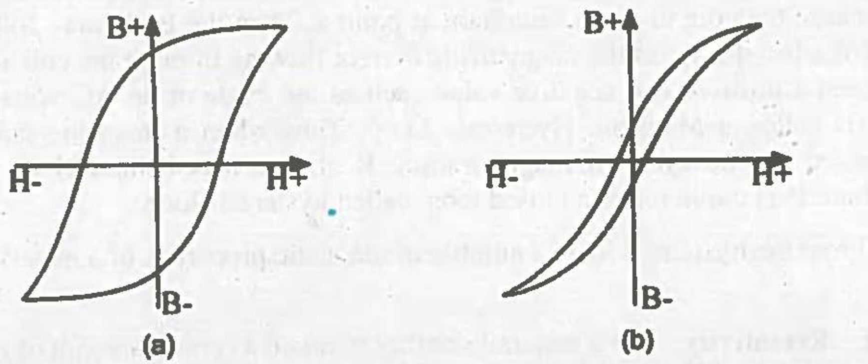


Figure 16.18: (a) the hysteresis loop for hard magnetic material suitable for permanent magnet; (b) the hysteresis loop for soft magnetic material suitable for a transformer core.

Materials with narrow hysteresis loops are easily magnetised and de-magnetised and known as soft magnetic materials.

Magnetic Hysteresis results in the dissipation of wasted energy in the form of heat with the energy wasted being in proportion to the area of the magnetic hysteresis loop. Hysteresis losses will always be a problem in AC transformers where the current is constantly changing direction and thus the magnetic poles in the core will cause losses because they constantly reverse direction

Hard magnetic materials:

In order to create a permanent magnet, a material with a very fat hysteresis loop may be used (figure 16.18a). Such materials, once magnetised, are very difficult to demagnetise and when the magnetising force is removed a substantial magnetic flux density remains. These materials are known as hard magnetic materials. Its examples are Tungsten steel, Cunife, Cobalt rare earth 1, Sintered ferrite 3, and Sintered alnico 8 etc.,

Materials with fat hysteresis loops are difficult to de-magnetise and magnetic flux density remains even magnetising force is removed are known as hard magnetic materials.

Key points

- In a crystalline solid, the particles (ion, molecule or atoms) are arranged in definite geometric pattern in the three dimensional network. This is known as long range order.
- Polycrystalline is a material made up of *many small single crystals* (also called crystallites or grains). Metals are examples of polycrystalline solids.
- Amorphous (Non-crystalline) Solid is composed of randomly orientated atoms, ions, or molecules that do not form defined patterns or lattice structures.
- Geometrical analysis of crystal structure is made by referring to an imaginary array of points in space.
- *The collection of infinite number of points in a periodic arrangement is called a lattice.*
- *The smallest geometric figure or unit whose periodic repetition in two or three dimension form a crystal is called unit cell.*
- Crystal structure is obtained when a basis is added at each point in the lattice.
- The ratio of stress to strain is constant and it is called elastic modulus
- The ratio of the magnitude of external force F to the area of cross-section A is called as tensile stress.
- Young's modulus is defined as the ratio of tensile stress to tensile strain.
- The shear stress is defined as the ratio of tangential deforming force F to the area A of the face being sheared.
- The bulk modulus is defined as the ratio of volume stress to volume strain.
- For small deformations the stress and strain are proportional to each other. This is known as Hooke's law.
- Hardness is a measure of how easily a material can be scratched or indented.
- A material that has a tendency to break easily is called brittle.

- Stiffness is the ability to resist bending.
- The work done when a wire is stretched results in energy being stored in it, called strain energy.
- According to band theory of solids the energy level in an atom can be divided into two main bands namely conductions band and valance band
- *A superconductor is a material that can conduct electricity or transport electrons from one atom to another with no resistance.*
- According to modern view, the magnetic properties of a substance are due to the motions of electrons (orbital and spin) in the atoms
- Magnetic materials are classified into three categories *viz.* diamagnetic, paramagnetic and ferromagnetic.
- When a magnetic material is subjected to a cycle of magnetism, it is found that flux density B in the material lags behind the applied magnetizing force H. This phenomenon is known as hysteresis.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. A wire is stretched to double of its length. Its strain is
   (a) 2 (b) 1 (c) 0 (d) 0.5
2. Which of the modulus of elasticity is involved in compressing a rod to decrease its length?
   (a). young's modulus (b). Bulk modulus
   (c). Modulus of elasticity (d). none of the above
3. Which one is Ferromagnetic in nature?
   (a) Soft iron (b) Nickle
   (c) Copper (d) None of these
4. If both the length and radius of the rod are doubled, then the modulus of elasticity will
   (a) increase (b) decrease
   (c) remains the same (d) doubled
5. Curie temperature is a point where
   (a) diamagnetism changes to paramagnetism
   (b) paramagnetism changes to diamagnetism
   (c) ferromagnetism changes to paramagnetism
   (d) paramagnetism changes to ferromagnetism
6. A cable breaks if stretched by more than 2mm. it is cut into two equal parts. How much either part can be stretched without breaking?
   (a) 25 m (b) 1mm (c) 2mm (d) 0.5m

Conceptual questions

1. Evaluate the importance of strength and stiffness in a design context.
2. Discuss the superconductivity of a conductor with the help of a curve.
3. Distinguish between crystalline, amorphous and polymer solids.

4. Define unit cell, basis and space lattice.
5. Differentiate between paramagnetic diamagnetic and ferromagnetic materials with suitable examples.
6. Distinguish between soft and hard substance by drawing its curves.
7. Explain Hook's Law and Modulus of Elasticity.
8. Is there any difference in the length of a 20 meter steel girder when standing vertically and horizontally?
9. Steel reinforcing is used in concrete beams to prevent cracking. Explain where the steel reinforcing should be placed in a concrete beam?
10. (a) What is meant by the elastic limit of a material.
    (b) In what way does a material behave if it obeys Hook's Law?
11. Cast-iron beams are used in bridge and building construction. The lower part of the beam is thicker than the upper part. Why is it better for the lower part of the beam to be thicker than the upper part? Given reasons for your answer including reference to the tensile and compressive strength.

Comprehensive questions

1. Explain the differences between tensile and compressive forces and how they affect equilibrium within a structure.
2. Draw and describe a stress/strain graph and identify the elastic region, plastic flow region, yield stress and ultimate tensile stress.
3. Describe the valence band, conduction band and forbidden energy gap with the help of energy level diagram.
4. Discuss the superconductivity of a conductor with the help of a curve.
5. Describe the mechanical properties of solids?
6. Describe Hysteresis loop for a ferromagnetic materials by drawing its curve for iron.
7. Explain strain energy in a deformed wire by drawing its graph.

Numerical Problems

1. A 1.50cm length of pianos wire with a diameter of 0.25cm is stretched by attaching a 10kg mass to one end. How far is the wire stretched?

   $(1.5 \times 10^{-4}m)$

2. A cable has a length of 12m and is stretched by 1.2×10^{-4}m when a stress of 8.0×10^{8} Nm^{-2} is applied. What is the strain energy per unit volume in the cable when the stress is applied?

   $(4 \times 10^{3}\ Jm^{-3})$

3. A cylindrical steel rod 0.50m long and 1cm in radius is subjected to a tensile force of 1×10^{4} N.

   (a) What is the tensile stress?

   (b) What is tensile strain?

   (c) By what amount does the rod stretch?

   ((a) $0.31 \times 10^{8}\ Nm^{-2}$ (b) 1.6×10^{-4} (c) 0.8×10^{-4}m)

4. A cable has an un-stretched length of 12m and it stretches by 1.2×10^{-4}m when a stress of $6.4 \times 10^{8}\ Nm^{-2}$ is applied. What is the strain energy per unit volume in the cable when this stress is applied? $(3.2 \times 10^{3}\ Jm^{-3})$

5. Young's Modulus for a particular wood is $1\times10^{10}\ Nm^{-2}$. A wooden chair has four legs each of length 42cm and cross sectional area $2\times10^{-3}m^{2}$. Hamza has a mass of 100Kg.

   (a)What is the stress on each leg of the chair when Hamza stands on the chair?

   (b) By what amount do the chair legs shrink when Hamza stands on the chair?

   ((a) $5\times10^{5}\ Nm^{-2}$, (b) 2.1×10^{-5}m)

6. A force of 1.5×10^{4}N causes a strain of 1.4×10^{-4} in a steel cable of cross-sectional area $4.8\times10^{-4}m^2$.

(a)What is the Young's Modulus of the steel cable?

(b) The stress strain graph is linear for this cable. Calculate the strain energy per unit volume stored in the cable when the cable has a strain of 1×10^{-4}.

((a) $2.2\times10^{11}Nm^{-2}$, (b) $1.5\times10^{3}Jm^{-3}$)

UNIT 17

.... Electronics ......

After studying this chapter the students will be able to:

- distinguish between intrinsic and extrinsic semiconductors.
- distinguish between P & N type substances.
- explain the concept of holes and electrons in semiconductors.
- explain how electrons and holes flow across a junction.
- describe a PN junction and discuss its forward and reverse biasing.
- define rectification and describe the use of diodes for half and full wave rectifications.
- distinguish PNP & NPN transistors.
- describe the operations of transistors.
- deduce current equation and apply it to solve problems on transistors.
- explain the use of transistors as a switch and an amplifier.

Figure 17.1: Semiconductors are materials having an intermediate electric conductivity between conductors and insulators. Semiconductors are essential to the good functioning of many modern appliances and are key products in the electronic systems.

Transistor is the basis of the integrated circuits that run our computers and many modern technologies, including programmable controllers. Many modern technologies use electro-mechanical principles to interface real world sensors and outputs to microprocessors, temperature controllers.

This unit increases students' understanding of the applications and uses of physics.

17.1 Intrinsic Semiconductor

A perfect, pure semiconductor crystal containing no impurities is called an intrinsic semiconductor. A semiconductor is considered to be pure when there is less than one impurity atom in a billion host atoms. In a silicon crystal at absolute zero temperature, the bonding arrangement may be represented by a two dimensional model, sketched in Fig17.2. In reality, the semiconductor is a three dimensional solid and the sharing of valence electrons occurs between nearest neighbour atoms in three dimensions.

In the Fig17.2: the hatched circles represent the core of the silicon atom. The four valence electrons are shown by the small black dots. The probability of valence electrons being in any place between the bonding atoms is indicated by the dotted curves. When temperature approaches to 0K, then all valence electrons are strongly bound to their atoms and they spend most of the time between neighbouring atoms. Since, all the valence electrons are engaged in covalent bonds, the bonds are complete. Free electrons do not exist in these solids. Consequently, the semiconductor nearly at 0K cannot conduct and behave as a perfect insulator.

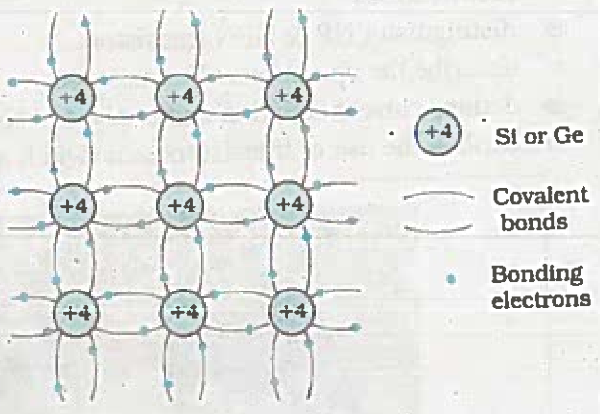


Figure 17.2 : two dimensional representation of Si or Ge structure. +4 symbols indicate inner cores of Si or Ge.

The energy band structure of a semiconductor is characterized by a valence band and a conduction band separated by the bandgap E_g as shown in the fig17.3:

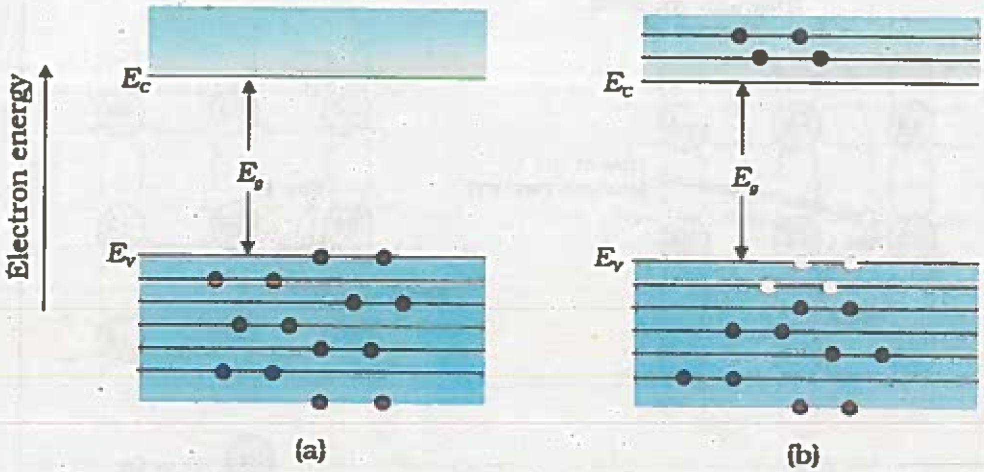


Figure 17.3: bandgap energy Eg (b) electron jumping from valence band to conduction band.

The electrons completely occupy the valence band leaving the conduction band vacant. As all the states in the valence band are full, electrons cannot be excited within the band. Electrons in the valence band do not possess enough energy to jump into the conduction band. Therefore, an externally applied electric field cannot cause a flow of current and the semiconductor nearly at 0K behaves as an insulator.

17.2 Intrinsic Semiconductor at Room Temperature

At higher temperatures, the finite thermal energy causes each atom in the crystal to vibrate about its mean position. When the vibration becomes violent, some of the electrons acquire sufficient energy and break away from the covalent bonds. The electrons librated from bonds become free electrons which vibrate more randomly in the empty spaces that exist in the atoms at fixed positions.

From an energy band point of view, it means that some of the electrons convert part of their thermal energy into potential energy. Those electrons, which acquire potential energy equal to, or in excess of, the bandgap energy E_g are excited from the valence band to the conduction band.

Thus, the *bandgap energy E_g is the minimum amount of energy required to excite an electron from valence band to conduction band.* E_g is a characteristic of the material. The number of electrons excited to the conduction band depends on the amount of thermal energy received by the crystal.

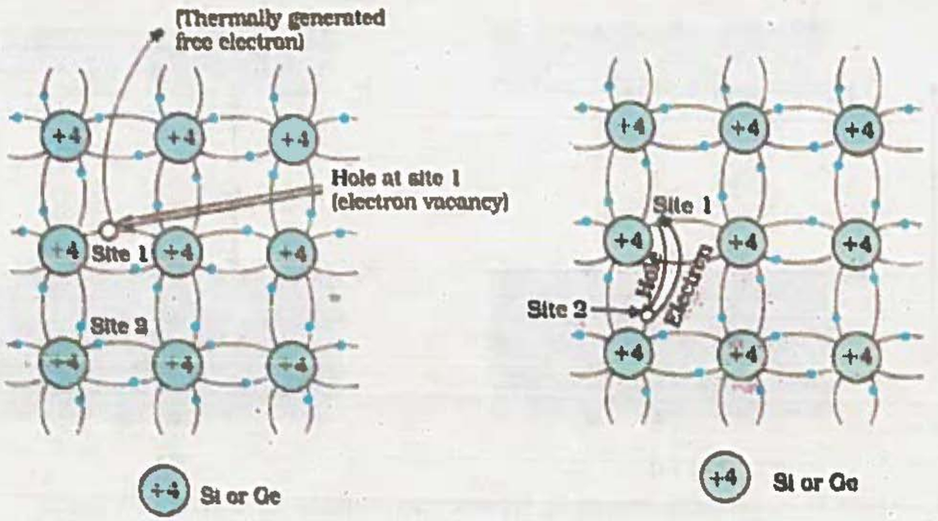


Figure 17.4: model of generation of hole at site 1 and conduction electron due to thermal energy. (b) movement of hole from site 1 to 2.

Let us now consider the situation in the valence band. When a covalent bond breaks and electron jumps to the conduction band, an empty state arises in the valence band. The electrons in the conduction band and the electrons in the valence band can be excited to upper vacant levels, within the respective bands. Therefore, if an electric field is applied, current flows in the crystal at ordinary temperatures. We may therefore, define intrinsic semiconductors as such materials in which conduction arises from thermally (or optically) excited electrons.

The motion of valence electrons in the valence band is customarily described in terms of a fictitious particle called hole which has positive charge $+e$ and its mass is equal to that of an electron m_e.

17.3 INTRINSIC CARRIERS

In pure semiconductors, a single event of bond breaking leads to two carriers, namely an electron and a hole. The electron and hole are created as a pair and the phenomenon is called electron-hole pair generation. The thermal generation is one possible mechanism for pair generation. At any temperature T, the number of electrons generated will be therefore equal to the number of holes generated. If N denotes the number density of electrons in the conduction band and P the number density of holes in the valence band, then,

$$N = P = N_i$$

where, N_i is called the intrinsic density or intrinsic concentration.

After the generation, the carriers move independently. The electron moves in the conduction band and the hole moves in the valence band.

We can also define a semiconductor as *intrinsic semiconductor in which free electrons and holes are created only by excitation of electrons from the valence band to the conduction band.*

17.3.1 DOPING OF IMPURITIES

The intrinsic semiconductors have low conductivity which is of little interest. But, when a small amount of impurity is added to semiconductor crystal then it greatly increases the conductivity of the intrinsic semiconductor.

An addition of impurity into an intrinsic semiconductor is called doping.

The impurity added is called a dopant. A semiconductor doped with impurity atoms is called an extrinsic semiconductor. Pentavalent elements from Group V or trivalent elements from Group III are used as dopants. The atoms belonging to these two groups are nearly of the same size as silicon or germanium atoms and easily substitute themselves in place of some of the host atoms in the semiconductor crystal. Thus, they are substitutional impurities and do not cause any distortion in the original crystal structure.

Two types of extrinsic semiconductors, namely N-type and P-type semiconductors are produced depending upon the group of impurity atom.

17.3.2 N-TYPE SEMICONDUCTOR

An N-type semiconductor is produced when a pure semiconductor is doped with a pentavalent impurity. Phosphorous and arsenic are the dopants normally used. Let us examine the effect of these pentavalent impurities on the carrier concentration and electrical conductivity. Suppose a phosphorous atom is doped with a silicon atom in the crystal. Phosphorus atom has five valence electrons. Out of the five electrons, only four can participate in the bonding, since there are only four bonds as shown in Fig. 17.5 (b). The fifth electron does not form a bond and remains loosely bound to the atom.

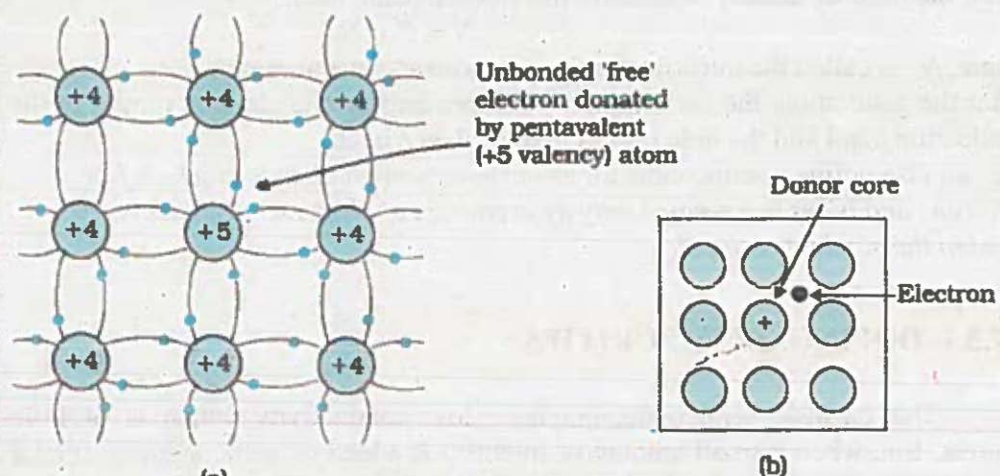


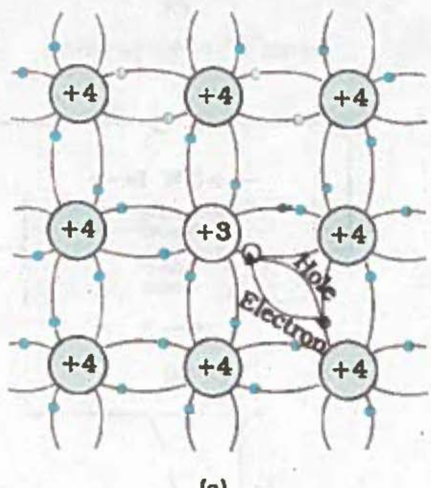
Figure 17.5 : doping of semiconductors

The impurity atoms which contribute electrons to the conduction band are called donor atoms. They produce electrons without producing holes in the valence band. At very low temperature, the donor atoms are not ionized and the conduction band is empty. At slightly elected temperatures, the donor electrons populate the conduction band. At ordinary temperatures, some electrons from the valence band are also excited into the conduction band through the intrinsic process. The holes in the valence band are smaller at ordinary temperatures. The electrons which are

in majority are called majority carriers whereas the holes are called minority carriers since they are very small in number. As the current is mainly carried by electrons which are negative charge carriers, the semiconductor is called an N-type extrinsic semiconductor, where N indicates the negative sign of the majority carriers.

17.3.3 P-TYPE SEMICONDUCTOR

A P-type semiconductor is obtained by doping an intrinsic semiconductor with trivalent elements such as boron and aluminum. When a trivalent impurity atom is added with a silicon atom, it falls short of one electron for completing the four covalent bonds with its neighbors. The substitution of a host silicon atom with, say, a boron atom does not disturb the neutral environment around the boron atom. However, when an electron from a neighboring atom acquires energy and jumps to form a bond, it leaves behind a hole. The boron atom having acquired an additional electron becomes a negative ion. The hole can move freely in the valence band whereas the impurity ion is immobile.



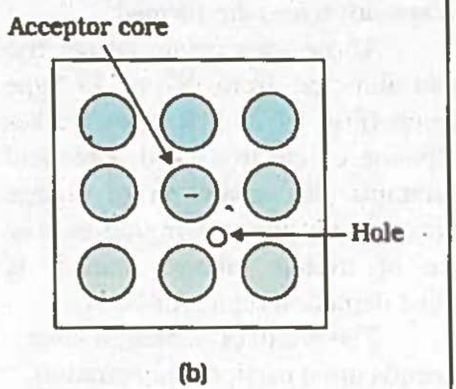


Figure 17.6: semiconductors doping

The impurity atoms which accept electrons from the valence band are known as acceptor atoms. The acceptor impurity atoms produce holes without the simultaneous generation of the electrons, in the conduction band. At ordinary temperatures, holes are produced due to intrinsic process also, by promoting

electrons from valence band to conduction band. The result is that holes out number electrons in the semiconductor. Therefore, holes constitute the majority carriers and electrons are minority carriers in this type of semiconductor. As positively charged carriers are mainly contributing to the conduction process, this type of semiconductor is known as a P-type extrinsic semiconductor where P signifies positive sign of the majority carriers.

17.4 PN JUNCTION

Junction diode is formed by placing a P-type crystal with in contact with N-type crystal and subjected to high pressure so that it becomes a single piece. PN junction has special properties such as rectification. At the junction, electrons from N-type fill the vacancies in P-region; thus negative and positive ions are formed.

These ions create an electric field directed from N- to P- type region (Fig. 17.7 a). It stops further diffusion of electrons and holes and maintains the separation of charge carriers. The junction region is now free of mobile charges and it is called depletion region or layer.

The width of depletion layer depends upon carrier concentration. Thus, a potential wall or barrier is formed at the junction.

The symbol of PN junction (diode) is shown in Fig17.7.

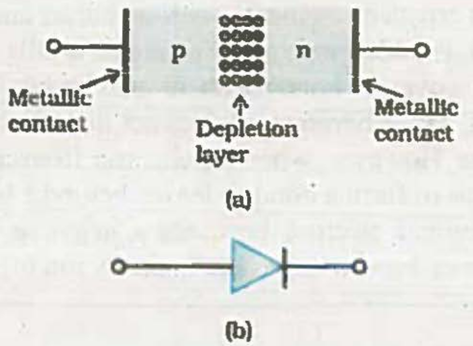


Figure 17.7 : PN junction

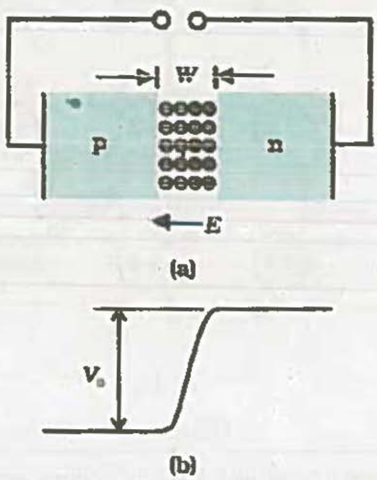


Figure 17.8 : PN junction potential barrier

When P-type region is connected with positive potential with respect to N-type region and the potential drop is increased slowly, the junction barrier-height decreases.

At one point, called as knee voltage, majority charge carriers cross the junction and the current flows, Fig17.9. This is called forward bias and the current is called forward current.

On the other hand, if P-type region is made negative with respect to N-type, no majority charge carriers cross the junction and hence there is no current. A small amount of current flows due to minority charge carriers. This biasing is called the reverse bias and the current is called as reverse current. The reverse current is generally of the order of few microamperes.

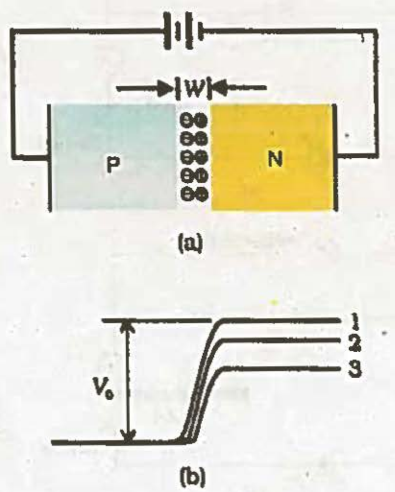


Figure 17.9: P-N junction diode under forward bias current.

17.4.1 *VI* Characteristics of PN Junction

The *VI* characteristics of PN junction are shown in fig17.11. The knee voltage is 0.3V for germanium and 0.7V for silicon. From the *VI* characteristics, we see that the forward current increases rapidly beyond the knee voltage. There exists a maximum current limit for junction, which is decided by power ratio of the junction. Beyond that, the junction is destroyed. In reverse bias, if voltage is increased, due to available energy, covalent bonds break and large number of electrons are released.

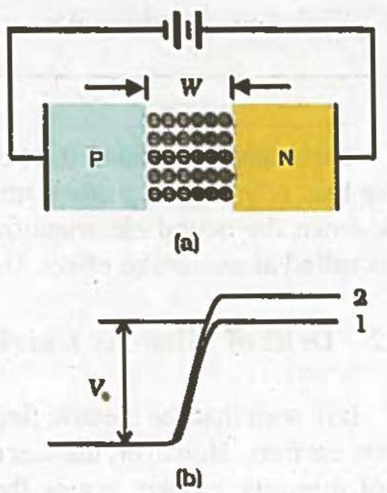


Figure 17.10: p-n junction diode under reverse bias current.

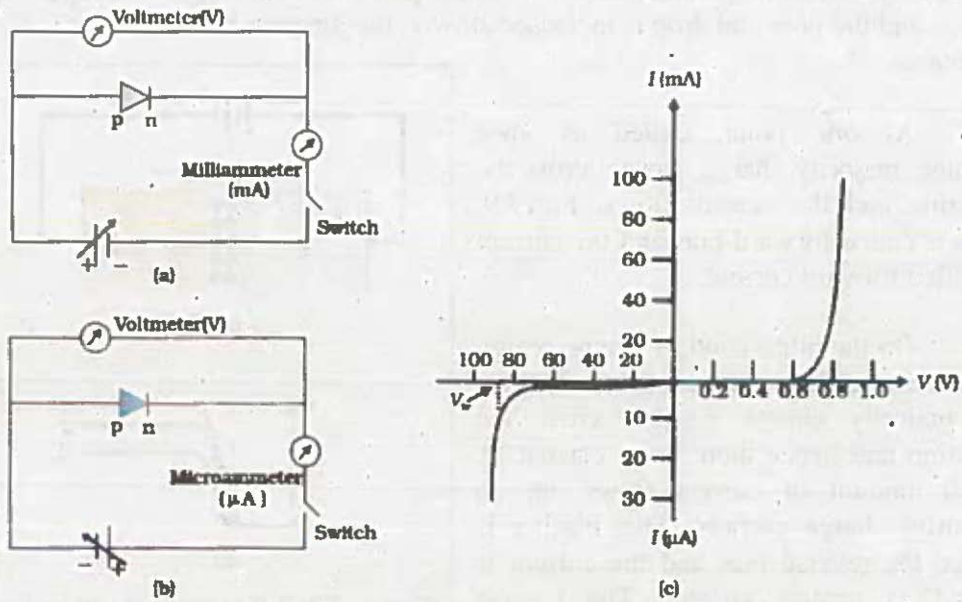


Figure 17.11: Experimental circuit arrangement for studying V-I characteristics of a p-n junction diode (a) in forward bias , (b) in reverse bias. (c) Typical V-I characteristics of a silicon diode.

This causes a sudden increase in current. This is called as zener effect. If reverse bias is increased further, minority charge carriers attain high velocity and knock down the bound electrons from covalent bonds and the current increases. This is called as avalanche effect. Using these effects, zener diodes are formed.

17.4.2 Drift of Minority Carrier

It is seen that the electric field across the junction prevents the diffusion of majority carriers. However, the electric field has the right direction to promote the flow of minority carriers across the junction. Electrons arriving at the junction from the bulk of P-region are assisted by the electric field to move into N-region. Similarly, holes in the N-region are helped to move into P-region. As a consequence, an electric current flows across the junction.

As the current is caused by an electric field it is a drift current. The net drift current through the junction is due to electron and hole which is given by.

$$I_{(drift)} = I_e + I_h$$

The minority carriers are generated through breaking of covalent bonds.

17.5 Rectification

Due to efficiency and safety reasons, alternating current (AC) is used for providing electrical power. But some electronic devices like electronically tune radio and TV receivers require DC for their operation. The DC supplies like cells, batteries etc. are expensive, low power, and short-lived. Therefore generally a DC supply is generated using an A.C. supply.

The conversion of AC into DC is called the rectification and a device used for rectification is called the rectifier.

Diodes provide compact, inexpensive means of rectification therefore it can be used as a rectifier. As we have seen, when diode is forward biased it allows the current to pass and in reverse bias it (almost) stops the current. Thus it can be used as unidirectional device (or rectifier). For most power applications, half-wave rectification is insufficient for task.
If we need to rectify AC power to obtain full use of both half-cycles of sine wave, different rectifier circuit configuration must be used. Such circuit is called full-wave rectifier.

17.5.1 Half wave rectifier

Simplest kind of rectifier circuit is half-wave rectifier. Fig17.12 (a) shows Half wave rectifier circuit which uses transformer to couple the ac input voltage from the source to the rectifier. Transformer coupling provides two advantages. First, it allows the source voltage to be stepped up or stepped down as needed. Second, the ac source is electrically isolated from the rectifier, thus preventing a shock hazard in the secondary circuit. The voltage across the secondary of the transformer, i.e. between A and B is represented as $V = V_m \sin\omega t$ and graphically it is shown in Fig 17.12 b. V_m is the peak value of the alternating voltage.

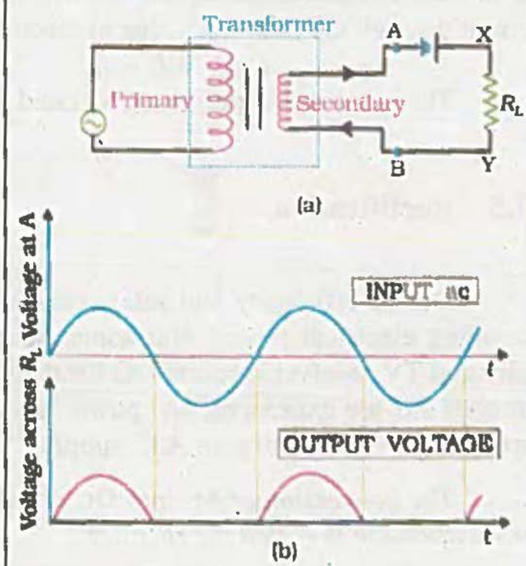


Figure 17.12 : (a) Half-wave rectifier circuit, (b) Input ac voltage and output voltage waveforms from the rectifier circuit.

During the positive half cycle of AC, the point A becomes positive with respect to B. The diode is forward biased and the current flows through the load. For negative half cycle, point A becomes negative with respect to B and the diode is reveres biased. Practically, no current flows through the load. Thus, only half wave is rectified and it is called half-wave rectifier. During the negative half cycle of AC, diode is reverse biased and the total voltage appears across the diode. Peak inverse voltage (PIV) is the maximum voltage V_m that the rectifying diodes has to withstand, when it is reversed-biased. During negative half-cycle of the input voltage, the diode is reversed biased, no current flows through the load resistance. R_L and so causes no voltage drop across load resistance R_L and consequently the whole of the input voltage appears across the diode. Thus the maximum voltage that appears across the diode is equal to the peak value V_{max}. Thus for a half-wave rectifier

$$PIV = V_{max}$$

17.5.2 Full-wave rectifier

One kind of full-wave rectifier, called center-tap design, uses transformer with center-tapped secondary winding and two diodes in alternate switching mode as shown in fig17.13. A full wave rectifier allows unidirectional (one way) current through the load during the entire cycle of the input cycle, whereas a half wave rectifier allows current through the load only during one half of the cycle.

The diodes D_1 and D_2 operate in alternate switching mode. For the first half cycle point, A becomes positive with respect to B and B becomes positive with respect to C. Thus, D_1 is forward biased and D_2 is reverse biased. The current through load is only due to D_1 while current due to D_2 is zero. For the second half cycle, point C becomes positive with respects to B and B becomes positive with respect to A. Now D_2 is forward biased ON and D_1 is reverse biased OFF. The current flows due to D_2. For the first half cycle, the current is due to D_1 and for the second half cycle, the current is due to D_2. Thus full wave is rectified.

One disadvantage of this full-wave rectifier design is necessity of transformer with center-tapped secondary winding.

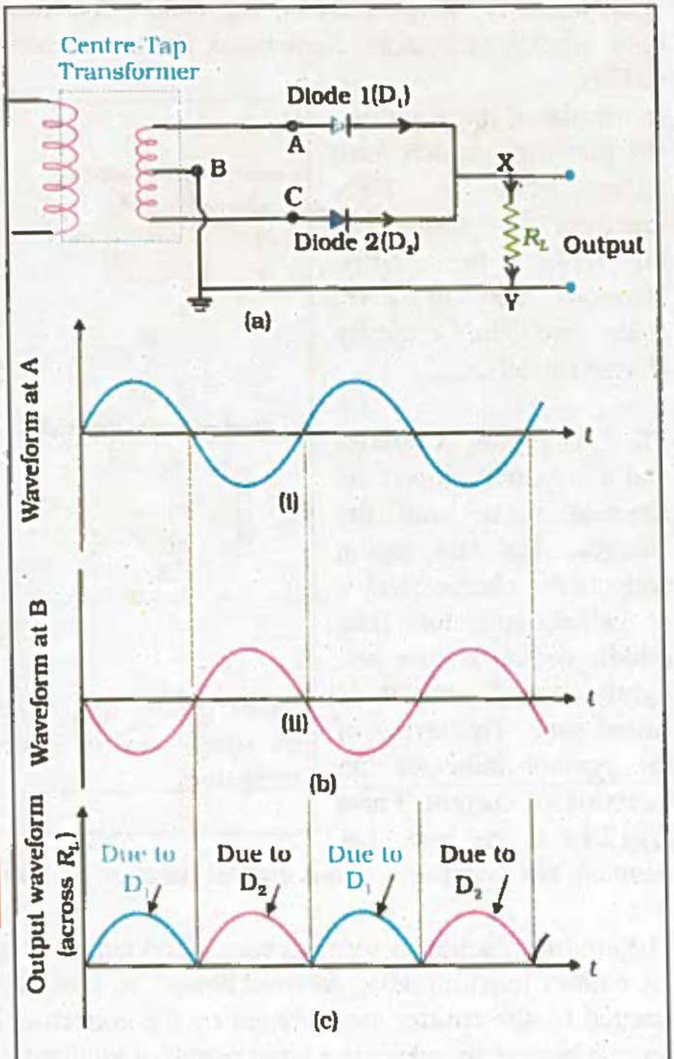


Figure 17.13:(a) A Full-wave rectifier circuit; (b) Input wave forms given to the diode D_1 at A and to the diode D_2 at B; (c) Output waveform across the load *RL* connected in the full-wave rectifier circuit.

17.6 Transistor

A transistor consists of three regions of doped semiconductors in which the current flowing is modulated by the voltage or current applied to one or more electrodes.

Modern transistors are of two types: bipolar, whose function depends upon both (majority and minority) charge carriers and unipolar, whose function depends upon majority charge carriers, e.g. field effect transistors. Here we shall study only bipolar transistors. Sometimes these are called bipolar junction transistor (BJTs).

In transistor, there are two PN junctions, which form either PNP or NPN transistor is shown in Fig17.14. In NPN, electrons and in PNP, holes are the majority charge carriers.

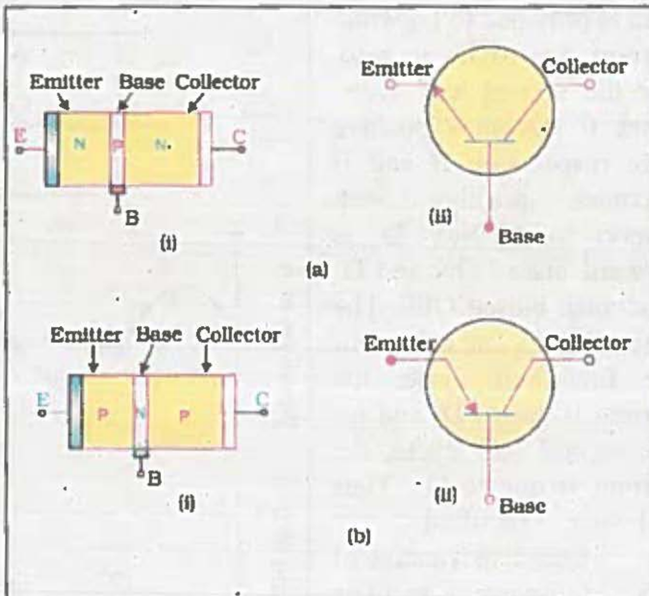


Figure 17.14: (a) N-P-N transistor and P-N-P transistor, and (b) Symbols for N-P-N and P-N-P transistors.

The first region is emitter and it is heavily doped. Its function is to emit the charges. The last region collects the charges and it is called collector. The middle region is thin and lightly doped and it is called base. The arrow in the symbol indicates the direction of current. From Fig17.14(a) we see that there are two junctions, one is emitter junction and other is collector junction.

The emitter junction is forward biased and collector junction is reveres biased. As the emitter junction being forward biased, so it offers low resistance. The charges emitted by the emitter are attracted by the collector. The collector junction being reverse biased, there exists a large potential gradient, which attracts the charges. If the collector is open (means not connected to power supply) charges return via base region. Most of the charges are collected by collector, therefore the collector current I_C is almost equal to the emitter current I_E.

Some charges, which cannot reach the collector, move via base, constituting the base current I_B, which is small. Thus addition of base current and collector current is equal to the emitter current.

Therefore,

$$I_E = I_B + I_C$$

The base is made thin and it is lightly doped, therefore, the recombination of charges is less and the transit time of charges is also less. There is no difference in operation of PNP and NPN transistor except the polarity of biasing. In most of the cases NPN, transistors are preferred because mobility of electrons is three times more than that of holes and therefore the operation is faster.

17.7 Types of configurations

In a transistor circuit, one electrode is common to both input and output circuits. Therefore there exist three types of configurations: common base (CB), common emitter (CE) and common collector (CC), as shown in Fig17.15.

17.7.1 Common base configuration

When a base is common to both input and output circuits, it is a common base configuration. Figure shows the circuit diagram and input and output characteristics of CB configuration.

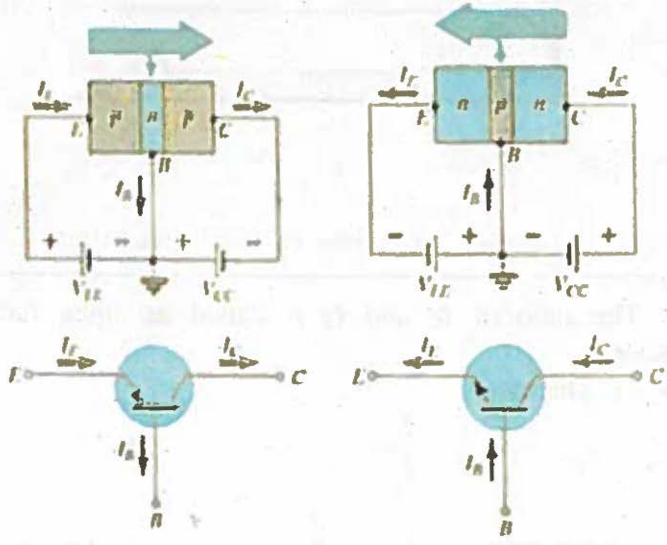


Figure 17.15: : transistors in CB mode: (a) circuit

The variation in the emitter current (I_E) with respect to change in the base-to-emitter voltage (V_{BE}) at the constant collector-to-base voltage (V_{CB}) is input characteristics and variation in the collector current (I_C) with change in collector-to-base voltage (V_{CB}) at the constant emitter current (I_E) is output characteristics. The output characteristic has three regions of operation, namely, active, cut-off and saturation. When the base-emitter junction is forward biased and the collector–base junction is revers biased, it is active region. When both, collector-base and base-emitter junctions are reverse biased it is cut-off region. The output current is zero in this case. When both the junctions are forward biased, it is saturation region. Figure shows these regions.

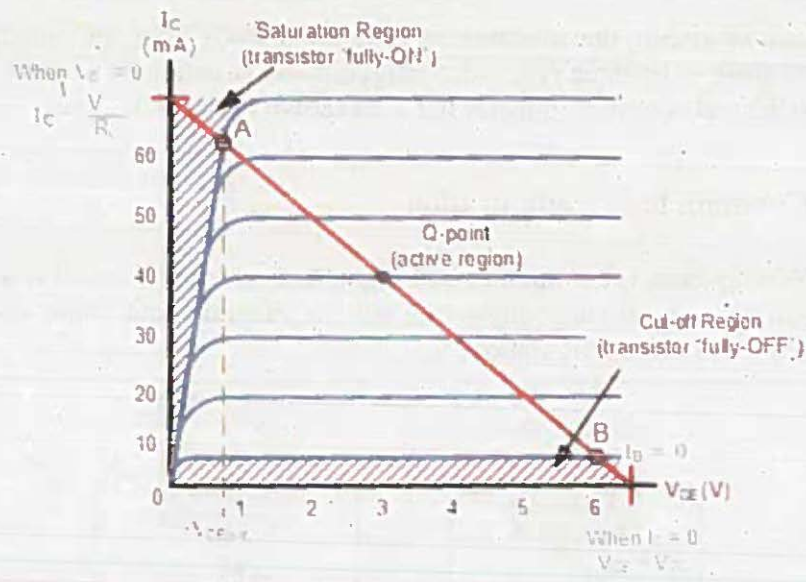


Figure 17.15: : transistors in CB mode: (b) output characteristics.

Alpha factor:. The ratio of I_C and I_E is called as alpha factor. It is an amplification factor.

Since $I_C \approx I_E$, $\alpha \approx 1$. Therefore,

$$\alpha_{static} = \frac{I_C}{I_E} \quad ...(17.3)$$

and $$\alpha_{dynamic} = \frac{\Delta I_C}{\Delta I_E}$$

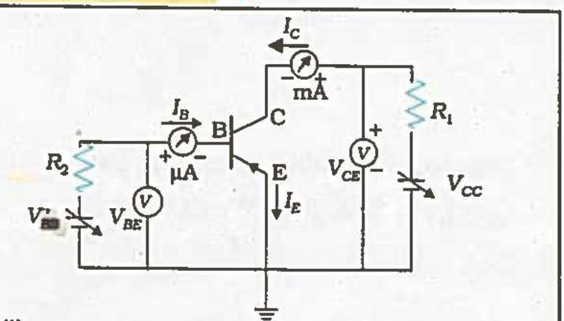
17.7.2 Common emitter configuration

In common emitter configuration the base emitter junction is forward based while base-collector junction is reverse biased.

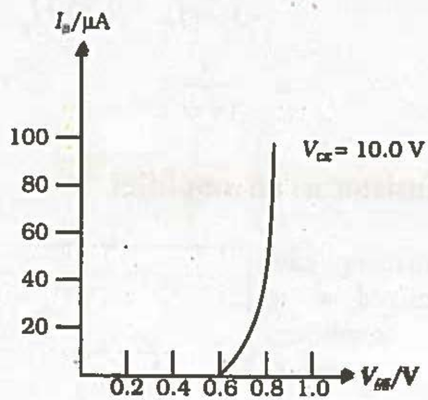
Fig17.16: shows circuit diagram, input and output characteristics, respectively. The variation in base current (I_B) with change in base-to-emitter voltage (V_{BE}) at constant collector-to-emitter voltage (V_{CE}) are input characteristics and variation in collector current (I_C) with change in collector-to-emitter voltage (V_{CE}) at constant base current (I_B) are output characteristics.

Fig: 17.16 b shows active, cut-off and saturation region of output characteristics.

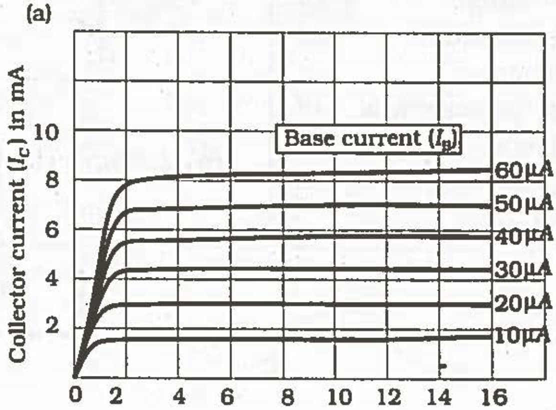
Beta factor: The ratio of collector current I_c and base current I_B is called as beta factor.



(i) Circuit arrangement for CE configuration.



(a)



(b) Collector to emitter voltage (V_{CE}) in volts

Figure 17.16: Transistor in CE mode: (i) circuit (a) input characteristics (b) output characteristics.

That is:

$$\beta_{static} = \frac{I_C}{I_B} \qquad ...(17.4)$$

and

$$\beta_{dynamic} = \frac{\Delta I_C}{\Delta I_B}$$

The beta factor is called as current gain or current amplification factor. Generally it ranges from 50 to 400. We can write,

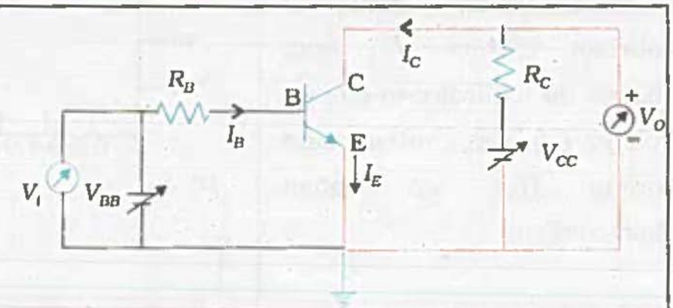
$$\beta = \frac{I_C}{I_B} \qquad ...(17.5)$$

$$= \frac{I_C}{I_E - I_C} = \frac{I_C / I_E}{1 - I_C / I_E}$$

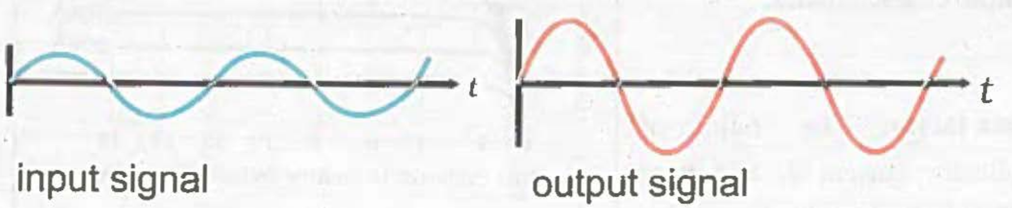
$$= \frac{\alpha}{1-\alpha} \qquad ...(17.6)$$

17.7.3 Transistor as an amplifier

A transistor can be characterized as a current amplifier, having many applications for amplification and switching. The arrangement of common emitter amplifier is shown in fig17.17.



(a) A simple circuit of a CE-transistor amplifier.



(b) Figure 17.17:

In which the input voltage appears across base and emitter, and the output voltage appears across the collector and emitter. i.e. the emitter terminal is shared by the input and output. An increase in voltage or current level is called as amplification. The transistor can be used as an amplifier. A small change in input (voltage or current) produces large change in output (voltage or current). If β is 100 then the change in collector current is 100 times the change in base current. The small change in base current produces large change in collector current.
Thus transistor acts as an amplifier. The energy required for amplification is taken from the power supply. In most of the cases, CE mode is preferred because its current and voltage gains are high and power gain is the highest.
At the input, the emitter junction is forward biased therefore input resistance is small and at the output, the collector junction is reverse biased therefore output resistance is high. The current is transferred from low-to-high resistance circuit.

17.7.4 Transistor as a switch

Transistor is used in a great variety of circuits. Transistor switches form the basis of all electronic computers. With the switch closed, base current flow causing collector to flow.
The output voltage is $V_{CE} = 0$ V (Fig17.18. **(a)**). The battery voltage is dropped across the load causing the collector voltage to fall to a very low value.
The transistor is said to be saturated. With the switch open no base current flow, therefore no collector current can flow. The transistor is said to be cut off. $V_{CE} \approx +V$ (Fig17.18 **(b)**).
These two states are described as 0 and 1, or low and high.

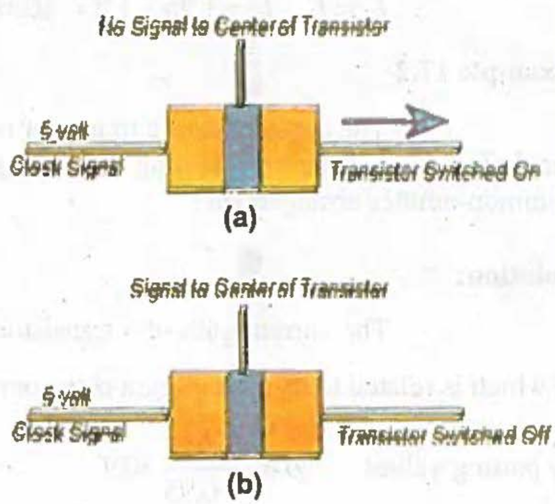


Figure 17.18: transistor on off switch

Example 17.1

A transistor is connected in a CE configuration. The collector supply voltage is 10V and the voltage drop across the 500Ω connected in the collector circuit is 0.6V. If α = 0.96, find the (a) collector-emitter voltage (b) base current, and (c) the emitter current.

Solution:

To collector current, $I_C = \frac{V_c}{R_c} = \frac{0.6}{500}$

$= 1.2\text{ mA}$

(1) Collector-emitter voltage $V_{CE} = V_{CC} - V_C = 10\text{-}0.6$

$=9.4\text{V}$

(2)

$$\alpha = \frac{I_c}{I_E} \quad or \quad I_E = \frac{I_C}{\alpha} = \frac{1.2}{0.96} = 1.25\text{ mA}$$

(3) $I_E = I_B + I_C$ or

$I_B = I_E - I_C = 1.25 - 1.2 = .05\text{mA}$

Example 17.2

The constant α of a transistor is 0.95. What would be the change in the collector current corresponding to a change of 0.4mA in the base current in a common-emitter arrangement?

Solution:

The current gain of a transistor in common-emitter arrangement is β, which is related to its current-gain α in common-base arrangement $\beta = \frac{\alpha}{1-\alpha}$

by putting values $\beta = \frac{0.95}{1-0.95} = 19$

But, β is the ratio of change in collector current to the change in base current.

$$\beta = \frac{\Delta I_c}{\Delta I_b}$$

$$or\, \Delta I_c = \beta \times \Delta I_b = 19 \times 0.4 \times 10^{-3}\text{A} = 7.6\text{mA}$$

Digital Electronics

Digital calculators, watches, modern communication systems and computers are widely used in everyday life. All persons working in various fields related to electronics must understand the performance of Digital Electronic Circuits. All sizes of computers, as we know, perform complicated task with fantastic speed and accuracy. At stores, the cash register read out digital display digital clock and watches flash the time in all city shops and restaurants. Most automobiles use microprocessors to control engine functions. Aircraft's defense sectors, factory machines and modern diagnostic in medical science are controlled by digital circuits.

The inexpensive fabrication of integrated circuits (ICs) has made the subject Digital Electronics easy to study. One small IC can perform the task of thousands of Transistors Diodes and Resistors. Many ICs are used to construct Digital Circuits. This is an exciting and rapidly growing field, which uses several principles for the working of computers, Communication systems, Digital machinery's etc.

Any device working under Digital Techniques are called Digital Systems and the Electronic Network used to make them operational is called Digital Circuits. The subject as a whole is often referred as Modern Digital Electronics.

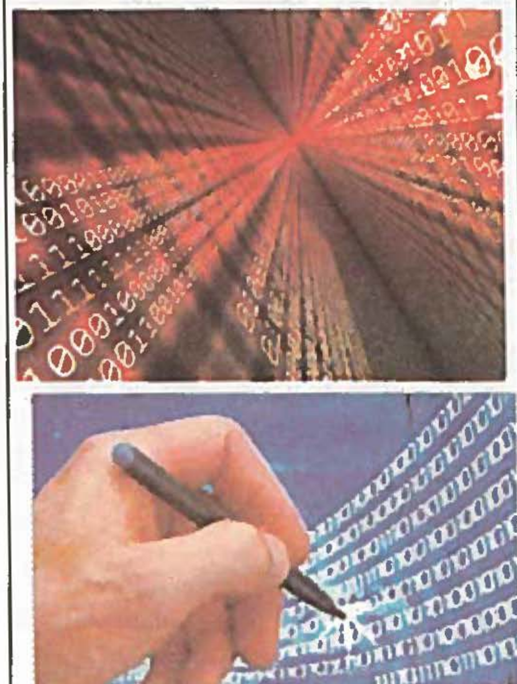


Figure 17.19

17.8 Optoelectronic junction devices

In semiconductor diodes carriers are generated by photons (photo-excitation). All such devices are called optoelectronic devices. Three commonly used such diodes are

(i) Photodiodes
(ii) Light emitting diodes (LED)
(iii) Photovoltaic

(i) Photo Diode

A Photodiode is a P-N junction diode, operated under reverse bias. When the photodiode is illuminated with light with energy ($h\nu$) greater than the energy gap (E_g) of the semiconductor, then electron-hole pairs are generated due to the absorption of photons. The diode is fabricated such that the generation of electrons holes pairs takes place in or near the depletion region of the diode. Due to electric field of the junction, electrons and holes are separated before they recombine. Electrons are collected on N-side and holes are collected on P-side giving rise to an e.m.f. When an external load is connected, current flows. The magnitude of the photocurrent depends on the intensity of incident light (photo current is proportional to incident light intensity). Hence photodiode can be used for the detection of optical signals.

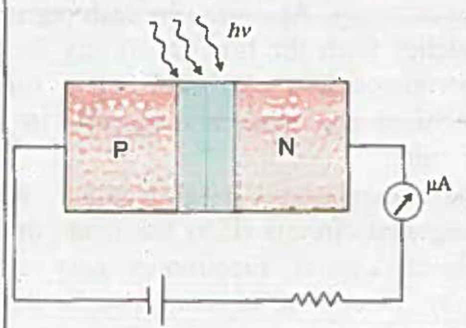


Figure 17.20: An illuminated photodiode under reverse bias

(ii) Light emitting diode

It is a heavily doped P-N junction which under forward bias emits spontaneous radiation.

When the diode is forward biased, electrons are sent from N → P (where they are minority carriers) and holes are sent

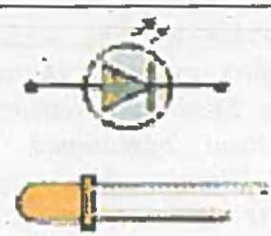


Figure17. 21: LED

from P → N (where they are minority carriers). At the junction boundary the concentration of minority carriers increases compared to the equilibrium concentration (i.e., when there is no bias). On either side of the junction, excess minority carriers are there which recombine with majority carriers near the junction. As a result energy is released in the form of photons. The diode is encapsulated with a transparent cover so that emitted light can come out (diode converts electrical energy into light). These LEDs are widely used in remote controls, burglar alarm systems, optical communication, etc. LEDs have fast action, long life and endurance and fast on-off switching capability.

(iii) Solar cell

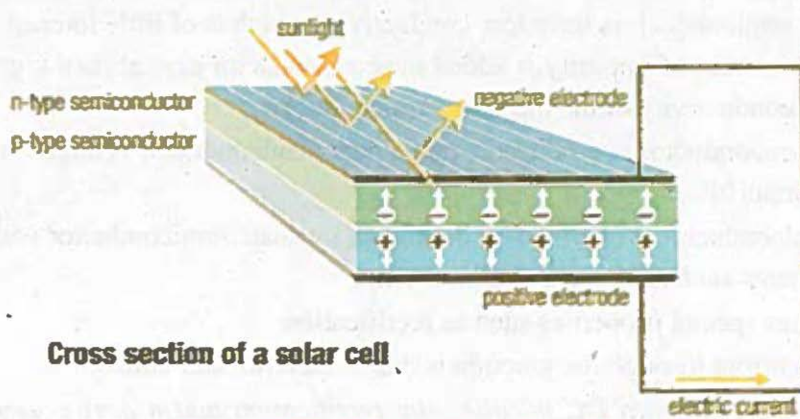


Figure 17.22 : P-N junction solar cell

A solar cell is also a P-N junction which generates e.m.f. when solar radiation falls on the P-N junction. It works on the same principle (photovoltaic effect) as the photodiode, except that no external bias is applied and the junction area is kept much larger. A simple P-N junction solar cell is shown in Fig17.22. The generation of emf by a solar cell, when light falls on, it is due to the (i) generation of e-h pairs (ii) separation of electrons and holes .(Electrons are swept to N-side and holes to P-side) (iii) the electrons reaching the N-side are collected by the front contact and holes reaching P-side are collected by the back contact. Thus P-side becomes positive and N-side becomes negative giving rise to photo voltage. When an external load is connected as shown in the Fig. 17.22 (a) a photocurrent I_L flows through the load. Solar cells are used to power electronic devices in satellites and space vehicles and calculators.

Key points

Semiconductors are materials having an intermediate electric conductivity between conductors and insulators.

- Chemically pure semiconductors are known as intrinsic semiconductors.
- In pure semiconductors, a single event of bond breaking leads to two carriers, namely an electron and a hole.
- The minority carriers are generated through breaking of covalent bonds.
- The intrinsic semiconductors have low conductivity which is of little interest. But, when a small amount of impurity is added to semiconductor crystal then it greatly increases the conductivity of the intrinsic semiconductor.
- An N-type semiconductor is produced when a pure semiconductor is doped with a pentavalent impurity.
- A P-type semiconductor is obtained by doping an intrinsic semiconductor with trivalent elements such as boron and aluminum.
- PN junction has special properties such as rectification.
- The net drift current through the junction is due to electron and hole.
- *The conversion of AC into DC is called the rectification and a device used for rectification is called the rectifier.*
- In transistor, there are two PN junctions, which form either PNP or NPN transistor.
- An increase in voltage or current level is called amplification. The transistor can be used as an amplifier.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. In an N-type silicon, which of the following statement is true:
   (a) Electrons are majority carriers and trivalent atoms are the dopants.
   (b) Electrons are minority carriers and pentavalent atoms are the dopants.
   (c) Holes are minority carriers and pentavalent atoms are the dopants.
   (d) Holes are majority carriers and trivalent atoms are the dopants.
2. The reverse saturation current in a PN junction diode is only due to
   (a) majority carriers (b) minority carriers
   (c) acceptor ions (d) donor ions
3. Improper biasing of a transistor circuit produces
   (a) heavy loading of emitter current
   (b) distortion in the output signal
   (c) excessive heat at collector terminal
   (d) faulty location of load line
4. When transistors are used in digital circuits they usually operate in the:
   a. active region
   b. breakdown region
   c. saturation and cutoff regions
   d. linear region
5. Most of the electrons in the base of an NPN transistor flow:
   a. out of the base lead b. into the collector
   c. into the emit d. into the base supply
6. In a transistor, collector current is controlled by:
   a. collector voltage b. base current
   c. collector resistance d. all of the above

Conceptual Questions

1. Explain the formation of depletion region in a PN-junction.

2. Explain why in a transistor (a) the base is thin lightly doped and (b) the collector is large in size.
3. Explain why the base current is weak as compared to collector current?
4. Why the emitter base junction is forward biased and collector base junction is reverse biased?
5. Draw the diagram of NPN and PNP transistors and explain how it works.
6. Distinguish between N-type semiconductors and P-type semiconductors.
7. A P-type semiconductor has a large number of holes but still it is electrically neutral. Why?
8. Explain why CE configuration is widely used in amplifier circuits?
9. Why transistor is called current amplification device?
10. A doped semiconductor has 10^{10} silicon atoms and 10 trivalent atoms. If the temperature is 25C°, how many free electrons and holes are there inside the semiconductor?

Comprehensive questions

1. Describe the energy band structure of insulator, semiconductor and conductor.
2. Explain the significance of depletion layer in an equilibrium state in a PN-junction. Give energy band diagram.
3. Explain how PN-junction acts as a half-wave rectifier.
4. Explain the working of transistor as an amplifier?
5. Draw the circuit for a half wave rectifier and full wave rectifier.
6. Compare the advantages and disadvantages of full wave rectifier and half wave rectifier.
7. Deduce the relation between α and β of a transistor.
8. Explain what is meant by the following terms.

   (a) P-type and N-type materials
   (b) Doping of semiconductors
   (c) P-N junction
   (d) Forward biasing
   (e) Reverse biasing
   (f) Minority carriers
   (g) Majority carriers

9. Discuss the conductivity of extrinsic semiconductor and its band gap energy.
10. Explain the formation of depletion region in a PN-junction.
11. What causes majority carriers to flow at the moment when P-region and N-region are brought together? Why does this flow not continue until all the carriers have recombined?
12. Discuss the carrier's movements across the emitter base and collector base junctions.
13. What is the effect of increasing the junction temperature of a diode on the reverse saturation current?
14. In a transistor the emitter and collector are of the same type of semiconducting material. Yet they cannot be interchanging in a circuit connection. Explain.
15. Is the frequency content of the output of a half wave rectifier and full wave rectifier the same? Explain.
16. Describe the advantages of digital electronics.

Numerical Problems

1. In a certain circuit, the transistor has a collector current of 10 mA and a base current of 40 μ A. What is the current gain of the transistor?

   (250)

2. The current flowing into the base of a transistor is 100 μ A. Find its collector current I_C, its emitter current I_E, and the ratio I_C / I_E if the value of current gain β is 100.

   (10 mA, 10.1mA, 0.99)

3. A transistor is connected in CE configuration. The voltage drop across the load resistance (R_C) 3 k Ω is 6 V. Find the base current. The current gain β of the transistor is 0.97.

   (2.06×10^{-3} A)

UNIT 18

...... Dawn of the Modern Physics.....

After studying this chapter students will be able to:

- distinguish between inertial and non-inertial frames of reference.
- describe the significance of Einstein's assumption of the constancy of the speed of light.
- identify that if c is constant then space and time become relative.
- explain qualitatively and quantitatively the consequence of special relativity in relation to:
  - the relativity of simultaneity
  - the equivalence between mass and energy
  - length contraction
  - time dilation
  - mass increase
- explain the implications of mass increase, time dilation and length contraction for space travel.
- describe the concept of black body radiation.
- describe how energy is distributed over the wavelength range for several values of source temperature.
- describe the Planck's hypothesis that radiation emitted and absorbed by the walls of a black body cavity is quantised.
- elaborate the particle nature of electromagnetic radiation.
- describe the phenomenon of photoelectric effect.
- solve problems and analyze information using: $E = hf$ and $c = f\lambda$.
- identify data sources, gather, process and present information to summarise the use of the photoelectric effect in solar cells & photocells

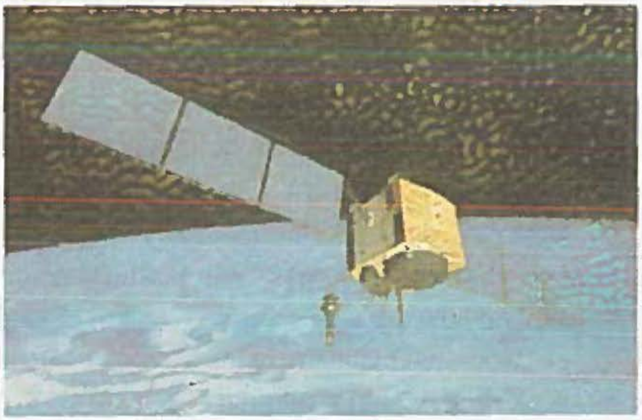
- describe the confirmation of de Broglie's proposal by Davisson and Germer experiment in which the diffraction of electrons by the surface layers of a crystal lattice was observed.
- describe the impact of de Broglie's proposal that any kind of particle has both wave and particle properties.
- explain the particle model of light in terms of photons with particular energy and frequency.
- describe Compton effect qualitatively.
- explain the phenomena of pair production and pair annihilation.
- explain how the very short wavelength of electrons, and the ability to use electrons and magnetic fields to focus them, allows electron microscope to achieve very high resolution.
- describe uncertainty principle.

Modern Physics came with the 20^{th} century and took over where the Newtonian Physics had fallen short. During the 19^{th} century, Newton's laws of physics were firmly established, the behaviour of all types of objects and systems could be predicted with very high degree of accuracy. Further the nature of light had been explained in terms of electromagnetic waves. Just about the time when the physicists started feeling that all the major problems of physics have been solved, there came a series of results of new experiments which could not be explained on the basis of the existing laws of physics. These experiments were concerned with extremely small objects and moving with extremely large velocities. Tremendous amount of hard work and thinking followed and it turned out that these results could only be explained if new concepts and new laws were introduced. Soon a new mechanics followed which were more general and more basic then the Newtonian mechanics. These developments meant complete overhaul and modernisation of the existing physics. Hence named Modern Physics.

Modern Physics does not discard Newtonian Physics. Modern Physics is a set of more general concepts and laws which readily change into the older concepts and laws for those objects and velocities with which we come across in every day life. It should further be pointed out that the so –called modern physics concepts are by no means the last word in physics. Even the 20th century physics has its problems and the coming years may witness a yet another revolution in the concepts and laws of physics.

This chapter is about the two great theories of modern physics, the theory of relativity and the quantum theory. Both theories, Discovered at the end of 19th century, revolutionized physics in the 20th century. The quantum theory is about observations, processes and interactions involving events at a sub microscopic scale. Information technology and communication would not have developed without the quantum theory which provide the theoretical basis for devices such as transistor and integrated circuits.

For your information



A GPS satellite is a satellite used by the NAVSTAR Global Positioning System (GPS) (Navigation System using Timing And Ranging). The Global Positioning System (GPS) is a space-based satellite navigation system that provides location and time information in all weather conditions, anywhere on or near the Earth. According to relativity theory, a moving clock appears to run slow with respect to a similar clock that is at rest. The satellites are constantly moving relative to observers on the Earth which causes them to run at a slightly faster rate than do clocks on the Earth's surface. A calculation using General Relativity predicts that the clocks in each GPS satellite should get ahead of ground-based clocks by 45 microseconds per day. So the role of special theory of relativity is important in Global poisoning system.

Relativity theory is about the nature of space, time, mass and energy. Nuclear power and discoveries such as quarks and black holes are the consequences of the theory of relativity.

18.1 Reference Frames

All motion must be measured in some particular reference frame, which we usually represent as a set of coordinate axis as shown in (fig 18.1)

Suppose two people walk hand –in –hand on a moving sidewalk in an airport. They might walk at 1.3 m/s with respect to a reference frame attached to the moving side walk and at 2.4 m/s with respect to reference frame attached to the building. The two reference frames are equally valid.

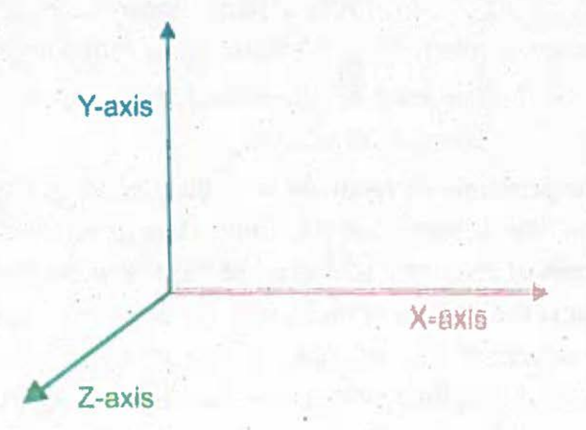


Figure 18.1 : coordinate axis

An inertial reference frame is one in which no accelerations are observed in the absence of external forces. In non-inertial reference frames, bodies have accelerations in the absence of applied forces because the reference frame itself is accelerating with respect to an inertial frame. In such frames, Newton's laws do not hold good unless appropriate pseudo force are added to the real force on the other hand Newton's laws hold good in all inertial reference frames with out the addition of fictitious forces.

For many purposes, the Earth's surface can be considered to be an inertial frames, even though strictly speaking it is not. The Earth's rotation causes phenomena such as the rotatory motion of hurricanes and trade wind, which in a

reference from attached to the Earth's surface, are acceleration not caused by the applied force.

Any reference frame that moves with constant velocity with respect to an inertial frame is itself inertial; if the acceleration of a body in one inertial frame is zero, its acceleration in any of the other inertial frames is also zero.

18. 2 Special Theory of Relativity

In 1905, Albert Einstein (1879 -1955) formulated his special theory of relativity on the basis of the following two postulates.

1. The laws of physics are the same in all inertial reference frames. (The principle of relativity)

The principle of relativity was first stated by Galileo and embodied in Newton's first law. It states that it is impossible to perform an experiment with in an inertial frame of reference to detect the motion of the frame of reference. The only way to detect the motion of an inertial frame of reference is by referring to another frame of reference. For example, if you are in a spacecraft far from any planet, star or other object, then you cannot observe that you are moving.

Your rockets have long been turned off and you are coasting along to your destination with uniform velocity. However, without referring to outside objects, it is impossible to measure or even detect your velocity.

2. The speed of light in vacuum is the same in all inertial reference frames, regardless of the motion of the source or the observer. (Principle of the constancy of the speed of light).

Imagine that you are sitting in a train facing forwards. The train is moving at the speed of light you hold up a mirror in front of you, at arm's length will you be able to see your reflection in the mirror? Yes, the reflection will be seen because, according to the principle of relativity, it would not be possible for any person in train to do any thing to detect the constant motion with which he or she is traveling. In classical physics, space (That is, displacement, position and velocity,

including the speed of light) can be relative to an observer, but time is an absolute quantity, passing identically for every body.

In the theory of relativity, which assumes that velocity of light is constant for all observers, then time is relative as well as space. In other words, time passes differently for different observers, depending upon how fast they are moving.

Check Point

What happens to the density of an object as its speed increases?

18.3 Consequences of Special Theory of Relativity

Some of the applications of the postulates of the special theory of relativity are summarized in the following without going into their mathematical derivations.

The relativity of simultaneity:

If two events in different places are judged by one observer to be simultaneous then they will not generally be judged to be simultaneous by another observer in different reference frame in relative motion. In other words, whether or not two events are seen by you to be simultaneous depends upon where you are standing.

Let a train is fitted with light operated doors. The light in the centre of the roof, and is operated by a train traveler standing in the middle of the floor. When the train is traveling at half the speed of light, the train traveler turns on the light. The light travels forwards and backwards with equal speed and reach both doors at the same time. The doors then open, and the train traveler sees them opening simultaneously. An observer standing outside the train watches that happen, but sees the back door opening before the front, This is because the back door is advancing on the light waves coming from the light, while the front door moving away from the light waves.

The Equivalence Between Mass and Energy:

The rest mass of an object is equivalent to certain quantity of energy. Mass can be converted into energy under extraordinary circumstances and, conversely, energy can be converted into mass. For example, part of the mass is converted into energy in nuclear fission reactions. When a particle and its anti –particle collide, the entire mass is converted into energy.

Einstein's famous equation expresses the equivalence between energy, E and mass, m : $E = mc^2$. The amount of energy given off in a nuclear transmutation is related by this relation to the amount of mass lost. In special theory of relativity, the law of conservation of Energy and the law of conservation of mass have been replaced by the law of conservation of mass–Energy.

Check Point

It is said that Einstein, asked the question, "What would I see in a mirror, is carried in my hand and at the speed of light"? How would you answer this question?

Example 18.1

The rest mass of an electron is $9.11\times10^{-31}\text{kg}$. Calculate the corresponding rest energy.

Solution:

Rest mass energy of electron

$$E = m_o c^2$$

$$E = (9.11\times10^{-31}\text{kg})(3\times10^{8}\ \text{m/s})^2$$

$$E = 8.199\times10^{-14}\,\text{j}$$

$$E = 0.512\,\text{MeV}$$

Length Contraction:

The length of an object measured within its rest frame is called proper length (L_o). Observers in different reference frames in relative motion will always measure the length (L) to be shorter.

$$L = L_o\sqrt{1-\frac{v^2}{c^2}} \qquad ...(18.1)$$

Let a train that is measured to be 100 meters long when at rest, travels at 80% of the speed of light (0.8c). A person inside the train will measure the length of the train to be 100m. A person standing by the side of the track will observe the train to be just 60 meters long. This effect of relativity i,e. shortening of length in direction of motion is called length contraction

Example 18.2

A spaceship is measured 100 m long while it is at rest with respect to an observer of this spaceship now flies by the observer with a speed of 0.99c. What length will the observer find for the spaceship?

Solution:

We know from equation 18.1

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 100\,\mathrm{m}\sqrt{1-\frac{(0.99c)^2}{c^2}}$$

$$L = 14\,\mathrm{m}$$

Time Dilation:

The time taken for an event to occur within its rest frame is called proper time (t_o). Observers in different reference frames in a relative motion will always judge the time taken (t) to be longer.

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad ...(18.2)$$

For example: A traveler on a train with a speed of 0.8c, picks up and open a newspaper. The event takes 1.0 s as measured by the train traveler. As observed by a person standing by the side of the track the event takes 1.7 s.

Mass Dilation:

Another consequence of the theory of special relativity is that the mass of a moving objects increases as its velocity increases. This is the phenomenon of mass dilation. It is another expression of the mass energy equivalence and is represented mathematically as:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad ...(18.3)$$

Where m= relativistic mass of particle.

m_o =rest mass of particle

v=the velocity of the particle relative to a stationary observer.

c = speed of light.

This effect is noticeable only at relativistic speed. As an object is accelerated close to the speed of light its mass increases. The more massive it become, the more energy that has to be used to give it the same acceleration, making further acceleration more and more difficult. The energy that is put into attempted acceleration is instead converted into mass. The total energy of an object is then its K.E plus the energy embodied in its mass.

To accelerate even the smallest body to the speed of light would require an infinite amount of energy. Thus material object are limited to speed less then the speed of light.

Check Point

Since mass is a form of energy, can we conclude that a compressed spring has more mass than the same spring when it is not

Example 18.3

Superman, who has an exceptionally strong arm, throws a fast ball with a speed of 0.9c. If the rest mass of the ball is 0.5 kg, what is its mass in flight?

Solution:

Using equation 18.3 we have

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{0.5\text{ kg}}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} = 1.15\text{kg}$$

Application of time dilation and length contraction for space travel:

Let a spaceship is traveling to a star at a half of the speed light. Let it take eight years to reach to the star, from the point of view of the observers on Earth. From the Earth's point of view the clocks on the spaceship are moving slowly, so that less time passes on the spaceship compared to the Earth.

From the point of view of the spaceship" occupants, the length of the journey has contracted to a significantly shorter distance, which they cover in less time. Hence the occupants of the spaceship recorded seven years to reach to their destination, rather than eight years. Hence the current maximum velocities do not allow for viable interstellar travel because the travel times are prohibitively long.

18.4 Black Body Radiation:

An object at any temperature is known to emit radiation sometimes referred to as thermal radiation. The characteristics of this radiation depends on the temperature and properties of the object. At low temperature, the wavelength of the thermal radiation are mainly in the infrared region and hence are not observed by the eye. As the temperature of the object is increased, it eventually begins to glow red. At sufficiently high temperatures, it appears to be white, as the glow of the hot tungsten filament of a light bulb.

A careful study of thermal radiation shows that it consists of a continuous distribution of wavelengths from the infrared, visible, and ultraviolet portions of the spectrum. From a classical viewpoint, thermal radiations (electromagnetic waves) originate from accelerated charged particles near the surface of the object, which emit radiation much like small antennas. The thermally agitated charges can have a distribution of acceleration, which accounts for the continuous spectrum of radiation emitted by the object. By the end of the 19th century, it had become apparent that the classical theory of thermal radiation was inadequate. The basic problem was in understanding the observed distribution of wavelength in the radiation emitted by a black body. By definition a black body is an ideal system that absorbs all radiation incident on it.

For your Information

The radiation represents a conversion of a body's thermal energy into electromagnetic energy, and is therefore called thermal radiation.

Person's energy is radiated away in the form of infrared energy.

A good approximation to a black body is the inside of a hollow object, as shown in Fig 18.2. The nature of the radiation emitted through a small hole leading to the cavity depends only on the temperature of the cavity walls. Experimental data for the distribution of energy for black body radiation at three different temperatures are shown in fig 18.3.

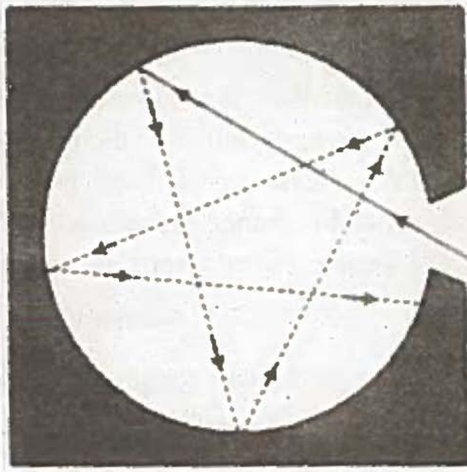


Figure 18.2: black body

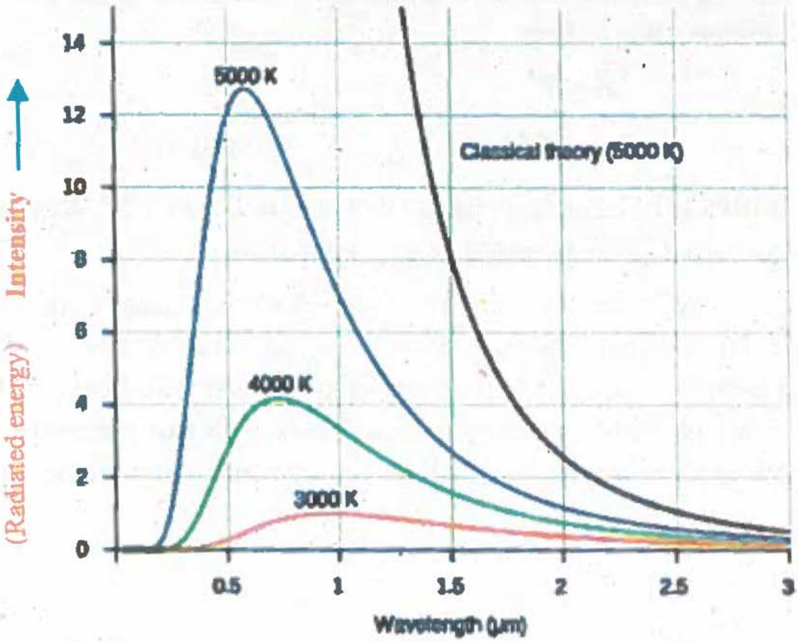


Fig 18.3 intensity of black body radiation versus wavelength at three different temperature

Note that the total radiation emitted (the area under the curve) increases with increasing temperature.

Note that the radiated energy varies with wavelength and temperature. As the temperature of the black body increases, the total amount of energy it emits increases. Also, with increasing temperature, the peak of the distribution shifts to shorter wavelengths. This shift was found to obey the following relationship, called wein's displacement law:

$$\lambda_{max}T = \text{constant} = 0.2898\times10^{-2}\,\text{m}\cdot\text{k} \cdots\cdots\cdots 18.4$$

Where λ_{max} is the wavelength which the curve peaks and T is the absolute temperature of the object emitting radiation.

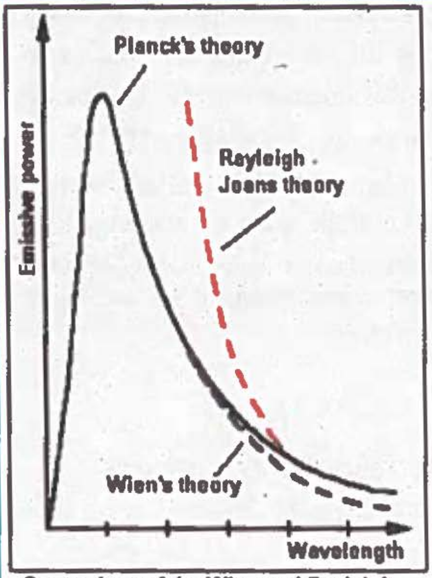
Also the area under each curve represents the total energy E radiated per second per square meter over all wavelengths at a particular temperature. It is found that area is directly proportional to the fourth power of absolute temperature T. Thus

$$E \propto T^4$$

$$E = \sigma T^4 \qquad \cdots(18.5)$$

Where σ is called Steffen's constant. Its value is $5.67\times10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$ and the above relation is known as Steffan –Bolts Mann law.

A series of attempts were made to explain the black body spectrum. For example, Raleigh –jean advanced a theory that could predict the long wavelength region but failed completely at short wavelengths. On the other hand, Wein's presented a theory, which agrees with experimental curve in the short wavelength region but fails in the long wavelength region, Fig 18.4. (a)



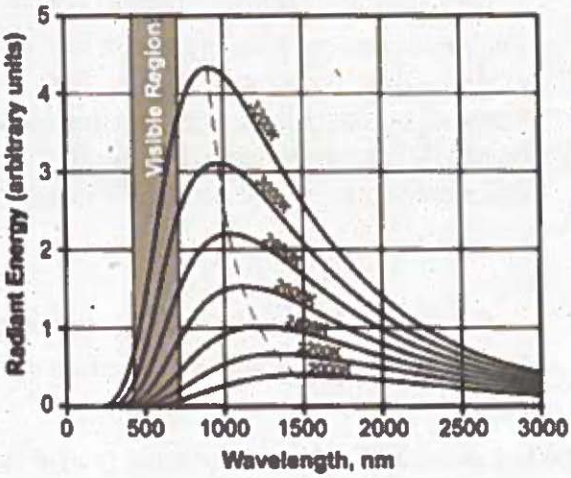


Figure 18.4: (a) (b)

Plank's Quantum Theory:

In 1900, Plank discovered a formula for blackbody radiation that was in complete agreement with experiments at all wavelengths. In his theory, plank made two assumptions, which at that time were bold and controversial, concerning the nature of the oscillating charges of the cavity walls.

1. The vibrating molecules which emitted the radiation have only certain discrete amount of energy, E_n given by.

$$E_n = n\,hf \qquad \cdots(18.6)$$

$n = 1, 2, 3, \cdots\cdots$

Where n is a positive integer called a quantum number and f is the frequency of vibration of the molecules. The energies of the molecules are said to be quantized, and the allowed energy state are called quantum states. The factor "h" is a constant, known as plank's constant. given by $h = 6.626\times10^{-34}$ js.

2. The molecules emit energy in discrete units of light energy is called quanta or photon. They do so by jumping from one quantum state to another. If the quantum number changes by one unit then energy of that the molecule is equal to hf. Hence, the energy of a light quantum, corresponding to the energy difference between two adjacent levels, is given by.

$$E = hf \qquad ...(18.7)$$

The molecule will radiate or absorbed energy only it changes quantum states. If it remain in one quantum state, no energy is absorbed or emitted.

The key point in Plank's theory is the radical assumption of quantized energy state, this development marked the birth of the quantum theory.

Example 18.4

The temperature of the skin is approximately 35°C. What is the wavelength at which the peak occurs in the radiation emitted from the skin?

Solution:

$$\lambda_{max} T = 0.2898\times10^{-2}\ m\cdot k$$

Solving for λ_{max} noting that 35°C is corresponds to an absolute temperature of 308k ,

We have ,

$$\lambda_{max} = \frac{0.2898\times10^{-2} m\cdot k}{308k}$$

$$\lambda_{max} = 9.4\,\mu m$$

This radiation is in the infrared region of the spectrum.

18.5 Photoelectric Effect

We know that metals, when heated, emit electrons. Can electrons be released from metals by light? It has been observed that metals, when exposed to electromagnetic radiations such as x – rays, γ – rays, visible, and infra –red light, emit electrons. This phenomenon is called photoelectric effect and the emitted electrons are called photoelectrons because they are liberated by means of light. The first discovery of this phenomenon was made by Hertz, who was also the first to produce the electromagnetic waves predicted by Maxwell.

(Fig 18.5) is a schematic diagram of an apparatus in which the photoelectric effect can occur. An evacuated glass tube contains a metal plate, *C*, connected to the negative terminal of a battery.

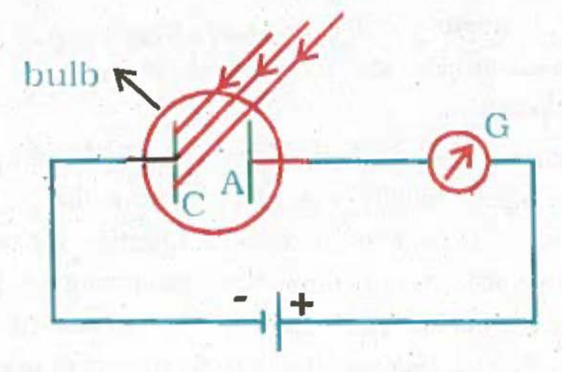


Figure 18.5 : circuit diagram for observing the photoelectric effect.

Another metal plate, A, is maintained at a positive potential by the battery. When the tube is kept in dark, the galvanometer, G, reads zero, indicating that there is no current in the circuit. However, when monochromatic light of the appropriate wavelength shine on plate, *C*, a current is detected by the galvanometer, indicating a flow of charges across the gap between *C* and A. The current associated with this process arises from electrons emitted from the negative plate and collected at the positive plate.

A plot of the photo-electric effect current versus the potential difference, *V*, between A and *C* is shown in (fig 18.6.a) for three light intensities.

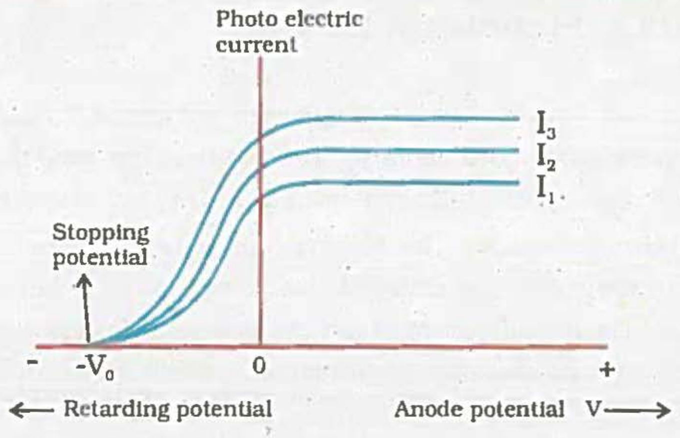


Figure 18.6 (a): Variation of photoelectric current with potential difference.

Note that for large values of V, the current reaches a maximum value, corresponding to the case where all photo-electrons are collected at A.

In addition, the current increases as the incident light intensity increases, as you might expect. Finally, when V is negative, that is, when the battery in the circuit is reversed to make C positive and A negative, the photoelectrons are repelled by the negative plate A. only those electrons having a K.E greater than eV will reach A, where e is the charge on electron. Furthermore, if V is greater than or equal to V_o, called the stopping potential, no electrons will reach A and the current will be. zero.

The stopping potential is independent of the radiation intensity. The maximum K.E of the photoelectrons is related to the stopping potential through the relation.

$$K \cdot E_{max} = eV_o \qquad (18.7)$$

Effect of intensity of incident radiation on photo electric current

Keeping the frequency of the incident radiation and the potential difference between the cathode and the anode at constant values, the intensity of incident radiation is varied. The corresponding photoelectric current is measured in the microammeter.

It is found that the photo electric current increases linearly with the intensity of incident radiation (Fig 18.6. b). Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, it implies that the number of photoelectrons emitted per second is proportional to the intensity of incident radiation.

Experimental Results:

This experiment yields the following interesting results.

1. Brighter light causes an increase in current (more electrons ejected) but does not cause the individual electrons to gain higher energies. In other words, the maximum K.E of the electrons is independent of the intensity of the light. Classically more intense light has larger amplitude and thus delivers more energy. That should not only enable a larger number of electrons to escape from the metal; it should also enable the electrons emitted to have more K.E.

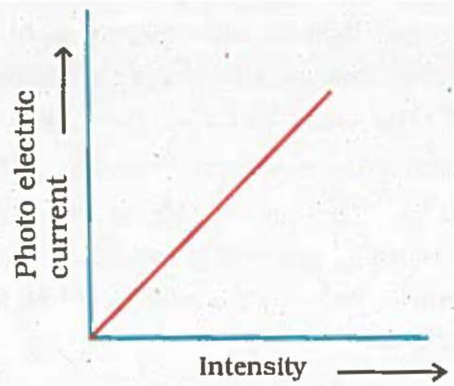


Figure 18.6 (b): Variation of photoelectric current with intensity of incident radiation.

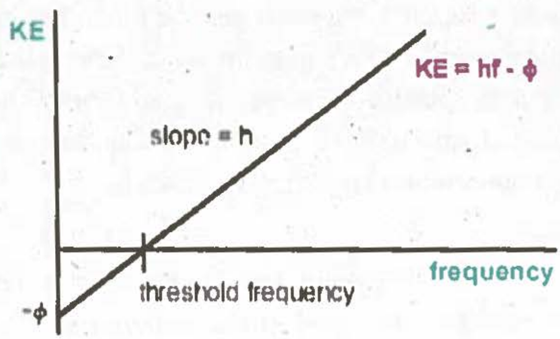


Figure 18.7: Variation of stopping potential with frequency of incident radiation.

2. The maximum K.E of emitted electrons depends on the frequency of the incident radiation (Fig 18.7).

Thus, if the incident light is very dim (low intensity) but high frequency, electrons with large K.E are released. Classically, there is no explanation for a frequency dependence.

3. For a given metal, there is a threshold frequency f_0. If the frequency of the incident light is below the threshold frequency, no electrons are emitted no matter what the intensity of the incident light is. Again, classical physics has no explanation for the frequency dependence.
4. Electrons are emitted from the surface almost instantaneously (less than 10^{-9}s after the surface is illuminated), even at low light intensities. Classically, one would expect that the electrons would require some time to absorbs the incident radiation before they acquire enough K.E to escape from the metal.

Photon Theory of Photoelectric Effect:

A successful explanation of the photo electric effect was given by Einstein in 1905, the same year he published his special theory of relativity. In his photoelectric effect paper, for which he received the Nobel Prize in 1921, Einstein extended plank's concept of quantization to the electromagnetic waves. He assumed that light of freqûency f can be considered to be a stream of photons. Each photon has an energy E, given by

$$E = hf$$

Einstein considered light to be much like a stream of particles traveling through space rather than wave. Where each particle could be absorbed as a unit by an electron. Furthermore, Einstein argued that when the photon's energy is transferred to an electron in a metal, the energy acquired by the electron must be hf. However, the electron must also pass through the metal surface in order to be emitted and some energy is required to overcome this barrier. The amount of energy ϕ required to escape the electron from metal surface is known as the work function of the substance and is of the order of a few electron volt for metals. Hence in order to conserve energy, the maximum K.E of the ejected

photoelectrons is the difference between the photon energy and the work function of the metal, $K \cdot E_{max} = hf - \phi$ ...(18.9) Where, $\phi = hf_o$

That is, the excess energy $hf - \phi$ equal to the maximum K.E the liberated electron can have outside the surface. (Eq 18.9) is called Einstein's photo electric equation. When K.E of the photo electron is zero. The frequency f is equal to threshold frequency f_o, hence the Eq (18.9) becomes. $0 = hf_o - \phi \Rightarrow hf_o = \phi$

Hence we can also write photoelectric Eq as $K \cdot E_{max} = hf - hf_o$ ...(18.9a)

The Eq(18.9a) holds good only for those electrons which come out with full surplus energy.

Example 18.5

A sodium surface is illuminated with light of wavelength 300 nm. The work function for sodium is 2.46 eV. Find

a. The K.E of the ejected electrons and

b. The cut-off wavelength for sodium.

Solution:

$$E = hf = \frac{hc}{\lambda}$$

$$E = \frac{(6.63\times10^{-34}\ \text{j.s})(3\times10^{8}\text{m/s})}{300\times10^{-9}\text{m}}$$

$$= 6.63\times10^{-19}\text{J}$$

$$\text{or } = \frac{6.63\times10^{-19}\text{J}}{1.60\times10^{-19}\text{J/eV}} = 4.14\text{ eV}$$

$$k\cdot E_{max} = hf - \phi$$

$$k\cdot E_{max} = 4.14\text{eV} - 2.46\text{eV}$$

$$k\cdot E_{max} = 1.68\text{eV}$$

b.

$$\phi = 2\cdot46\text{ eV} = (2.46\text{ eV})(1.60\times10^{-19}\text{j/eV}) = 3.94\times10^{-19}\text{j}$$

Hence

$$\lambda_c = \frac{hc}{\phi} = \frac{(6.63\times10^{-34}\text{j.s})(3\times10^{8}\text{m/s})}{3.94\times10^{-19}\text{j}}$$

$$\lambda_c = 5.05\times10^{-7}\text{m} = 505\text{nm}$$

This wavelength is in the green region of the visible spectrum.

Applications of the photo electric Effect

Photocell

A photocell is based on photo electric effect. A simple photocell is shown in (fig 18.8). It consists of an evacuated glass bulb with a thin anode rod and cathode of an appropriate metal surface. The material of the cathode is selected to suit to the frequency range of incident radiation over which the cell is operated. For example sodium or potassium cathode emits electrons for infrared light and some other metals respond to ultraviolet radiation. When photo–emissive surface is exposed to appropriate light, clectron are emitted and a current flows in the external circuit which increases with the increase in light intensity. The current stops when the light beam is interrupted. The cell has wide range of applications.

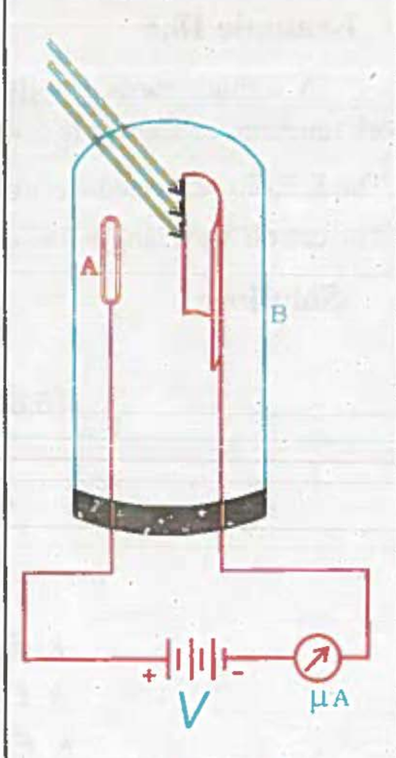


Figure 18.8: Photocell

Some of these are to operate.

i. Security system
ii. Counting system
iii. Automatic door system
iv. Automatic street lighting
v. Exposure meter for photography
vi. Sound track of movies.

Solar Cell:

A solar cell is a type of photo–cell whose aim is to obtain energy from solar radiation, either on the Earth's surface or in space. A recent method uses a bloomed resellers to focus the light from a large area into a cell. Even with simple silicon cells this technique can give effective efficiencies over 20%. The largest solar power station being built (2006) is in southern Portugal; designed to produce 11 Mw, it consists of 52000 photovoltaic panels steered always to point to the sun during day light. But trapping solar energy by direct absorption in solid and liquid materials is just as important in research area as those described; such cells are for cheaper than ones that use semi-conductors.

Example 18.6

The stopping potential to prevent electrons from flowing across a photo electric cell is 4.0 V. What maximum K.E is given to the electrons by the incident light?

Solution:

$$K\cdot E_{max} = eV$$
$$= 1.6\times10^{-19}\,c \times 4\,J\,c^{-1}$$
$$= 6.4\times10^{-19}\,J$$

18.6 Compton's Effect

Further justification for the photon theory of light came from an experiment conducted by Arthur H. Compton in 1923 in his experiment Compton directed a beam of X–rays of wavelength λ toward a block of graphite. He found that the scattered X–rays had a slightly longer wavelength, λ' than the incident X–rays, and hence the energies of the scattered rays were lower. The amount of energy reduction depended on the angle at which the X–rays were scattered. The change in wavelength, $\Delta\lambda$, between a scattered X–ray and an incident X–rays is called the Compton shift.

In order to explain this effect, Compton assumed that if a photon behaves like particle, its collision with other particles is similar to that between two billiard balls. Hence, both energy and momentum must be conserved. If the incident photon collides with an electron initially at rest, as in (fig 18.9), the photon transfers some of its energy and momentum to the electron.

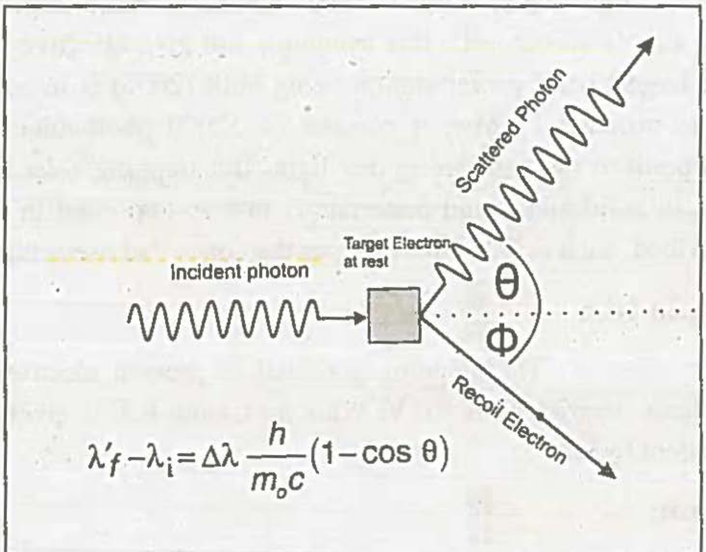


Figure 18.9: incident photon collision with electron

Consequently, the energy and frequency of the scattered photon are lowered and its wavelength increases.

Applying relativistic energy conservation to collision described in (fig 18.9), we have

$$hf = K \cdot E + hf' \qquad ...(18.10)$$

Where hf is the energy of the incident photon, K.E is the kinetic energy given to the recoiling electron, and hf' is the energy of scattered photon. Conservation of momentum requires:

$$\overline{p} = \overline{p_o} + \overline{p'} \qquad ...(18.11)$$

Where p_o and p' represents the momentum of scattered photon and recoiling electron

According to classical electromagnetic Theory, EM waves carry momentum of magnitude E/c, where E is the energy of waves and c is the speed

of light. In the photon picture, each photon carries a little bit of that momentum in proportion to the amount of energy it carries.

The momentum of a photon is

$$p = \frac{\text{photon energy}}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad ...(18.12)$$

Using the incident photon's direction as the x –axis, we can separate this into two components equations.

Along x –axis $$\frac{h}{\lambda} = p_e \cos\theta + \frac{h}{\lambda'}\cos\phi \qquad \cdots(18.13)$$

Along y –axis $$0 = -p_e \sin\theta + \frac{h}{\lambda}\sin\phi \qquad \cdots(18.14)$$

From Eq (18.12), (18.13) and (18.14), Compton derived this relationship.

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta) \qquad \cdots(18.15)$$

The quantity $\frac{h}{m_o c}$ is known as the Compton wavelength because it has the dimension of a wavelength it has a value 0.00243 nm. In term of frequency (Eq 18.15) can be written as

$$\frac{1}{f'} = \frac{1}{f} + \frac{h}{m_o c^2}(1 - \cos\theta) \qquad \cdots(18.16)$$

Example 18.7

X –rays of wavelength $\lambda = 0.20\,\text{nm}$ are scattered from a block of carbon. The scattered X –rays are observed at an angle of $45°$ to the incident beam calculate the wavelength of the scattered X –rays at this angle.

Solution:

The shift in wavelength of the scattered X –rays is given,

$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta)$$

$$= \frac{6.663\times10^{-34}\,J.s}{(9.11\times10^{-31}\,kg)(3\times10^{8}m/s)}(1\text{-}\cos 45°)$$

$$= 7.11\times10^{-13}\,m = 0.000711\,nm$$

Hence, the wavelength of the scattered X –ray at this angle is

$$\lambda' = \Delta\lambda + \lambda$$

$$\lambda' = 0.000711nm + 0.20nm$$

$$\lambda' = 0.200711nm$$

18.7 Pair Production

An energetic photon can create a positron and an electron where no such particles existed before. The photon is totally absorbed in this process. Energy must be conserved in any process, so in order for pair production to occur,

$$E_{Photon} = E_{electron} + E_{positron} \qquad \cdots(18.17)$$

The total energy of a particle with mass m is the sum of its kinetic energy and its rest mass energy. A particle of mass m has rest energy.

$$E = m_o c^2 \qquad \cdots(18.18)$$

Thus, a photon must have energy of at least $2m_oc^2$ in order to create an electron –positron pair. If the photon energy is greater than $2m_oc^2$, 1.02 MeV the excess energy appears as kinetic energy of the electron and positron. A photon is massless and thus, has no rest energy; the total energy of a photon is

$$E = hf = \frac{hc}{\lambda}.$$

Charges must also be conserved in this process. Note it is impossible for a photon to produce a single electron, a single positron, two electrons, or two positron, because the photon have zero charge and then charge will not be conserved.

Momentum must also be conserved.

$$\frac{hf}{c} = mv_{e^-} + mv_{e^+} \qquad \cdots 18.19$$

Where $\frac{hf}{c}$ is the momentum of photon, mv_{e+} is the momentum of positron and mv_{e^-} is the momentum of electron.

Pair production can only occur when the photon passes near a massive particle such as an atomic nucleus (Fig 18.10). The recoil of the massive particle satisfies momentum conservation without carrying off a significant amount of energy, so our assumption that all the energy of the photon goes into the electron –positron pair is a good approximation.

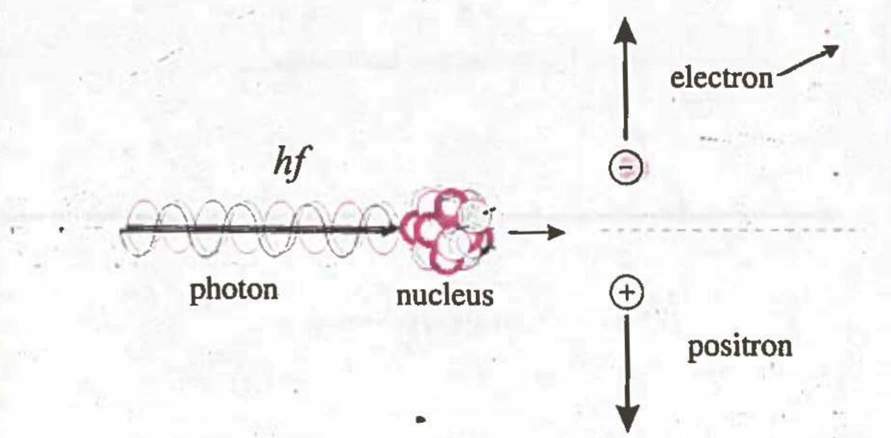
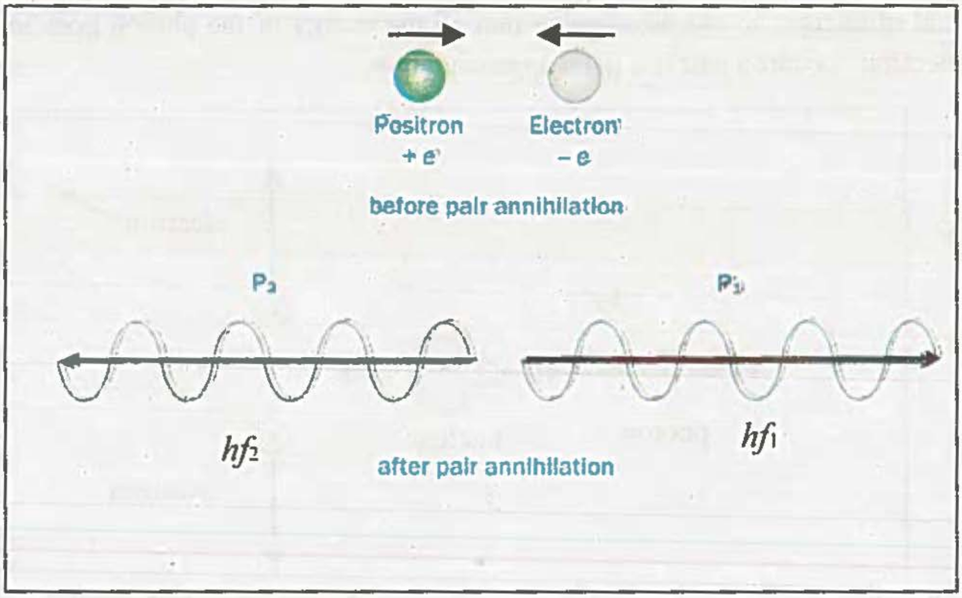


Figure 18.10: pair production, a photon passes near an atomic nucleus. After collision the photon vanishes by creating an electron–positron pair.

18.8 Pair Annihilation

Pair annihilation is a process in which an electron –positron pair produces two photons, which is the inverse of pair production. Pair annihilation cannot create just one photon, because it is required to conserve both energy and

momentum. The total energy of the two photons must be equal to the total energy of the electron –positron pair. Ordinarily the kinetic energies of the electron and positron are negligible compared to their rest energies, so for simplicity we assume they are at rest; then their total energy is just their rest energy, $2m_oc^2$, and their total momentum is zero. Annihilation of the pair then produces two photons, each with energy $E = hf = m_oc^2 = 511\,\text{keV}$, traveling in opposite direction (Fig 18.11). Besides confirming the photon model of EM radiation, pair annihilation and pair production clearly illustrate Einstein's idea about mass and rest energy.



Example 18.8

Find the threshold wavelength for a photon to produce an electron –position pair.

Solution:

The minimum photon energy to create an electron –positron pair is

$$E = 2m_oc^2 = 1.022\,\text{MeV}$$

Now to find the wave length of a photon with this energy.

$$E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\,\text{J.s}\times3\times10^{8}\,\text{m/s}}{\left(1.022\times10^{6}\,\text{eV}\right)}$$

$$= \frac{1240\ \text{eV-nm}}{1.022\times10^{6}\,\text{eV}} = 0.00121\ \text{nm}$$

18.9 The Wave Nature of Particles

We have seen that light has particle characteristics as well as wave characteristics. In 1924, the French physicist Louis de Broglie reasoned that if light, which characteristically demonstrated pure wave properties, can show to have particle or matter properties, then matter, which characteristically demonstrated of pure particle properties, must have wave properties. He postulated that particles of matter obey a wave equation just as photon does.

Compton's investigation showed that the momentum p of a photon is described by the equation $p = \frac{h}{\lambda}$ where h is plank's constant and λ is the wavelength of the photon. This means that λ would be expressed,

$$\lambda = \frac{h}{p} \qquad \cdots(18.19)$$

As nature reveals symmetry, de Broglie asserted that (Eq: 18.19) is a completely general formula that applies to material particles as well as to photons. The momentum of a particle of mass m and velocity "V" is

$$p = mv$$

and consequently its de Broglie wave wavelength is

$$\lambda = \frac{h}{mv} \qquad \cdots 18.20$$

The greater the particle's momentum, the shorter its wave length.In (Eq 18.20) m is the relativistic mass.

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Thus, de Broglie contended that the relationship between matter and electromagnetic radiation is more intrinsic than was previously believed.

de Broglie's equation for the wavelength of matter waves explains why the wave nature of large particles is not observable. Consider the de Broglie wavelength of a cricket ball of approximate mass 0.25 kg when it leaves a bat with a speed of 20 m/s

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63\times10^{-34}\text{ j.s}}{0.25\text{kg}\times20\text{ m/s}} = 1.3\times10^{-34}\text{m}$$

This wavelength is far too small to be observed.

On the other hand, calculate the de Broglie wavelength of an electron moving with a typical speed of 10^6 m/s,

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.6\times10^{-34}\text{ j.s}}{9.1\times10^{-31}\text{kg}\times10^{6}\text{ m/s}} = 7.3\times10^{-10}\text{m}$$

This wavelength approximates the distance between the atoms in a crystal. It makes the wavelength suitable for diffraction and interference. Thus, the wavelengths of very small particle of matter are readily observable.

18.9.1 Davisson and Germer Experiment

· The de Broglie relation was confirmed by Davisson and Germer. They bombarded electrons on a nickel crystal and measured the intensity of the

electron beam scattered from the crystal at various angle (Fig 18.12). Electrons emitted from a filament were accelerated through a potential difference applied between the filament and anode in an electron gun. The accelerated electrons passed through slits and collided with the nickel crystal. The electrons scattered at various angles were detected by the detector D. The whole apparatus was enclosed in a vacuum chamber. It was observed that at certain angles, depending upon the energy of the electrons, the intensity of the scattered beam was large and at other angle it was small (selective reflection).

The results obtained from this experiment were explained by treating the beam of electrons as wave of wavelength given by de Broglie's expression. The diffraction of electrons from the crystal was similar to that of x – rays from crystals. The wave property of particles (electrons, neutrons, atoms and even molecules) has been verified experimentally. The diffraction of electrons and neutrons is used to study the structure of crystals in a similar manner as is done with the help of x –rays.

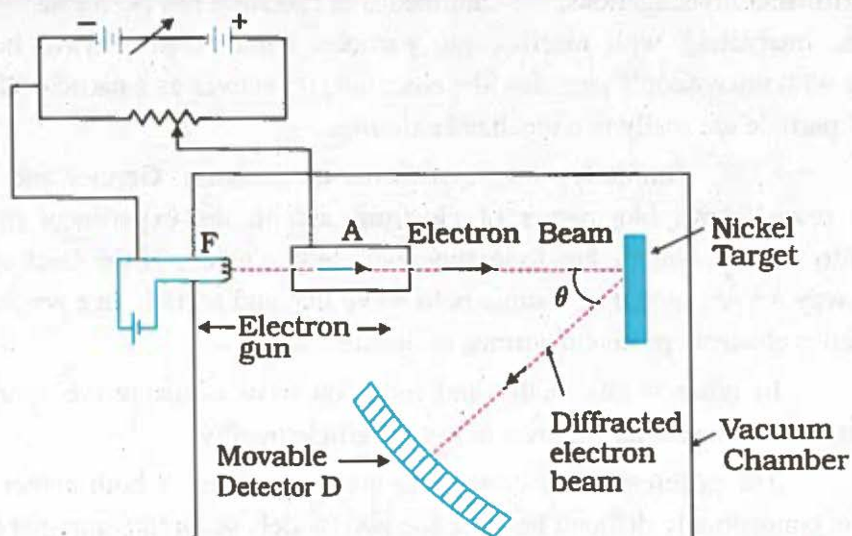


Figure 18.12 :Davisson and Germer experiment illustration

18.9.2 The wave –Particle Duality

Is the radiation emitted by an atom a particle or a wave? A noncommittal answer to this question is that it is both, a wave as well as a particle. The experimental observation of interference and diffraction of light, and their successful interpretation on the basis of the wave theory, suggest that light is a wave. On the other hand, the experiments on photo-electric effect can be interpreted only in term of the quantum theory.

According to wave theory, the radiant energy spreads out continuously in the form of waves but, according to the quantum theory, the radiant energy spread out in discrete packets (or quanta), each having the energy hf. The two theories are apparently contradictory. One group of experimental facts can be explained by one theory and the other group by the second theory. The physicists were faced with a dilemma. One theory could not be rejected in favors of the other.

Could both be right? With the development of quantum mechanics and experimental investigations, the dual nature of radiation has been established. Radiations interacting with macroscopic particles behaves as a wave but in encounter with microscopic particles like electrons, it behaves as a particle. Hence wave and particle are really two idealized extremes.

Similarly, the experiments of davisson, Germer and G P Thomson reveal wave like nature of electrons and in the experiment of J.J. Thomson to find the e/m we had to assume particle like nature of the electron. In the same way we are forced to assume both wave like and particle like properties for all matter: electron, proton, neutrons, molecules etc.

In other words matter and radiation have a dual wave –particle nature and this new concept is known as wave particle duality.

The problem of understanding the dual nature of both matter and radiation is conceptually difficult because the two models seem to contradict each other. This problem as it applies to light has been discussed. Niel Bohr helped to resolve this problem in his principle of complementarity, which states that the wave and particle models of either matter or radiation complement each other.

Neither model can be used exclusively to adequately describe matter or radiation. A complete understanding is obtained only if the two models are combined in a complementary manner.

Check Point

Why does the existence of cut-off frequency favour a particle theory for light rather than a wave theory?

18.10 Electron Microscope

A practical device that is based on the wave characteristics of electron is the electron microscope. (Fig.18.13) which is in many respects similar to an ordinary compound microscope. One important difference between the two is that the electron microscope has a much greater resolving power because electron can be accelerated to very high kinetic energies, giving them a very short wavelength. Any microscope is capable of detecting details that are comparable in size to the wavelength of radiation used to illuminate the object. Typically, the wavelength of electrons are about 100 times shorter than those of the visible light used in optical microscopes. As a result, electron microscope, are able to distinguish details about 100 times smaller. In operation, a beam of electrons falls on a thin slice of the material to be examined. The section to be examined must be very thin, typically a few hundred angstroms, in order to minimize undesirable effects, such as absorption or scattering of the electrons. The electron beam is controlled by electro-static or magnetic deflection, which acts on the charges to focus the beam to an image. Rather than examining the image through an eyepiece as in an ordinary microscope, a magnetic lens forms an image on a fluorescent screen. The fluorescent screen is necessary because the image produced would not otherwise be visible.

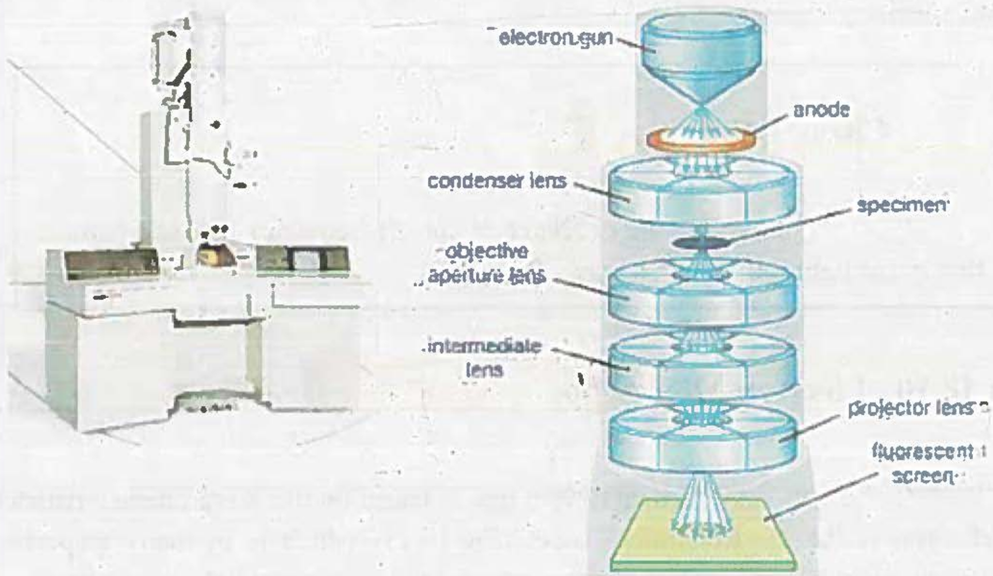


Figure 18.13 : electron microscope ray diagram

Example 18.9

Determine the wavelength of an electron that has been accelerated through a potential difference of 100 V.

Solution:

Gain in K·E of the electron in falling through a potential difference of V volts is

$$\frac{1}{2}mv^2 = eV$$

$$= \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2\times1\cdot6\times10^{-19}c\times100V}{9\cdot1\times10^{-31}kg}}$$

$$= 5.9\times10^6\ m/s$$

The de –Broglie wavelength of electron is

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63\times10^{-34}\ js}{(9.1\times10^{-31}kg)\times(5.9\times10^6 m/sec)} = 1\cdot2\times10^{-10}m$$

$$\lambda = 0.12\ nm$$

Example 18.10

A particle of mass 5.0 mg moves with speed of 8.0 m/s. Calculate de Broglie wavelength.

Solution:

$$m = 5.0mg = 5.0\times10^{-6}kg$$
$$v = 8.0m/s$$
$$h = 6.63\times10^{-34}\ js$$

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \lambda = \frac{6.63\times10^{-34}\ js}{5\cdot0\times10^{-6}kg\times8.0m/s}$$

$$\Rightarrow \lambda = 1.66\times10^{-29}m$$

18.11 Uncertainty Principle

Suppose some one starts thinking of designing a super microscope to see an electron and take observation to know its position and momentum at a particular instant. The question is whether it is possible to make such observations. We shall see below that it is fundamentally impossible to make such observations even if one succeeds to construct an ideal instrument for this purpose.

In order to see an electron we use light of wavelength λ. The light consists of photons –each having a momentum h/λ. When one of these photons hits the electron, the photon will be scattered and the original momentum of the electron will be changed.

One cannot predict the exact change in the momentum Δp of the electron. But one can say that change of momentum of the electron will be of the same order as the momentum of the photon itself, Hence,

$$\Delta p \approx \frac{h}{\lambda} \qquad \cdots 18.20$$

The equation gives the uncertainty in the momentum.

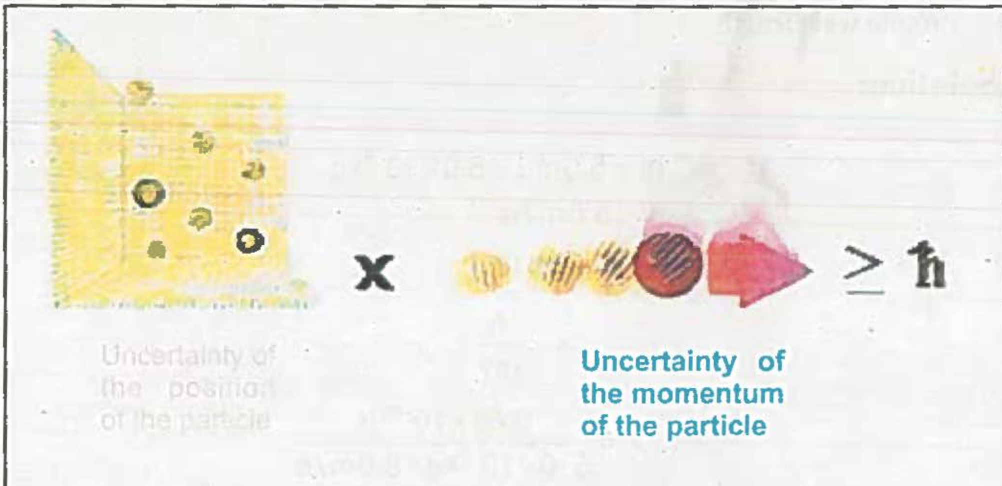


Figure 18.14 : illustrations of uncertainty principle

In order to reduce the uncertainty in momentum, one must use light of large wavelength. Now the uncertainty in determining the position of the electron will be of the order of wavelength of light. Hence uncertainty in position is

$$\Delta x \approx \lambda \qquad \cdots(18.21)$$

In order to reduce the uncertainty in position, one must use light of shorter wavelength.

If one use light of short wavelength, the accuracy in position measurement increases but the accuracy in the measurement of momentum decreases. On the other hand if one use light of large wavelength, the accuracy in momentum measurement increases but the accuracy in position measurement decreases. Multiplying the two equations, we get.

$$(\Delta p)(\Delta x) \approx h \qquad \cdots(18.22)$$

This is one form of the uncertainty principle. It states that the product of the uncertainty Δp in the momentum of a body at some instant and the uncertainty in its position Δx at the same instant is approximately equal to plank's constant. This means that it is impossible to measure the position and momentum of the electron simultaneously with perfect accuracy even with an ideal instrument.

As h is a very small quantity and, therefore, in the case of large objects with which we come across in our daily life, the limitation imposed on measurements by the uncertainty principle is negligible, but when we are working with sub atomic particles, these limitations are not negligible.

Another form of uncertainty principle can be obtained through similar reasoning.

$$(\Delta E)(\Delta t) \approx h \qquad \cdots(18.23)$$

Which states that the product of the uncertainty in a measured amount of energy and the time available for the measurement is approximately equal to plank's constant.

The uncertainly principle tells us that it is impossible to know everything about a particle. There will be uncertainty about its exact momentum at a given position and its exact energy at a given time.

Example 18.11

An electron is placed in a box about the size of an atom that is about 1×10^{-10} m. What is the velocity of the electron?

Solution:

Using uncertainty principle

$$\Delta P \Delta x \approx h$$

$$\Delta P \approx \frac{h}{\Delta x}$$

$$m\Delta v \approx \frac{h}{\Delta x}$$

$$\Delta v \approx \frac{h}{m\Delta x}$$

$$\Delta v \approx \frac{6.63\times10^{-34}\,j\cdot s}{9.1\times10^{-31}\,kg\times1\times10^{-10}m}$$

$$\Delta v \approx 7.29\times10^{6}\,m\,s^{-1}.$$

Key points

- Frame of reference is a coordinate system such as the OXYZ system, which is required to describe the relative position of an object.
- A reference frame moving with constant velocity is called an inertial frame of reference.
- A frame of reference that is accelerating is a non –inertial frame of reference.
- Special theory of relativity is based on two postulates.
    a. The laws of physics are the same in all inertial frames.
    b. The speed of light in free space has the same value for all observers, regardless of their state of motion.
- $E = mc^2$ is an important result of special theory of relativity.
- A black body is a solid block having a hollow cavity within. It has small hole and the radiation can enter or escape only through this hole.
- Stephen Boltz Mann law states that total energy radiated over all wave lengths at a particular temperature is directly proportional to the fourth power of that Kelvin temperature.
- The emission of electrons from a metal surface when exposed to light is called photo electric effect. The emitted electrons are known as photo electrons.
- When x –rays are scattered by loosely bound electrons from a graphite target, it is known as Compton effect.
- The change of very high energy photon into electron –positron pair is called pair production.
- When a positron comes close to an electron, they annihilate and produce two photons in the r–rays range. It is called annihilation of matter.

- Radiation and matter exhibit particle as well as wave like properties. This is known as wave –particle duality.
- Position and momentum of a particle cannot both be measured simultaneously with perfect accuracy. There is always fundamental uncertainty associated with any measurement. It is a consequence of the wave particle duality of matter and radiation. It is known as Heisenberg uncertainty principle.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. If the K.E of a free electron doubles, its de Broglie wavelength changes by the factor.

   a. $\sqrt{2}$ b. $\frac{1}{\sqrt{2}}$ c. 2 d. $\frac{1}{2}$

2. Einstein's Photoelectric equation is $E_k = hf - \phi$ in this equation E_k refers to

   a. K.E of all the emitted electrons

   b. Mean K.E of emitted electrons

   c. Maximum K.E of emitted electrons

   d. Minimum K.E of emitted electrons

3. De –Broglie waves are associated with

   a. Moving charged particles only

   b. Moving neutral particles only

   c. All moving particles

   d. All particles whether in motion or at rest

4. A perfect absorber must also be perfect

   a. Cavity b. Source of radiation

   c. Radiator d. None of them

5. Pair production occurs only when energy of photon is at least equal to

   a. 1.02 keV b. 1.02 eV

   c. 1.02 MeV d. 1.02 GeV

6. Pair production cannot take place in vacuum because.

a. Mass is not conserved
b. Momentum is not conserved
c. Energy is not conserved
d. Charge is not conserved

7. The positron has charge which is in magnitude equal to the charge on

a. Electron
b. Proton
c. β – particle
d. All

8. We can never accurately describes all aspects of subatomic particles simultaneously. It is correct according to

a. Uncertainty Principle
b. De –Broglie Theory
c. Einstein Theory
d. Photo electric effect

9. An electron microscope employs which to one of the following principles?

a. Electron have a wave nature
b. Electrons can be focused by an electric field.
c. Electrons can be focused by a magnetic field.
d. All of the above

Conceptual Questions

1. Imagine a world in which $c = 50\ m/s$. How would the every day events appear to us?
2. Both zarak and samina are twenty years old. Zarak leaves earth in a space craft moving at .8*c*, while samina remains on the earth. Zark returns from a trip to star 30th light years from earth, which one will be of greater age. Explain?
3. Which has more energy, a photon of ultraviolet radiation or a photon of yellow light? Explain.
4. Some stars are observed to be reddish, and some are blue. Which stars have the higher surface temperature? Explain.
5. An electron and a proton are accelerated from rest through the same potential difference. Which particle has the longer wavelength? Explain.

6. All objects radiate energy. Explain why, then, are we not able to see objects in a dark room.
7. If the photo electric effect is observed for one metal, can you conclude that the effect will also be observed for another metal under the same conditions?
8. Explain why it is impossible for a particle with mass to move faster than the speed of light.
9. Use photon model to explain why the ultraviolet radiation is harmful to your skin while visible light is not.
10. Explain why the annihilation of an electron and positron creates a pair of photons rather than a single photon.
11. When a particle's K.E increases, what happen to its de Broglie wavelength?
12. Explain why we can experimentally; observe the wave like properties of electrons, but not of billiard ball?
13. Does a light bulb at a temperature of 2500 K produce as white a light as the sun at 6000K? Explain.
14. A beam of red light and a beam of blue light have exactly the same energy. Which light contains the greater number of photons?
15. In Compton scattering experiment, an electron is accelerated straight ahead in the direction of the incident X –ray photon. Which way does the scattered photon move? Explain.
16. Why must the rest mass of a photon be zero? Explain.
17. What happens to total radiation from black body if its absolute temperature is doubled?
18. Why don't we observe Compton's effect with visible light.
19. If the following particles all have the same K.E, which has the shortest wavelength? Electron, alpha particle, neutron and proton?
20. If an electron and a proton have the same de Broglie wavelength, which particle has greater speed?

21. Why ultraviolet radiation is harmful to skin while visible light is not?
22. An incandescent light bulb is connected to a dimmer switch. When the bulbs operate at full power, it appears white, but as it is dimmed it looks more and more red. Explain?

Comprehensive Questions

1. State Einstein's postulate of the special theory of relativity. Discuss its various results.
2. What are the main features of the thermal radiation from a black body? Discuss plank's quantum theory and its importance in physics.
3. What are the main feature of photoelectric effect? Discuss the failure of classical physics and success of photon concept in explaining this effect.
4. What is Compton's effect. Develop a mathematical relation for the Compton's wave shift.
5. Write note on pair production and annihilation of matter.
6. What is de Broglie hypothesis? Describe an experiment to show that particle has wave characteristics.
7. What is meant by wave–particle duality? Explain.
8. State and explain Heisenberg's uncertainty principle. Justify the validity of this principle by a thought experiment.

Numerical Problems

1. The length of a space ship is measure to be exactly one –third of its proper length. What is the speed of the space ship relative to the observer?

[0.9428c]

2. The time period of a pendulum is measured to be 3s in inertial frame of the pendulum. What is the period when measured by an observer moving with a speed of 0.95c with respect to the pendulum?

[9.6s]

3. An electron, which has a mass 9.11×10^{-31}kg, moves with a speed of $0.75c$. Find its relativistic momentum and compare, this value with the momentum calculated from classical expression.

(3.10×10^{-22} kgms^{-1}, 2.1×10^{-22} kgms^{-1}, 50%)

4. An electron moves with a speed of $v = 0.85c$. Find its total energy and K.E in electron volt.

[0.970MeV, 0.459 MeV]

5. The rest mass of a proton is 1.673×10^{-27} kg. At what speed would the mass of the proton be tripled ?

[0.9428c]

6. At what fraction of speed of light must a particle move so that it K.E is one and a half times its rest energy?

[0.916c]

7. A metal, whose work function is 3.0 eV, is illuminated by light of wavelength 3×10^{-7}m. Calculate (a) The threshold frequency, (b) The maximum energy of photoelectrons (c) The stopping potential.

[0.72×10^{15} Hz, 1.16eV , 1.16V]

8. The thermal radiation from the sun peaks in the visible part of the spectrum. Estimate the temperature of the sun.

[5800 K]

9. A 50 keV $X-\text{ray}$ is scattered through an angle of $90°$. What is the energy of the $X-\text{ray}$ after Compton scattering?

[45.5keV]

10. Calculate the wavelength of de Broglie waves associated with electrons accelerated through a potential difference of 200V.

[0.86 A°]

11. An electron is accelerated through a potential difference of $50V$. Calculate its de Broglie Wavelength.

$\left[\lambda = 1\cdot 74 \times A^0\right]$

12. The speed of an electron is measured to be $5\times10^3 m/s$ to an accuracy of 0.003%. Find the uncertainty in determining the position of this electron.

$\left[4.84\times10^{-3} m\right]$

13. The life time of an electron in an exited state is about $10^{-8}s$. What is its uncertainty in energy during this time?

$\left[6.6\times10^{-26} J\right]$

UNIT 19

.... Atomic Spectra ....

At the end of this chapter the student will be able to:

- describe and explain the origin of different types of optical spectra.
- show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines.
- explain how the uniqueness of the spectra of elements can be used to identify an element.
- analyse the significance of the hydrogen spectrum in the development of Bohr's model of the atom.
- explain hydrogen atom in terms of energy levels on the basis of Bohr Model.
- determine the ionization energy and various excitation energies of an atom using an energy level diagram.
- Solve problems and analyse information using $1/\lambda = R_H\,[1/p^2 - 1/n^2]$.
- understand that inner shell transitions in heavy elements result into emission of characteristic X-rays.
- explain the terms spontaneous emission, stimulated emission, meta stable states, population inversion and laser action.
- describe the structure and purpose of the main components of a He-Ne gas laser.

The beginning of the twentieth century saw the start of new branches of Physics – atomic structure and spectra which has a profound effect on revealing the inner mysteries of the structures of atoms.
The existence of line emission spectra from atomic gases is used to infer a structure of an atom in terms of discrete energy levels in atoms. J.J. Balmer in

1885 succeeded to devise an empirical formula which could explain the existence of the spectra of atomic hydrogen.

In this chapter we will study the line spectrum of hydrogen atom, the Bohr model of hydrogen atom, production of X-rays, working principle of CAT scanner and Laser.

For your Information

A CT scan stands for Computed Tomography scan. It is also known as a CAT (Computer Axial Tomography) scan. It is a medical imaging method that employs tomography. CT scanning is useful to get a very detailed 3-D image of certain parts of the body, such as soft tissues, the blood vessels, the lungs, the brain, abdomen, and bones.

19. 1 Atomic Spectra

When a substance is heated, its atoms absorb energy and are excited, i.e. some of its electrons jump to higher energy states. The electron stays there for a short duration $(10^{-8}\,s)$ and fall back to its lower energy state. In this process it emits a radiation called photon which is supposed to be a discrete packet of light energy. A photon is a particle of light having wave characteristics, i.e. it has an frequency and wavelength.

The frequency of emitted radiation or photons is equal to the frequency with which the electron bounces back and forth between the higher and the lower energy state.

In a solid, a liquid or a dense gas, the atoms are closely packed and are, therefore, not free to emit radiation because of interaction.

Thus we do not get discrete radiation but instead obtain only a continuous spectrum. The atoms of gas are however free to emit a radiation when excited. The emission spectrum of a gas is, therefore, discrete, having line spectrum.

Suppose an evacuated glass tube is filled with a gas such as neon, helium, or argon. If a potential difference between electrodes in the tube produces an electric current in the gas, the tube will emit light whose color is characteristic of the gas. If the emitted light is analyzed by passing it through a narrow slit and then through a spectroscope, a series of discrete lines is observed, each line corresponding to a different wavelength or color. We refer to such a series of lines as a line spectrum. The wavelengths contained in a given line spectrum are characteristic of the elements emitting light. Because no two elements emit the same line spectrum, this phenomenon represents a practical technique for identifying elements in chemical substance. The first such spectral series was found by J.J Balmer in 1885 in the course of a study of the visible part of the hydrogen spectrum.

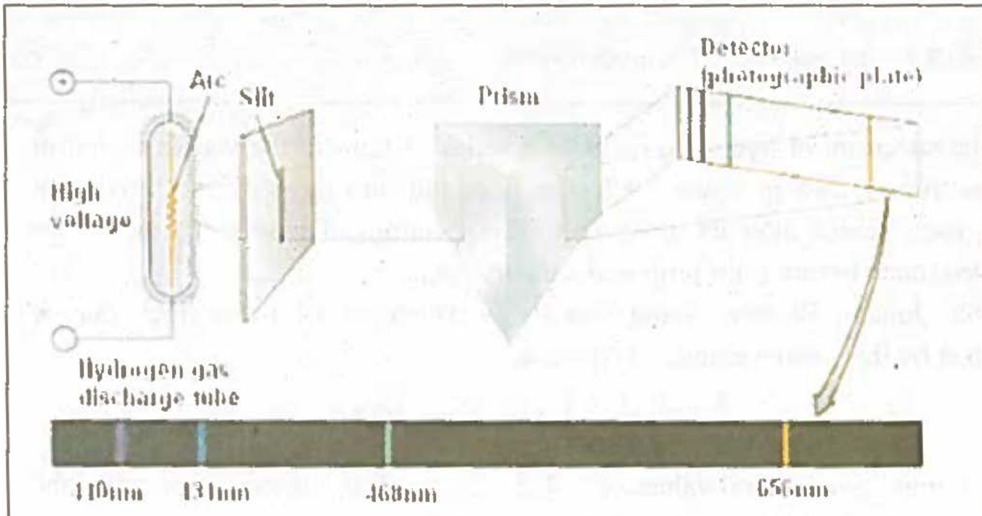


Figure 19.1 : line spectrum

19. 2 The Spectrum of Hydrogen Atom

When hydrogen gas is placed in a discharge tube, and a discharge is caused in it by means of high voltage across the tube, the gas becomes luminous and gives off a bluish –red light, Fig 19.2. This light can be analyzed by passing it through a dispersing device such as a prism or a grating. The spectrum of hydrogen atoms consists of a series of lines. Each line represents a wavelength of light given off by the light source.

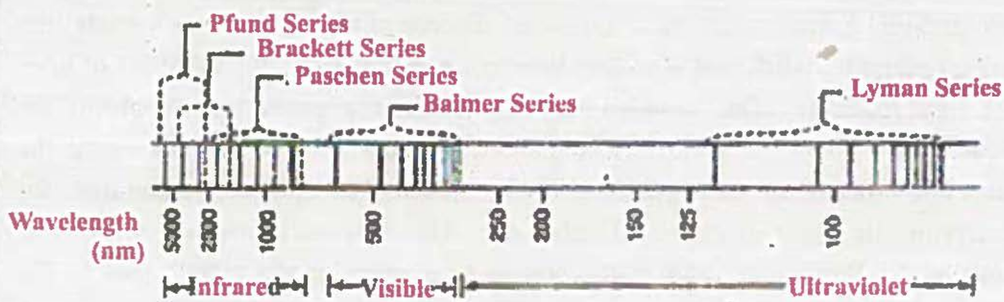


Figure 19.2 : line spectrum of hydrogen atom

The line spectrum of hydrogen includes a series of lines in the visible region of the spectrum shown in figure 19.2. The lines fall into three distinct groups or series, each named after its discoverer. The spectrum of atomic hydrogen was observed much before Bohr proposed a theory for it.

In 1885, Johann Balmer, found that the wavelengths of these lines can be described by this simple empirical equation.

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right) \qquad ...... 19.1$$

Where n may have integral values of 3, 4, 5...... and R is constant, now called the Rydberg constant. If the wavelength is in meters, R has the value.

$$R=1\cdot 0973732\times 10^7 m^{-1}$$

Balmer predicted that other series of lines might exist outside the visible region which would obey the equations given below.

Several years later, other were discovered. These spectra are called the Lyman, Paschen and Brackett series after their discoverers. The wave lengths of the lines in these series can be calculated by the following empirical formulas.

Lyman series $\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$ ......(19.2)

Where $n=1,2,3,4 5,......$

paschen series $\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$ ......(19.3)

Where $n=4,5,6,7,......$

Brachett series $\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$ ......(19.4)

Where $n=5,6,7...$

All the above formulas can be written in a general mathematical form as:

$$\frac{1}{\lambda_n}=R\left(\frac{1}{P^2}-\frac{1}{n^2}\right)......(19.5)$$

Where $P=1,2,3......$

And $n=P+1,P+2,P+3,......$

The Balmer series lies in the visible region of the spectrum, the paschen and Brakett series in the infra –red, and the Lyman series in the ultraviolet.

19. 3 Bohr Model of The Hydrogen Atom

In 1913, the Danish scientist Neil Bohr (1885 – 1963) proposed a theory of the hydrogen atom which contained a combination of ideas from classical physics, Plank's original quantum theory, Einstein's photon theory of light, and Rutherford's model of the atom. Bohr's model of the hydrogen atom contains some classical features as well as some revolutionary postulates that could not be justified within the frame work of classical physics. The Bohr model can be applied quite successfully to such hydrogen –like ions as single ionized helium

and doubly ionized lithium. However, the theory does not properly describe the spectra of more complex atoms and ions.

The basic postulates of the Bohr model of the hydrogen atom are as follows.

1. The electron move in circular orbits about the nucleus under the influence of the coulomb force of attraction between the electron and the positively charged nucleus.

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \quad ....(19.6)$$

Where $\frac{mv^2}{r}$ and $\frac{ke^2}{r^2}$ are centripetal and coulomb forces respectively.

2. Only those stationary orbits are allowed for which orbital angular momentum is equal to an integral multiple of $\frac{h}{2\pi}$.

$$mvr = n\frac{h}{2\pi} \quad ....(19.7)$$

Where "h" is Plank's constant and its value is $h = 6.6256\times10^{-34}$ j s.

3. The electron in stable orbit does not radiate energy as in the classical theory.

4. The atom radiates energy only when the electron jumps from one allowed stationary orbit to another. The frequency of the radiation obeys the condition.

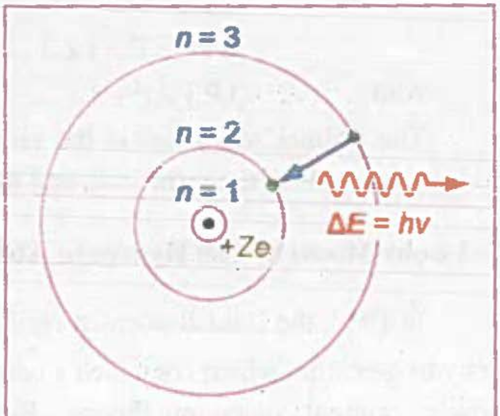


Figure 19.3 (a) : energy radiation due to jumping of electron from higher to lower orbit.

$$hf = E_n - E_p \quad ...(19.8)$$

Where E_n and E_p are higher and lower energy states respectively.

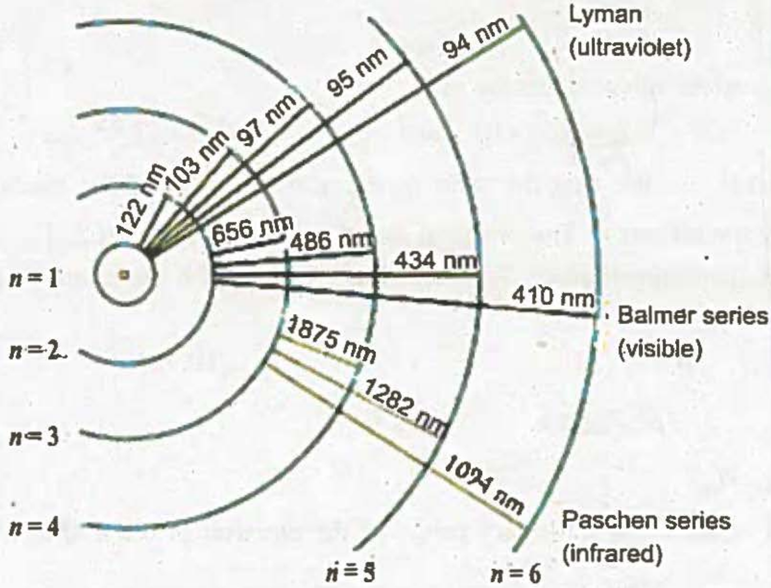


Figure 19.3 (b) : Lyman, Balmer, and paschen series.

The Radii of The Quantized Orbit

From Eq:(19.7)

$$v = \frac{nh}{2\pi m r_n} \qquad \ldots(19.9)$$

For electrons to stay in a circular orbit, the centripetal force is provided by the coulomb's force. Thus,

$$\frac{mv^2}{r_n} = \frac{ke^2}{r^2_{\ n}} \qquad \ldots\ldots(19.10)$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

From Eq: 19.9 put the value in Eq: 19.10. After simplification we get.

$$r_n = \frac{n^2h^2}{4\pi^2 kme^2} = n^2 r_1 \qquad \cdots 19\cdot 11$$

Where $$r_1 = \frac{h^2}{4\pi^2 kme^2}$$

By putting the values of constants we get

$$\text{Or} \quad r_1 = 0.53\times10^{-10}\,\text{m} \qquad \cdots 19.12$$

From Eq (19.11), we see that the radii of the allowed orbit of the electron are determined by the integer n. The orbits of the electrons are quantized. The integer n is called the quantum number. We label the orbits by the quantum number n. Thus,

$$r_n = n^2 r_1 \qquad \ldots(19.13)$$

$$r_n = n^2 \times .053\,\mathring{A} \qquad : n = 1,2,3$$

Where $1\,\mathring{A} = 10^{-10}\text{m}$

Thus the radii of different stationary orbits of the electron in the hydrogen atom are given by

$$r_n = r_1, 4r_1, 9r_1, 16r_1 \ldots\ldots$$

Energy of Electron in Quantized Orbit

Let us now compute the energy of the hydrogen system for a given n. The energy consists of the electron's kinetic energy and the electrostatic potential energy of the two charges. From Eq (19.10).

$$K\cdot E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{ke^2}{r_n} \qquad \ldots(19.14)$$

And its electrostatic energy is

$$P\cdot E = -\frac{ke^2}{r_n} \qquad \ldots(19.15)$$

So the total energy will be

$$E_n = K\cdot E + P\cdot E$$

$$E_n = \frac{ke^2}{2r_n} - \frac{ke^2}{r_n}$$

Hence

$$E_n = -\frac{1}{2}\frac{ke^2}{r_n} \qquad \text{...(19.16)}$$

By substituting the value of r_n from Eq: (19.11), we have

$$E_n = -\frac{2\pi^2 e^4 k^2 m}{h^2} \times \frac{1}{n^2}$$

Write the constant factor in the above equation as

$$\frac{2\pi^2 e^4 k^2 m}{h^2} = E_o$$

We get

$$E = -\frac{E_o}{n^2} \qquad \text{...(19.17)}$$

Thus the total energy is determined by the quantum number *n*. The total energy of the hydrogen atom is quantized. Label the energy *E* by the quantum number *n*, we have

$$E_n = \frac{-E_o}{n^2} \qquad \text{......(19.18)}$$

Where n=1,2,3..

The minus sign shows that the electron is bound to the nucleus and cannot escape from it. Substituting the value of various constants, we find that

$$E_o = 2.17 \times 10^{-18}\,\text{j} = +13.6\,\text{ev}$$

Thus

$$E_n = \frac{-13.6\,ev}{n^2} \qquad \text{......(19.19)}$$

The lowest stationary energy state, or ground state, corresponds to $n=1$ and has energy $E_1 = -13\cdot6$ ev. The next state, corresponding to $n=2$, has an energy $E_2 = -\frac{E_1}{4} = -3\cdot4\,\text{ev}$, and so on.

Hydrogen Emission Spectrum

The result derived above for the energy levels along with postulate 4 can be used to derive the expression for the wavelength of the hydrogen spectrum. Suppose that the electron in hydrogen atom is in the excited state "*n*" with energy

E_n and makes a transition to a lower state "P" with energy E_p, where $E_n > E_p$, then $hf = E_n - E_p$.

Where $$E_n = \frac{-E_o}{n^2} \text{ and } E_P = -\frac{E_0}{P^2}$$

Hence $$hf = -E_0\left[\frac{1}{n^2} - \frac{1}{P^2}\right]$$

Substituting for $f = \frac{c}{\lambda}$ we have,

$$\frac{1}{\lambda} = \frac{E_0}{hc}\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{P^2} - \frac{1}{n^2}\right) \qquad ....(19.20)$$

Where R_H is the Rydberg constant given by the equation.

$$R_H = \frac{E_0}{hc} = 1.0974\times10^7\ m^{-1} \qquad ...(19.21)$$

This value of R_H in Bohr model is in agreement with the value of the Rydberg's constant determined empirically by Balmer. Thus the Balmer empirical formula $\frac{1}{\lambda} = R\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$ and that derived from Bohr's theory $\frac{1}{\lambda} = R_H\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$ are actually the same. Thus, similar to the Balmer empirical formula, Bohr's theory can be used to compute the energies or wavelength of the transitions involved in the various emission series.

19.4 Energy –Level Diagram

According to Bohr's theory the total energy and the radii of the electron orbits in hydrogen atom are respectively given by the following relations.

$$E_n = \frac{-E_0}{n^2} = \frac{-13.6\text{ ev}}{n^2} \text{ and } r_n = r_1 n^2$$

When n = 1, the electron is in the first orbit; the energy is minimum and has the value $E_1 = -E_0 = -13.6\,\text{ev}$. When $n \to \infty$, then $r_n \to \infty$, $E_n \to 0$.

The electron become free from the nucleus. The atom is then said to ionized. It is convenient to represent the energy of the quantized states of the atom on an energy level diagram as shown in Fig 19.4.

The energy levels of the atom E_n are represented by a series of horizontal lines. Transition between the levels are represented by vertical arrows. When the electron is free from the atom and is at rest, both its kinetic and potential energies are zero at $n=\infty$ level.

The energy level diagram can be used to illustrate the origin of various spectral series observed in the emission spectrum of hydrogen. The transition from various energy level to the lowest level $(n=1)$ gives rise to lyman series. Balmer series occurs for transition ending at second energy level $(n=2)$. The paschen, bracket and pfund series occurs for transitions from various energy levels to the $r=3,4,5$ energy levels, respectively.

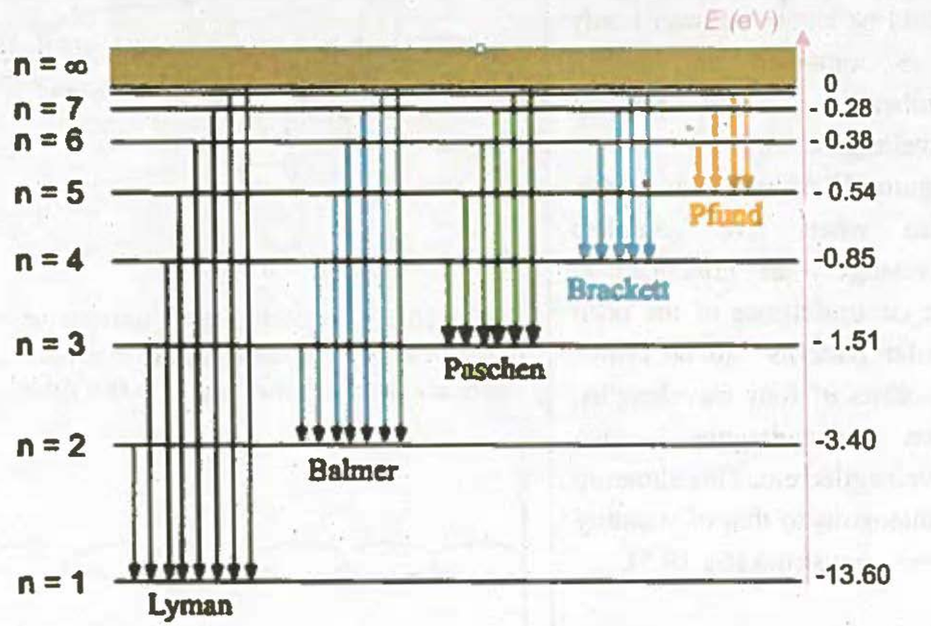


Figure 19.4 : energy level diagram Energy levels of the hydrogen atom with some of the transitions between them to give rise to the spectral lines

19.5 De –Broglie Waves And The Hydrogen Atom

One of the postulates made by Bohr in his theory of the hydrogen atom was that angular momentum of the electron is quantized in units of

$$\frac{h}{2\pi}, \quad \text{or} \quad mvr = n\frac{h}{2\pi}$$

For more than a decade following Bohr's publication, no one was able to explain why the angular momentum of the electron was restricted to these discrete values. Finally, de Broglie recognized a connection between his theory of the wave character of material properties and the quantization condition given above. De–Broglie assumed that an electron orbit would be stable (allowed) only if it contained an integral number of electron wavelengths.

Figure (19.5) demonstrate this point when five complete wavelengths are contained in one circumference of the orbit similar patterns can be drawn for orbits of four wavelengths, three wavelengths, two wavelengths, etc. This situation is analogous to that of standing waves on a string (fig 19.5).

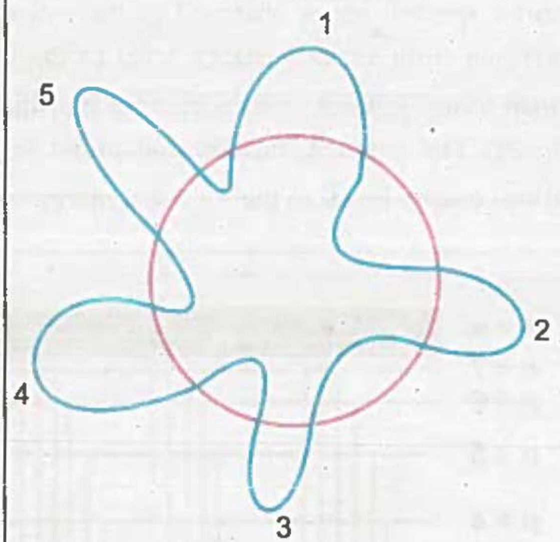


Figure 19.5 (a) : Standing wave pattern for an electron wave in a stable orbit of hydrogen. There are five full wavelengths in this orbit.

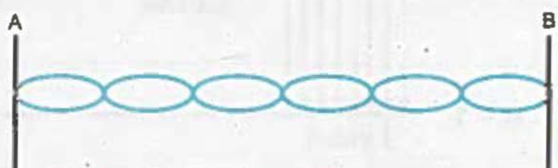


Figure 19.5 (b) : Standing waves pattern for a vibrating stretched string

Now imagine that the vibrating string is removed from its support at A and B and bent into a circular shape such that A and B are brought together. The end result is a pattern similar to that shown in (fig 19.5). Standing waves pattern for a vibrating stretched string fixed at its ends. This pattern has three full wavelengths.

In general, the condition for de –Broglie standing wave in an electron orbit is that the circumference must contain an integral multiple of electron wavelengths. We can express this condition as

$$2\pi r = n\lambda \qquad \cdots(19.21)$$

$$(n = 1,2,3......)$$

De –Broglie's equation for the wavelength of an electron in terms of its momentum.

$$\lambda = \frac{h}{mv} \qquad \cdots 19.22$$

Substituting λ in Eq:(19.21), we have,

$$2\pi r = n\frac{h}{mv}$$

$$mvr = n\frac{h}{2\pi}$$

This precisely explains the quantization of angular momentum condition imposed by Bohr in his original theory of hydrogen atom.

Limitations of Bohr's Theory

Bohr's theory successfully explains the spectra of simpler atoms or ions which contain only one electron e.g. hydrogen, singly ionized helium, doubly ionized lithium etc. But this theory fails to explain the spectra of many electrons atom. Also when a spectral line of hydrogen is examined more closely with high precision instruments it reveals a fine structure i.e. the spectral lines is found to consist of a number of closely spaced lines. Bohr's theory could not explain the fine structure of the spectral lines of hydrogen atom. Later researchers studied the effect of electric and magnetic fields on spectral lines.A spectral line was found to split into a number of lines under the influence of magnetic field (Zeeman Effect) and electric field (Stark effect). Bohr's theory cannot explain these two effects.

Check Points

What postulate of Bohr's model is justified by de –Broglie?

Example 19.1

The electron in the hydrogen atom makes a transition from n = 2 energy state to the ground state $n=1$. Find the wavelength of the emitted photon.

Solution:

We can use the equation

$$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=\frac{3R}{4}$$

$$\lambda=\frac{4}{3R}$$

$$\lambda=\frac{4}{3\left(1.097\times10^7\right)}$$

$$\lambda=1.215\times10^{-7}\text{m}=121.5\text{ nm}$$

Example 19.2

The Balmer series for hydrogen atom corresponds to electronic transitions that terminate in the state of quantum number n = 2. Find the longest wavelength of photon emitted.

Solution:

The longest –wavelength in the Balmer series result from the transition.

From $n=3$ to $n=2$.

$$\frac{1}{\lambda_{max}}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5}{36}R$$

$$\lambda_{max}=\frac{36}{5R}$$

$$\lambda_{max}=\frac{36}{5\left(1.097\times10^7\right)}=656.3\text{ nm}$$

Example 19.3

Find the shortest wavelength photon in the Balmer series.

Solution:

The shortest –wavelength photon in the Balmer series is emitted when the electron makes transition from $n=\infty$ to $n=2$. Therefore,

$$\frac{1}{\lambda_{min}}=R\left(\frac{1}{2^2}-\frac{1}{\infty}\right)=\frac{R}{4}$$

$$\lambda_{min}=\frac{4}{R}=\frac{4}{1.097\times10^7}=364\cdot6\text{ nm}$$

19.6 Excitation and Ionization Potential

The Bohar model as well as the current quantum mechanical model of an atom predicts that the total energy of an electron in an atom is quantized. The allowed energies are given by a relation of the form.

$$E_n=\frac{-E_0}{n^2}; \qquad n=1,2,3......$$

The state $n=1$ is called ground state, while states with $n=2,3,4,......$ are called exited states. When energy is supplied to the atom, then an electron in the atom reaches one of its exited states. The atom in an exited state cannot stay for a long time. The electron in an exited atom soon returns to lower energy levels by emitting photons.

Excitation Energy

The energy required to move electron from its ground state to an exited state is known as excitation energy. For example the first and second excitation energies of hydrogen atom are calculated to be.

$$\frac{-E_0}{2^2}-(-E_0)=\frac{3}{4}E_0=\frac{3}{4}(13.6\,\text{eV})=10.2\,\text{eV}_0$$

$$\frac{-E_0}{3^2}-(-E_0)=\frac{8}{9}E_0=\frac{8}{9}(13.6\text{ eV})=12.1\text{ eV}_0$$

Excitation Potential

The potential difference V in volts applied to an electron in its ground state to get an amount of energy equal to the excitation energy of the electron in the atom is called excitation potential of the atom. For example, the first and second excitation potential of H –atom are respectively 10.2 V and 12.1 V.

Ionization Energy

If an atom absorbs sufficient amount of energy, an electron may be raised to a level $n = \infty$. The electron then becomes free from the attractive force of the nucleus, i.e., the electron is removed from the atom. An atom which has lost one or more electrons is said to be ionized. The minimum energy required to remove an electron from its ground state is called ionization energy of the atom. But the energy of the electron in the initial (ground) state is E_o, and its energy in the final (ionized) state is zero. Thus the ionization energy of the atom is $\{0-(-E_0)\} = E_0$. This means that the ionization energy of the atom is numerically equal to the ground state energy of the atom. For example, the ionization energy of H –atom is 13.6 eV.

Ionization Potential

The potential difference applied to an electron to provide it the requisite amount of ionization energy is called ionization potential.

Example 19 .4

When a hydrogen atom is bombarded, the atom may be raise into a higher energy state. As the excited electron fall back to the lower energy levels, light is emitted. What are the three longest wavelength spectral lines emitted by the hydrogen atom as it returns to the $n = 1$ state from higher energy states?

Solution:

$$n=2 \rightarrow n=1: \quad \Delta E_{2,1} = -3.4-(-13.6) = 10.2\,\text{eV}$$

$$n=3 \rightarrow n=1: \quad \Delta E_{3,1} = -1.5-(-13.6) = 12.1\,\text{eV}$$

$$n=4 \rightarrow n=1: \quad \Delta E_{4,1} = -0.85-(-13.6) = 12.8\,\text{eV}$$

To find the corresponding wavelengths we can use $\Delta E = hf = \frac{hc}{\lambda}$.

For $n=2$ to $n=1$ transition

$$\lambda = \frac{hc}{\Delta E_{2,1}}$$

$$\lambda = \frac{6.63\times10^{-34}\,\mathrm{j\cdot s}\left(3\times10^{8}\,\mathrm{ms^{-1}}\right)}{(10.2\mathrm{eV})\left(1.60\times10^{-19}\,\mathrm{JeV^{-1}}\right)} = 121\ \mathrm{nm}$$

For $n=3$ to $n=1$ transition

$$\lambda = \frac{hc}{\Delta E_{3,1}}$$

$$\lambda = \frac{\left(6.63\times10^{-34}\,\mathrm{js}\right)\left(3\times10^{8}\,\mathrm{ms^{-1}}\right)}{(12.1\mathrm{eV})\left(1.60\times10^{-19}\,\mathrm{JeV^{-1}}\right)} = 102\ \mathrm{nm}$$

For $n=4$, to $n=1$ transition

$$\lambda = \frac{hc}{\Delta E_{4,1}}$$

$$\lambda = \frac{\left(6.63\times10^{-34}\,\mathrm{j\cdot s}\right)\left(3\times10^{8}\,\mathrm{ms^{-1}}\right)}{(12\cdot8\,\mathrm{eV})\left(1.60\times10^{-19}\right)\mathrm{JeV^{-1}}} = 96\cdot9\mathrm{nm}$$

These are the first three lines of the Lyman series.

19 .7 Inner shell Transition and Characteristic X–Rays

The transitions of electron in hydrogen atom results in the emission of spectral lines in the infrared, visible or ultraviolet region of electromagnetic spectrum due to small energy difference in the transition levels.

In heavy atoms, the electrons are assumed to be arranged in concentric shells labeled as K, L, M, N, O, etc. The K shell being closest to the nucleus, the L shell next, and so on. The inner shell electrons are tightly bound and large amount of energy is required for their displacement from their normal energy levels. When a heavy target material is bombarded with a beam of electrons, that has been accelerated by several k eV. Some of these electrons will collide with inner–shell electrons of the target and knock them out of their respective atoms.

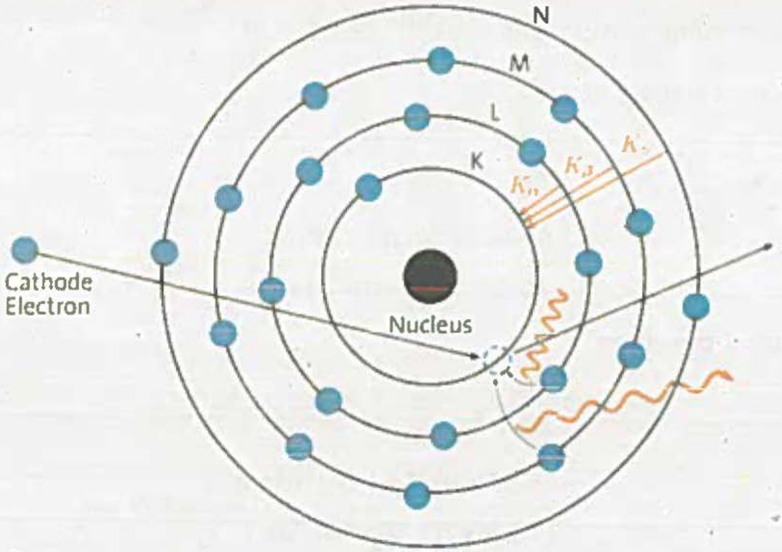


Figure 19.6 (a) :Inner shell Transition

Let a K–shell electron is knocked out from an atom creating a vacancy in K–shell.

Then an electron from either, L, M, or N –shell will quickly jump down to fill the vacancy in the K–shell emitting the excess energy as x –rays photon.

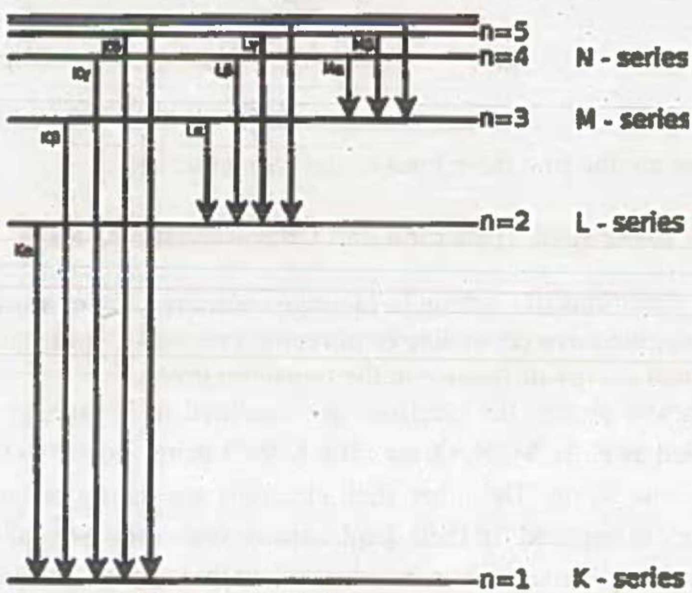


Figure 19.6 (b) : Characteristic X–Rays

These x–rays consists of series of specific wavelengths or frequencies and hence are called characteristic x–rays. An x–ray photon due to transition from L–shell to the vacancy in the K–shell is called K_α characteristic x –rays. The transition from M –and N –shell to the K –shell gives rise to K_β and K_γ characteristic x –rays respectively. The study of characteristic x –rays spectra has played a very important role in the study of atomic structure and the periodic table of elements.

19.7.1 Continuous x –rays

Another process that can rise the emission of x –rays, is illustrated in (fig 19 .7). Consider an electron traveling towards a target nucleus in the x –rays tube. The incident electron has coulomb interaction with orbital electrons as well as the positive nucleus. Because of the concentrated positive charge, the interaction with the nucleus is very strong. The force of attraction accelerates the electron. According to the classical theory of electromagnetism, an accelerated charge emits radiation called Bremsstrahlung, a Germen word meaning braking radiation. This Bremsstrahlung is called continues x –rays.

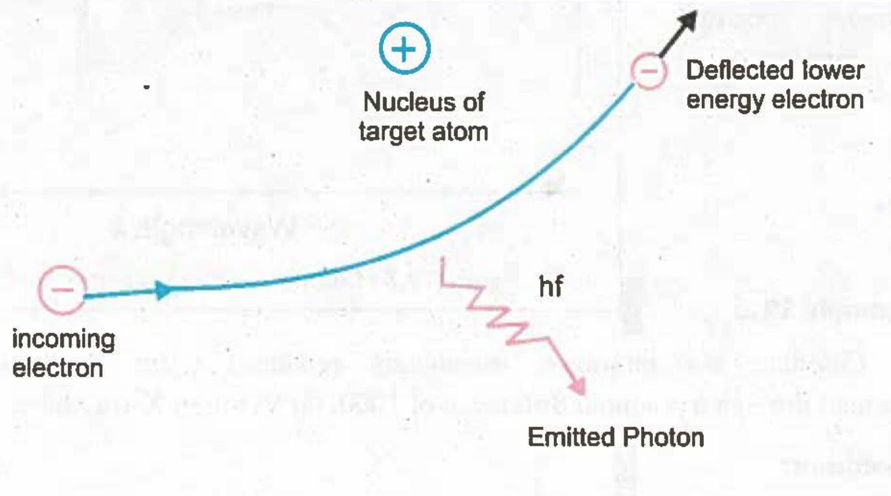


Figure 19.7 : electron moving towards nucleus

According to quantum theory, this radiation must appears in the form of photon. Since the radiated photon (shown in Fig: 19.7) carries energy, the electron must lose kinetic energy because of its encounter with the target nucleus. Let us consider an extreme example in which the electron losses all of its energy in a single collision. In this case, the initial energy of the electron (eV) is transformed completely into the energy of the photon (hf_{max}). In equation form we have

$$eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} \qquad \cdots(19.23)$$

Where eV is the energy of the electron after it has been accelerated through a potential difference of V volt and e is the charge on electron.

All radiation produced does not have the wavelength given in Eq:19.23 because many of the electrons are not stopped in a single collision.

This results in the production of the continuous spectrum of wavelengths.

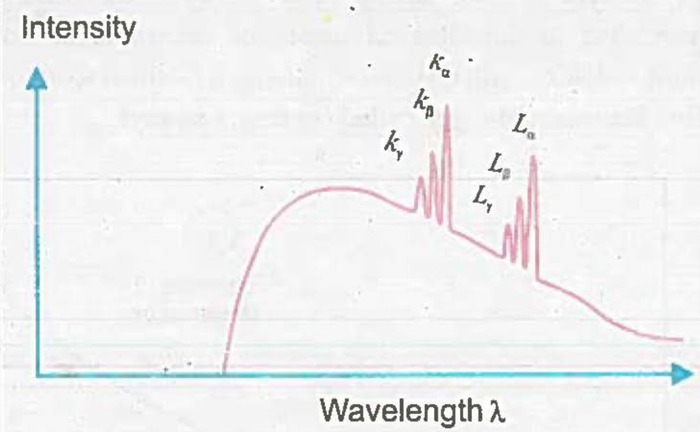


Figure 19.8 : Continuous x –rays curve.

Example 19 .5

Calculate the minimum wavelength produced when electrons are accelerated through a potential difference of 1000, 00 V, for an X –ray tube.

Solution:

Using (Eq 19.23)

$$\lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} = \frac{(6.63\times10^{-34}\,j\cdot s)(3\times10^{8}\,m/_{sec})}{(1.60\times10^{-19}c)(10^{5}v)}$$

$$\lambda_{min} = 1.24\times10^{-11}m$$

19.7.2 Production of X –Rays

Fig (19.9) shows an arrangement of producing X –rays. It consists a filaments F, heated by the current supplied from a battery B, emits electrons. This serves as a hot cathode. The anode is made of a solid copper bar "c". A high melting –point metal like platinum or tungsten is embedded at end of the copper rod and it serve as a target T.

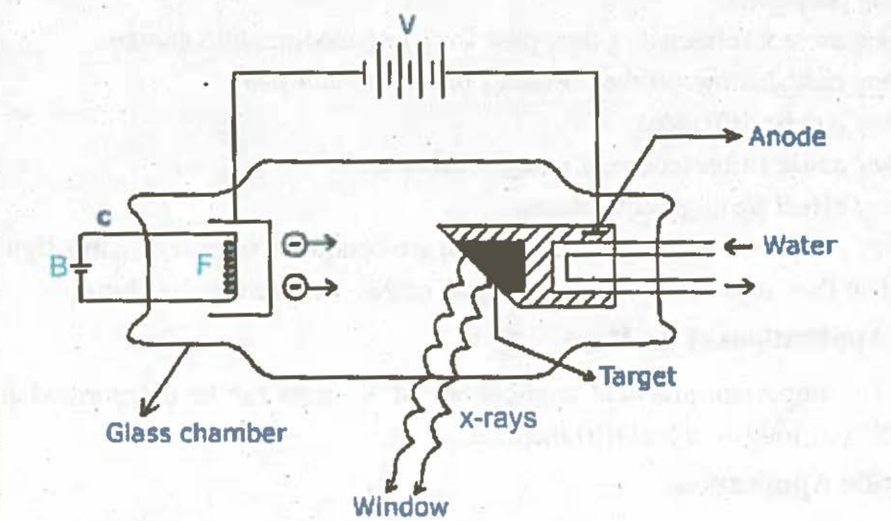


Figure 19.9 : X-rays device

The cathode and anode are enclosed inside an evacuated glass chamber and a high DC voltage of the order of 50,000 V is maintained between them. The electrons emitted from the cathode are accelerated by the high potential difference. The energetic electrons strike the target T and X–rays are produced.

It may be mentioned that a small part of the kinetic energy of the incident electrons is converted into X –rays, the rest is converted into heat. The target T becomes very hot and must, therefore, have a high melting point. The heat generated in target T is dissipated through the copper rod. Sometime the anode is cooled by water flowing behind the anode.

When such highly energetic electrons are suddenly stopped by target T, an intense beam of X –rays produced. These X –rays have large penetrating capacity and are called hard X –rays, while those with small penetrating power are called soft X–rays.

Properties of X–rays

Preliminary experimental investigations revealed that X –rays have the following properties.

1. They are not refracted as they pass from one medium into another.
2. They cast shadows of the obstacles placed in their path.
3. They can be diffracted.
4. They cause fluorescence in many substances.
5. They effect photographic plates.
6. They penetrate solid substances which are opaque to ordinary visible light.
7. When they pass through a solid, liquid or gas, they ionize the atoms.

Applications of X –Rays

The important practical applications of X –rays can be categorized as (i) Scientific (ii) Industrial and (iii) medical.

Scientific Applications

The diffraction of x –rays at crystals gave birth to x –ray crystallography. The Laue diffraction pattern can be used to determine the internal structure of the crystals. The spacing and dispositions of the atom of a crystal can be precisely determined.

Industrial Applications

Since X –rays penetrate the materials on which they are incident, they are used in industry to detect defects in metallic structures in big machines, railway tracks and bridges.

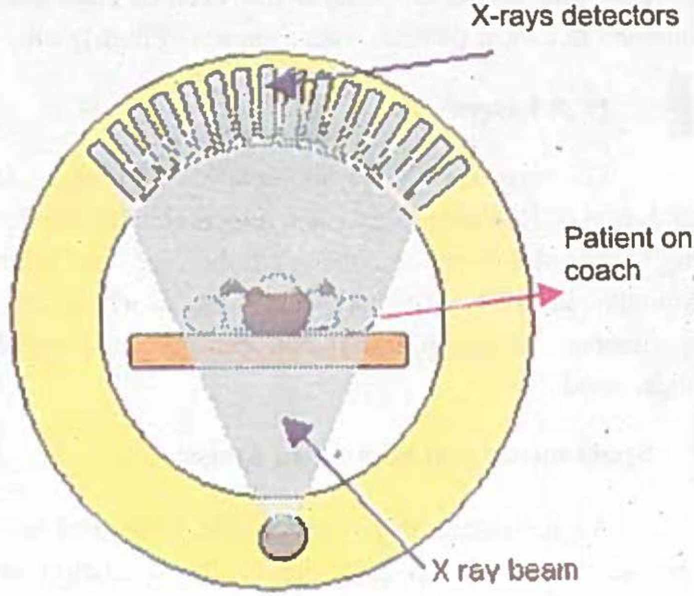
X –rays are used to analyse the compositions of alloys such as bronze, steel and artificial pearls. The structure of rubber and plastics can be analysed and controlled by X –rays studies.

Medical Applications

Almost immediately after their discovery by Roentgen, X –rays were used in hospitals in Vienna for surgical operations. Since bone is more opaque to x – rays than flesh, if x –rays are allowed to pass through a human body, the bones cast their shadow on the photographic plate. The X –ray photographs reveal fractures of bones or the presence of foreign bodies. X –rays can also be used for curing malignant tissues of the body. X –ray therapy has also been used for the treatment of cancer.

19.7.3 C T Scanner

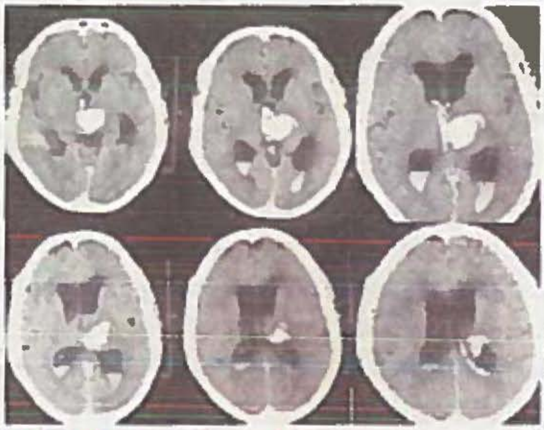
A 'normal' X- ray gives only limited information because it is rather like a shadow picture – fine detail within the image may be invisible especially if one organ lies in front of the region of the body being studied. To give a high quality images CT Scans are used to identify internal structures of various part of the human body. CT Scans Machine is 3D machine with computer model



19.9(b): CT scanner with the X-rays source and detectors

In the CT scanner there is one X-ray source but a large number of detectors.

The source and the detectors are mounted in a large ring-shaped machine and the patient is placed inside this on a couch as shown in figure. Each detector records an image and the source and detectors are then rotated around the patient to give views from a variety of direction. The image is called a tomogram. The couch and patient are then moved along the axis of the machine and another set of images is taken.


19.9(c): CT scan images

This large number of images (many hundreds) are then combined by a computer to give a composite detailed 3D image of the organs under investigation. The development of the CT scanner has been of enormous help in the study of the tumours in cancer patients where images of high quality are essential.

19 .8 Lasers

The term laser is an abbreviation of Light Amplification by Stimulated Emission of Radiation. Laser is a remarkable device that produces an intense and highly parallel beam of coherent light. The first laser was fabricated by T.H. Maiman in 1960. To understand the working of a laser, terms such as spontaneous emission, stimulated emission and population inversion must be understood.

Spontaneous and Stimulated Emission

We have seen that one possible method of exciting an atom is to send photons whose energy is equal to the excitation energy of the atom. The excitation energy ΔE is the difference between the two possible energy states of an atom. The excited atom wait for a brief period of about 10^{-8} s and then

spontaneously drops back to its lowest energy state, emitting light or photon of energy exactly equal to ΔE.

The only role of the passing photon is to give up its entire energy in exciting the electron to a higher energy state. This is a form of resonance in which a photon induce an upward transition.

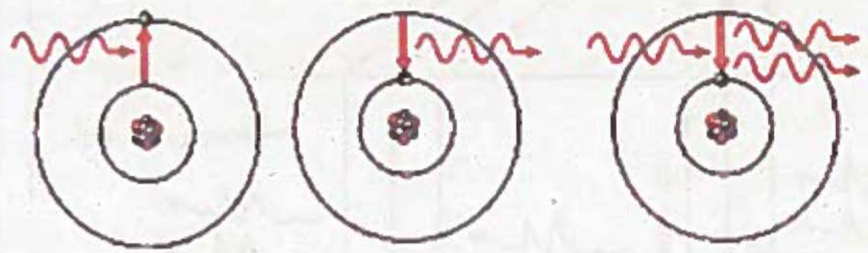


Figure 19. 10 : Electron falls back to its lower energy state E_1 and emitting a photon, thus duplicating the stimulated photon.

Can the photon play the opposite role, i. e. can it induce or stimulate the downward transition? The answer is, Yes. Imagine a photon of energy ΔE incident on an atom which is already excited, its excitation energy being equal to the energy ΔE of the photon.

The photon can stimulate the excited electron to fall back to the lowest energy state, instead of the excited electron waiting for 10^{-8} s for its spontaneous transition. This transition can then take place much sooner than 10^{-8} s. In this process a photon of energy ΔE is emitted and we already have the incident photon of the same energy because, now it is not absorbed. The emitted photons travel in exactly the same direction as the stimulated photon and are exactly in phase. (Fig 19. 10).

Population Inversion and Laser Action

Let us consider a simple case of a material whose atoms can reside in three different states as shown in fig 19 .11. State E_1 which is ground state, the excited

state E_3, in which the atoms can reside only for 10^{-8} s and the metastable state E_2, in which the atoms can reside for 10^{-3} s, much longer than 10^{-8} s.

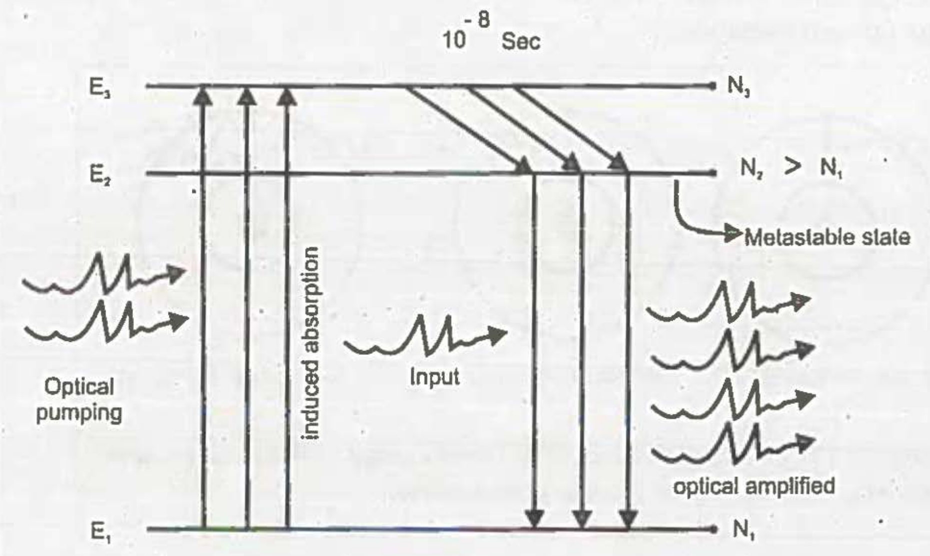


Figure 19.11 (a): electrons transition

A metastable state is an excited state in which an excited electron in unusually stable and from which the electron spontaneously falls to lower state only after relatively longer time.

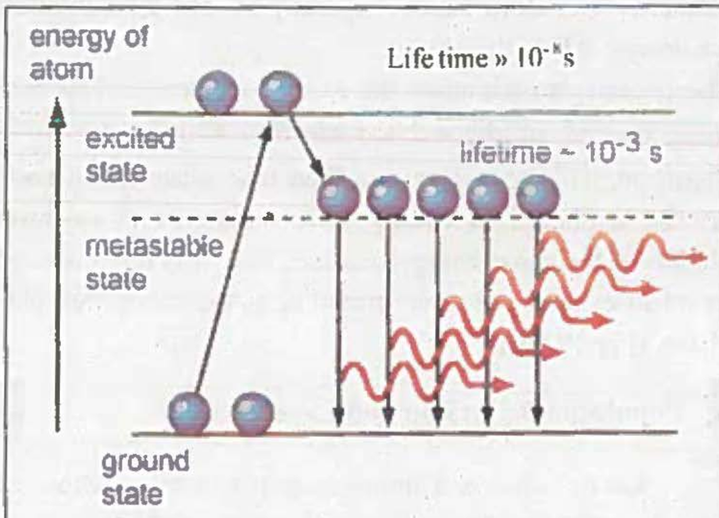


Figure 19.11 (b), metastable states

The transition from or to this state are difficult as compared to other excited states.

Hence, instead of direct excitation to this state, the electrons are excited to higher level for spontaneous fall to metastable state.

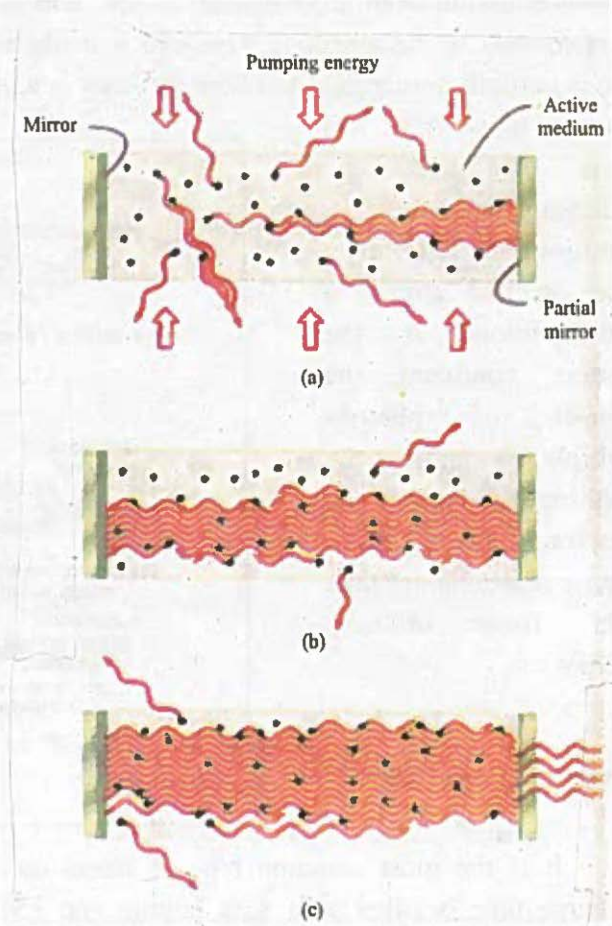


Figure 19.12 : photons move back and forth inside reflecting mirrors.

Also let us assume that the incident photons energy $hf = E_1 - E_3$ raise the atom from ground state E_1 to the excited state E_3, but the excited atoms do not decay back to E_1. Thus the only alternative for the atoms in the excited state E_3 is to decay spontaneously to state E_2. This eventually leads to the situation that the state E_2 contains more atoms than state, E_3. This situation is known as population inversion. Once the population inversion has been reached, the lasing action of a laser is simple to achieve.

The atoms in the metastable state E_2 are bombarded by photons of energy $hf = E_2 - E_1$, resulting in an induced emission, giving an intense, coherent beam in the direction of the incident photon.

The emitted photons must be confined in the assembly long enough to stimulate further emission from other excited atoms. This is achieved by using mirrors at the two ends of the assembly. One end is made totally reflecting, and the other end is partially transparent to allow the laser beam to escape (Fig 19.12). As the photons move back and forth between the reflecting mirrors they continue to stimulate other excited atoms to emit photons. As the process continues the number of photons multiply, and the resulting radiation is, therefore, much more intense and coherent than light from ordinary sources.

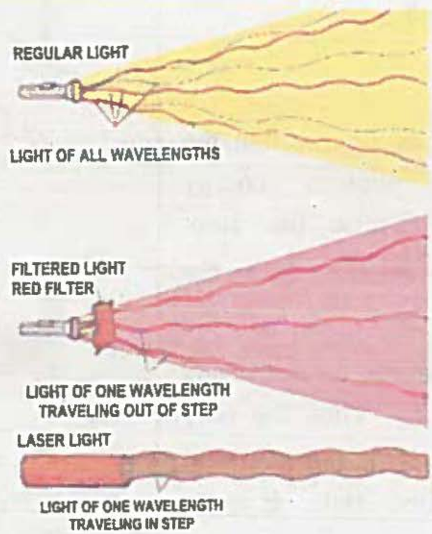


Figure 19.12 (d): comparison of light sources with laser.

Helium –Neon Laser

It is the most common type of lasers used in physics laboratories. Its discharge tube is filled with 85% helium and 15% neon gas. The neon is the lasing or active medium in this tube. By chance, helium and neon have nearly identical metastable states, respectively located 20.61 eV and 20.66 eV level. The high voltage electric discharge excites the electrons in some of the helium atoms to the 20.61 eV state. In this laser, population inversion in neon is achieved by direct collisions with same energy electrons of helium atoms.

Thus excited helium atoms collide with neon atoms, each transferring its own 20.61 eV of energy to an electron in the neon atom along with 0.05eV of K.E from the moving atom. As a result, the electrons in neon atoms are raised to the 20.66 eV state.

In this way, a population inversion is sustained in the neon gas relative to an energy level of 18.70 eV. Spontaneous emission from neon atoms initiate laser action and stimulated emission causes electrons in the neon to drop from 20.66 eV to the 18.70 eV level and red laser light of wavelength 632.8 nm corresponding to 1.96 eV energy is generated (Fig 19.13).

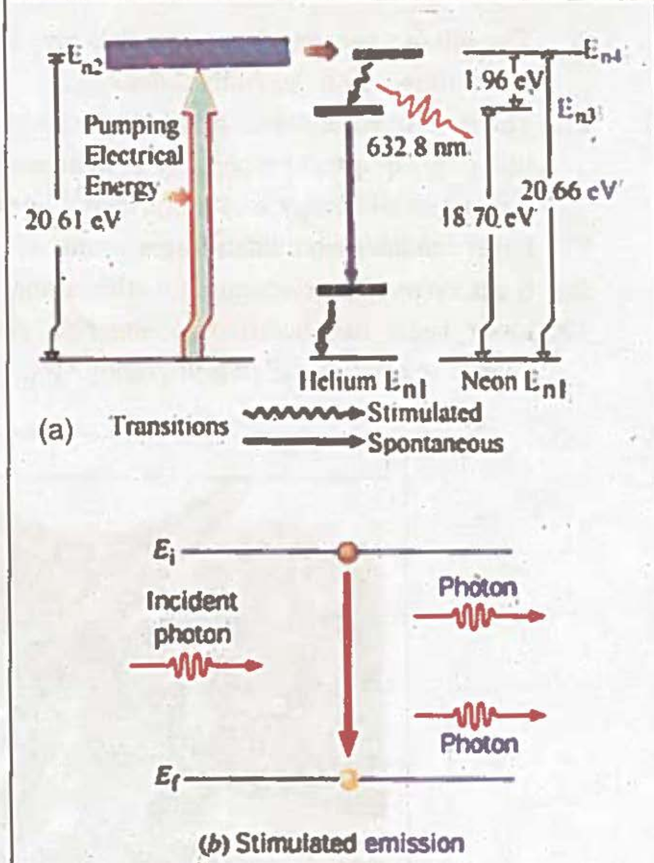


Figure 19.13 :

Uses of Laser

1. Laser beams are used as surgical tool for "welding" detached retains.
2. The narrow intense beam of laser can be used to destroy tissue in a localized area. Tiny organelles with a living cell have been destroyed by using laser to study how the absence of that organelle effects the behaviour of the cell.
3. Finely focused beam of laser has been used to destroy cancerous and pre –cancerous cells.
4. The heat of laser seals off capillaries and lymph vessels to prevent spread of the disease.

5. The intense heat produced in small area by a laser beam is also used for drilling tiny holes in hard materials.
6. The precise straightness of a laser beam is also useful to surveyors for lining up equipment especially in inaccessible locations.
7. It is potential energy source for including fusion reactions.
8. Laser can develop hidden finger prints.
9. It can be used in telecommunication along optical fibers.
10. Laser beam can be used to generate three dimensional images of objects in process called holography. Fig.19.14

Figure 19.14 : hologram

Key points

- When an atom gas or vapours at less than atmospheric pressure is suitably exited, usually by passing electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only.
- Postulates of Bohr's model of H –atom are:
  - i. An electron bound to the nucleus in an atom, can move around the nucleus in certain circular orbits without radiating. These orbits are called the discrete stationary states of the atom.
  - ii. Only those stationary states are allowed for which orbital angular momentum is, equal to an integral multiple of $\frac{h}{2\pi}$ i.e, $mvr = \frac{nh}{2\pi}$
  - iii. Whenever an electron makes a transition, i.e, jumps from high energy state E_n to a lower energy state E_p, a photon of energy hf is emitted so that $hf = E_n - E_p$.
- The transition of electrons in the hydrogen or other light elements results in the emission of spectral lines in the infrared, visible or ultraviolet region of electromagnetic spectrum due to small energy difference in the transition levels.
- The X–rays emitted in inner shell transition are called characteristics X–rays, because their energy depends upon the type of target materials.
- The X–rays that are emitted in all directions and with a continuous range of frequencies are known as continuous X–rays.
- Laser is the acronym for light amplification by stimulated Emission of Radiation.

- The incident photon absorbed by an atom in the ground state E_1, there by leaving the atom in the exited state E_2 called stimulated or induces absorption.
- Spontaneous or induced emission is that in which the atom emits a photon of energy $hf = E_2 - E_1$ in any arbitrary direction.
- Stimulated or induced emission is that in which the incident photon of energy $hf = E_2 - E_1$ induce the atom to decay by emitting a photon that travels in the direction of the incident photon. For each incident photon, we have two photons going in the same direction giving rise to an amplified as well as unidirectional coherent beam.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. If 13.6eV energy is required to ionize the hydrogen atom, then the required energy to remove an electron from n = 2 is
   a. 10.2eV b. 0 eV c. 3.4 eV d. 6.8 eV

2. For an atom of hydrogen atom the radius of the first orbit is given by,
   a. $\frac{h}{me^2}$ b. $\frac{me}{4h^2}$
   c. $\frac{h^2}{4\pi^2 kme^2}$ d. h^2me^2

3. The Balmer series is obtained when all the transition of electrons terminate on
   a. 1^{st} orbit b. 2^{nd} orbit c. 3^{rd} orbit d. 4^{th} orbit

4. In accordance with Bohr's theory the K.E of the electron is equal to
   a. $\frac{1}{2}\frac{Ze^2}{r}$ b. $\frac{Ze^2}{r}$ c. $\frac{Ze^2}{r^2}$ d. $\frac{1}{2}\frac{Ze^2}{r^2}$

5. According to Bohr's theory the radius of quantized orbit is given by
   a. $\frac{4\pi^2 m}{n^2h^2Ze^2}$ b. $\frac{n^2h^2}{4\pi^2 mZe^2}$
   c. $\frac{4\pi^2 m\,Ze^2 k}{n^2 h^2}$ d. $\frac{n^2h^2\,Ze^2}{4\pi^2 m}$

6. In the Bohr's model of the hydrogen atom, the lowest orbit corresponds to
   a. Infinite energy b. Maximum energy
   c. Minimum energy d. Zero energy

7. When an electron in an atom goes from a lower to higher orbit its
   a. K.E increases, P.E decreases b. K.E increases, P.E increases
   c. K.E decreases, P.E increases d. K.E decreases, P.E decreases

8. Frequency of X –rays depends upon
   a. Number of electrons striking target b. Accelerating potential
   c. Nature of the target d. Both b and c
9. Target material used in X –rays tube must have following properties.
   a. High atomic number and high melting point
   b. High atomic number and low melting point.
   c. Low atomic number and low melting point. d. High atomic number only
10. Laser is a device which can produce.
   a. Intense beam of light b. Coherent beam of light
   c Monochromatic beam of light d. All of the above

Conceptual Questions

1. Why does the spectrum of hydrogen consists of many lines even though a hydrogen atom has only a single electron?
2. Suppose that the electron in hydrogen atom obeyed classical mechanics rather quantum mechanics. Why would such a hypothetical atom emit a continuous spectrum rather than the observed line spectrum?
3. Can the electron in the ground state of hydrogen absorb a photon of energy (a) less than 13.6eV (b) greater than 13.6eV? Explain.
4. Why do solids give rise to continuous spectrum while hot gases give rise to line spectrum?
5. Explain the difference between laser light and light from an incandescent lamp.
6. Why Bohr extends quantum theory to the structure of the atom?
7. Why ${}_2He^4$ has larger ionization energy than H?
8. X –rays can emit electrons from metal surface and X –rays can be diffracted. Comment?
9. Why X-rays have different properties from light even though both originate from orbital transition of electrons in excited atoms?

10. What is meant by the statement that a laser beam is coherent, monochromatic and parallel?
11. What are laser knives?
12. Why we cannot see atom?
13. What meant by breaking radiation?
14. What is optical pumping?

Comprehensive Questions

1. Describe the spectrum of hydrogen atom in detail.
2. What are Bohr's postulates about hydrogen atom? Hence derive expression for the (a) radii of electron orbit (b) energy of the electron.
3. What do you understand by the terms normal state, Excited state, Excitation energy, ionization energy.
4. What are X-rays? Give an account of the properties, and uses of X-rays.
5. What is a laser? Explain the principle and operation of a laser. Describe some practical uses of lasers.

Numerical Problems

1. Find the shortest wavelength photon emitted in the lyman series of hydrogen. (91nm)
2. What is the wavelength of the second line of paschen series? $[1281.43\text{nm}]$.
3. Calculates the longest wavelength of radiation for the paschen series $[1875\text{ nm}]$.
4. The series limit wavelength of the Balmer series is emitted as the electron in the hydrogen atom falls from $n=\infty$ to the $n=2$ state. What is the wavelength of this line. Where ΔE=3.40eV. $[365\text{nm}]$.
5. A photon is emitted from a hydrogen atom, which undergoes a transition from that $n=3$ state to the $n=2$ state. Calculate (a) the energy (b) the wavelength, and (c) frequency of the emitted photon. $\left[(a)1.89\text{eV},(b)658\text{ nm }(c)4.56\times10^{14}\text{Hz}\right]$

6. Find the longest wavelength of light capable of ionizing a hydrogen atom. How much energy in needed to ionize a hydrogen atom? [91.2 nm, 13.6 eV].

7. Calculate the radius of the innermost orbital level of the hydrogen atom. $\left[5.3\times10^{-11}\text{m}\right]$.

8. (a) Determine the energy associated with the innermost orbit of the hydrogen atom (n=1). (b) Determine the energy associated with the second orbit of the hydrogen atom. (c) What energy does an incoming photon possess to raise an electron from first to the second allowed orbit of the hydrogen atom? $\left[(a)=-13.6\text{eV},\ (b)-3.4\text{eV},\ (c)-10.2\text{eV}\right]$.

9. An electron drops from the second energy level to the first energy level within an excited hydrogen atom (a) determine the energy of the photon emitted (b) calculate the frequency of the photon emitted (c) calculate the wavelength of the photon emitted.

$\left[(a)\ 10.2\ \text{eV}\ (b)\ 2.5\times10^{15}\text{Hz}\ (c)1.2\times10^{-7}\text{m}\right]$

10. An electron is in the first Bohr orbit of hydrogen. Find (a) the speed of the electron. (b) the time required for the electron to circle the nucleus.

$\left[(a)\ 2.19\times10^{6}\text{ms}^{-1},\ (b)1.52\times10^{-6}\ \text{s}\right]$

11. Electrons in an x –ray tube are accelerated through a potential difference of 3000 V. if these electrons were slowed down in a target, what will be the minimum wavelength of x –rays produced.

$\left[4.14\ \text{A}^{\circ}\right]$

12. Compute the potential difference through which an electron must be accelerate in order that the short –wave limit of the continuous x –ray spectrum shall be exactly 0.1nm.

[12,400V].

UNIT 20

.... Nuclear Physics ......

After studying this chapter students will be able to:

- describe a simple model for the atom to include protons, neutrons and electrons.
- determine the number of protons, neutrons and nucleons it contains for the specification of a nucleus in the form $_{Z}X^{A}$.
- explain that an element can exist in various isotopic forms each with a different number of neutrons.
- explain the use of mass spectrograph to demonstrate the existence of isotopes and to measure their relative abundance.
- define the terms unified mass scale, mass defect and calculate binding energy using Einstein's equation.
- illustrate graphically the variation of binding energy per nucleon with the mass number.
- explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission.
- identify that some nuclei are unstable, give out radiation to get rid of excess energy and are said to be radioactive.
- describe that an element may change into another element when radioactivity occurs.
- identify the spontaneous and random nature of nuclear decay.
- describe the term half-life and solve problems using the equation $\lambda=0.693/T_{1/2}$.
- determine the release of energy from different nuclear reactions.

- explain that atomic number and mass number conserve in nuclear reactions.
- describe energy and mass conservation in simple reactions and in radioactive decay.
- describe the phenomena of nuclear fission and fusion.
- describe the fission chain reaction.
- describe the function of various components of a nuclear reactor.
- describe the interaction of nuclear radiation with matter.
- describe the use of Geiger Muller counter and solid state detectors to detect the radiations.
- describe the basic forces of nature.
- describe the key features and components of the standard model of matter including hadrons, leptons and quarks.

For your information

The goals of Nuclear Physics is to discover, explore, and understand all forms of nuclear matter .Every star shines because of the energy provided by nuclear reactions taking place inside it. It is also nuclear reactions that drive the spectacular stellar explosions seen as supernovas, which create nearly all of the chemical elements. A supernova is the explosion of a star. In an instant, a star with many times the mass of our Sun can detonate with the energy of a billion suns. And then within just a few hours or day, it dims down again.

20 .1 Atomic Nucleus

Let us begin by reviewing a few fundamental facts that are probably already familiar. The nucleus is made up neutrons and protons, two particles which are about 1840 times more massive than electrons. They are spoken of collectively as nucleons Fig 20.1(a).

The number of protons in a nucleus is just equal to its atomic number z, and the total number of nucleons A is the integer closest to its mass number; hence the number of neutrons is A –Z. Thus the nucleus of " $_{11}Na^{23}$, a sodium atom which has atomic number 11 and mass number 23, contains 11 protons and 12 neutrons.

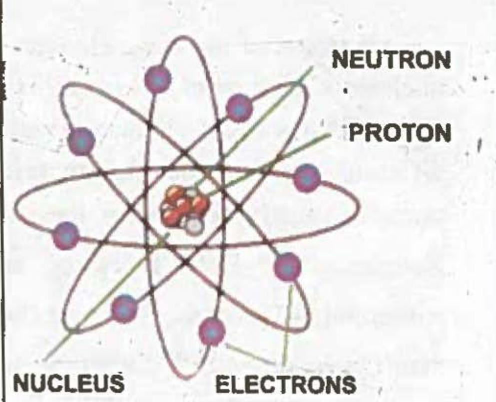


Figure 20.1(a): atomic nucleus

This is a relatively light nucleus; a typical heavy nucleus $_{92}U^{235}$, which is obviously contains 92 protons and 143 neutrons. The mass of the nucleus is very nearly equal to the mass of the atom; in kilograms it is the atomic weight divided by Avogadro's number, 6.03×10^{26}

A nuclide is a particular nucleus with a specified number of protons and neutrons. Any nuclide can be represented by its mass number A and atomic number z. For any element X its nuclide is written as $_{Z}X^{A}$. For example $_{1}H^{1}$ has $Z=1$ and A=1. $_{6}C^{12}$ has $Z=6$ and A=12.

The nucleus was first discovered in 1911 in experiment conducted by lord Rutherford and his students Geiger and Marsden on scattering of alpha particles by atom. He found that the scattering pattern could be explained of atoms consist of a small nucleus, deviation indicate that the nuclear size in of the order of 10^{-14} m. Since this is 10,000 times smaller than the diameter of atoms. The nucleus contains *Ze* charge, where Z is atomic number of the

element and e is the charge quantum $1.60\times10^{-19}C$. The mass of nucleus is of the order of $10^{-27}kg$. In nucleus, protons and neutrons are collectively called as nucleons.

20.2 Isotopes

All atoms of the same element contain the same number of protons in the nucleus of each atom. The number of neutrons in each atom of an element can differ. Atoms of an element which has the same atomic number Z, but have different mass number A, are referred to as isotopes. For example, natural uranium mostly consists of the isotopes $_{92}U^{238}$ and a small proportion of the isotopes $_{92}U^{235}$. Both types of atoms are uranium atoms, each nucleus containing 92 protons. However the isotope $_{92}U^{238}$ contains three more neutrons than the isotope $_{92}U^{235}$. Others examples are $_6C^{11}$;$_6C^{12}$;$_6C^{13}$; and $_6C^{14}$ are four isotopes of carbon, $_1H^1$;$_1H^2$ and $_1H^3$ are three isotopes of hydrogen etc.

Note that the number of electrons in an uncharged atom is equal to the number of protons in the nucleus. The chemical properties of an element are the same for all the isotopes of the element. This is because chemical reactions are determined by the electrons in an atom. Atoms of the same element undergoes the same chemical reactions because each atom has the same electron arrangement even if the atoms are different isotopes of the same elements.

20.3 Mass Spectrograph

It is a device with the help of which not only the isotopes of any element can be separated from one another but their masses can also be determined quite accurately. A mass spectrograph is based upon the principle that a beam of ions moving through electric and magnetic fields suffers a deflection that depends upon the charge and masses of the ions.

Hence ions of various masses are deflected differently. A spectrometer separates a mixture of ions into a spectrum of atoms having different masses.

A simple mass spectrograph is shown in (fig 20.1b). The atoms or molecules of the elements under investigation, in vapour form, are ionized in the ion source S. As a result of ionization, one electron is removed from the particles, leaving with a net positive charge +e. The positive ions, escaping the slit S_1 are accelerated through a potential difference V applied between two slits S_1 and S_2.

The ion passes through slit S_2 in the form of a narrow beam. The K.E of single charged ion at the slit S_2 will be given by

$$\frac{1}{2}mv^2 = Vq \qquad ...(20.1)$$

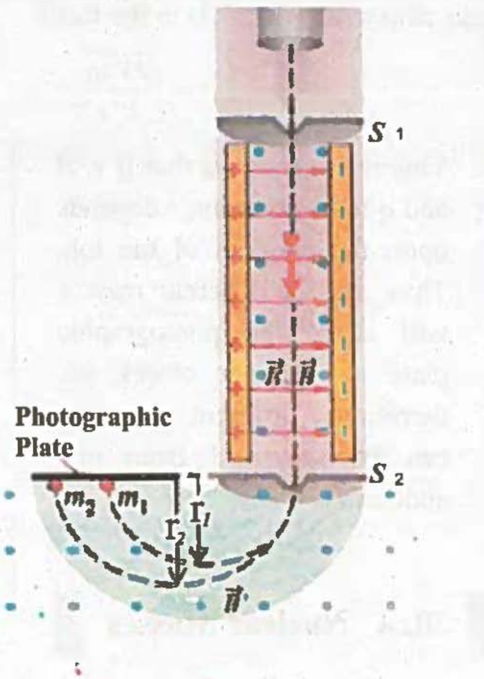


Figure 20.1(b): mass spectrograph

The ions are then subjected to a perpendicular and uniform magnetic field B in a vacuum chamber, where they are deflected in semi-circular paths towards a detector. The detector records the number of ions arriving per second. The centripetal force applied by magnetic fields is given by

$$Bqv = \frac{mv^2}{r} \qquad ...(20.2)$$

OR

$$m = \frac{B\,q\,r}{v}$$

Putting the value of v from Eq. 20.1, we get

$$m = \left(\frac{qr^2}{2V}\right)B^2 \qquad \ldots(20.3)$$

We can therefore, compute the mass m of the ion if *r*, *B*, *q* and *V* are known. We can also write (Eq 20.3) in the form.

$$r = \sqrt{\frac{2\,V\,m}{B^2 q}} \qquad \ldots(20.4)$$

This relation shows that if *v*, *B* and *q* to be constant, *r* depends upon the mass m of the ion. Thus ions of different masses will strike the photographic plate at different places, so, therefore, different isotopes can be separated from one another.

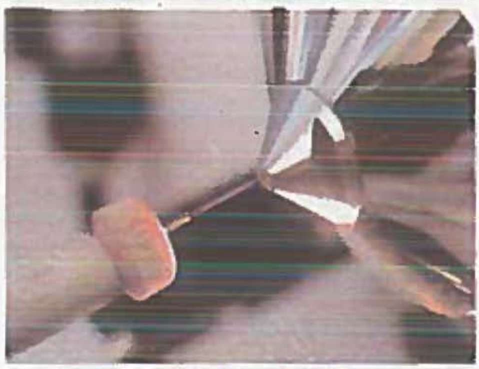


Figure 20.1 (c): Modern Mass Spectrometry instruments are used in the Drug Discovery and Development process.

20. 4 Nuclear Masses

It is known that a kilogram- mole of any element should contain Avogadro's number of atoms: 6.023×10^{26} atoms/ kg mole. Thus the mass of an atom or a nucleus is of the order of 10^{-27} kg. Since it is a small number, therefore, atomic and nuclear masses are expressed in term of unified (U) mass scale. The unified mass scale is a scale based on assigning a mass exactly 12 to rest mass of an atom of C^{12}. On this scale one, mass unit, called an atomic mass unit or a.m.u., is equal to $\frac{1}{12}$ of the mass of the carbon atom $_6C^{12}$. All other masses are then measured in this unit by comparison. The relation of a.m.u. or u to the kilogram is found as follows:

$$\text{Mass of } 6.23\times10^{26} \text{ atoms of } c^{12} = 12\,\text{kg}$$

$$\text{Mass of 1 atom of } C^{12} = \frac{12}{6.023\times10^{26}}\,\text{kg} = 1.660\times10^{-27}\,\text{kg}$$

It is often convenient, in nuclear physics to express certain masses in energy unit. According to Einstein mass–energy equivalence relation.

$$E = mc^2$$

$$1u = (1.660\times10^{-27}\ kg)(3\times10^{8}\ ms^{-2})^2$$

$$= 1.49\times10^{-10}\ j$$

Since $1eV = 1.60\times10^{-19}\ j$

$$1u = \frac{1.49\times10^{-10}}{1.60\times10^{-19}}\ eV = 9.31\times10^{8}\ eV$$

$$1u = 931\times10^{6}\ eV = 931\ MeV$$

The masses of electron, proton and neutron on u –scale are

$$m_e = 9.109\times10^{-31}\ kg = 5.485\times10^{-4}\ u = 0.51\ MeV$$

$$m_p = 1.673\times10^{-27}\ kg = 1.007u = 937\ MeV$$

$$m_n = 1.675\times10^{-27}\ kg = 1.008u = 938\ MeV$$

20. 5 Mass defect and binding energy

Why should a large unstable nucleus release energy when it fissions or a radioactive change takes place? The potential energy of a system depends on the position of the particles in the system, relative to each other. A stable system is one in which the potential energy of the system is at its lowest. When an unstable system becomes more stable, it changes to a state of lower potential energy. The protons and the neutrons in a nucleus are held together by a strong attractive force that prevents the protons pushing away from one another. To separate the protons and neutrons from one another, work would need to be done on them to overcome the strong nuclear force. The work needed to separate a nucleus into separate neutrons and protons is referred to as the binding energy of the nucleus fig (20.2). The greater the binding energy of a nucleus, the greater the work that would be needed to separate the neutrons and the protons in the nucleus from each other.

The mass of a nucleus is less than the mass of the same number of separate neutrons and protons. For example, the mass of a helium nucleus which consists of two protons and two neutrons is 0.8% less than the mass of two protons and two neutrons separated from each other. This difference is called the mass defect of the nucleus and is due to the protons and neutrons binding together when the nucleus was formed. The binding energy of the nucleus can be calculated from the mass defect using Einstein's famous equation $E = mc^2$.

Binding energy = mass defect $\times c^2$ ...(20.5)

Nuclear masses are usually expressed in atomic mass unit (u).

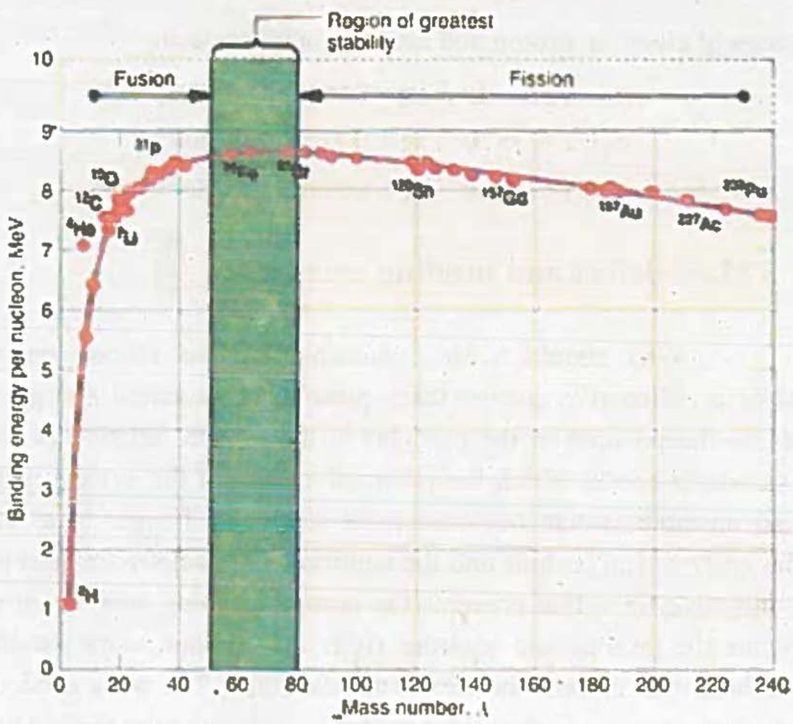


Figure 20.2 :binding energy curve

The binding energy per nucleon of a nucleus is the binding energy of a nucleus divided by the number of nucleons (i.e. protons and neutrons) in the nucleus.

This quantity is a measure of the stability of a nucleus. It can be easily calculated for any nucleus $_{Z}X^{A}$ of known mass M by following the steps below:

1. The mass defect (in atomic mass unit) of the nucleus,

$$\Delta m = Z\, m_p + (A-Z)m_n - M_{(A,Z)} \qquad ...(20.6)$$

Or $\quad \Delta m = Zm_p + Nm_n - M_{(A,Z)}$

Where m_p is the mass of a proton and m_N is the mass of a neutron.

2. The binding energy E_b(in MeV) = $931 \times \Delta m$

3. The binding energy per nucleon = $\frac{E_b}{A}$ (Packing fraction)

Note: Z = The number of protons in the nucleus,

A = The number of neutrons and protons,

So A − Z is the number of neutrons in nucleus.

Example 20.1

The mass of a $_{92}U^{235}$ nucleus is 234.99333U. The mass of a proton = 1.00728 u and mass of a neutron = 1.00867u. Calculate the binding energy per nucleons of a $_{92}U^{235}$ nucleus.

Solution:

Z = 92, A = 235

The number of neutrons = A–Z = 143

mass defect = (92×1.00728) + (143 ×1.00867) – (234.99333 = 1.91624 U

Binding energy = 1.91624 ×931 = 1784 MeV

Binding energy per nucleon = $\frac{1784}{235}$ = 7.6 MeV/ A

A graph of binding energy per nucleon number *A* is shown in fig 20.2 Remember that greater the binding energy per nucleon of a nucleus is, the more stable the nucleus is. The graph shows that

1. The binding energy per nucleon increases as "A" increases to a maximum of about 1MeV per nucleon at about "A" = 50 to 60 then decreases gradually.

2. The most stable nuclei are about $A = 50$ to 60 since this is where the binding energy per nucleons is greatest.
3. The binding energy per nucleon is increasesd when nuclear fission of a uranium 235 nucleus occurs.
4. The binding energy per nucleons is increased when light nuclei are fused together.

When a ${}_{92}U^{235}$ nucleus undergoes fission, the two fragment nuclei each comprise about half the number of nucleons. Therefore the binding energy per nucleon increases from about 7.5 MeV per nucleon for ${}_{92}U^{235}$ to about 8.8 MeV per nucleon for the fragments.

Thus the binding energy per nucleon increases by about 1 MeV for every nucleon ~~which means~~ that the energy released from the fission of a single fissionable nucleus is about 200 MeV. The mass of a ${}_{92}U^{235}$ nucleus is about 4×10^{-25} kg.

20. 6 Radioactivity

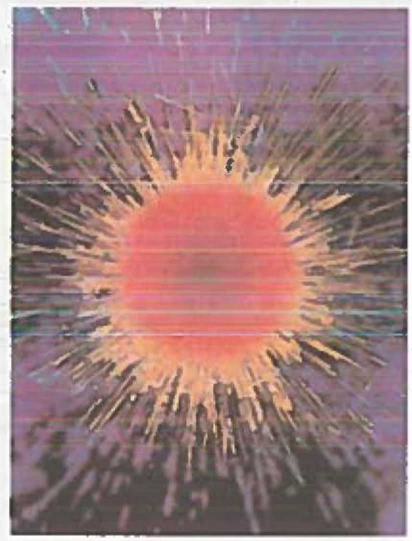


Figure 20.3 (a): nuclear radiations

Radioactivity was discovered accidentally in Paris by Henry Becquerel in 1896 when he was conducting research into the effects of x–rays on uranium compounds. He had discovered that certain substances exposed to X–rays glow and continued to glow when the X–rays machine was switched off. He wanted to know if the reverse effect was possible, namely emission of x–rays after the substances had been exposed to strong sunlight. In readiness for a sunny day, he placed a wrapped photographic plate in a drawer with a small quantity of uranium compound on it. After

several dull days, he decided to develop the plates, expecting to observe no more than a faint image of the compound.

He was therefore very surprised when he found a very strong image on plate. He realized that the uranium compound was emitting some form of radiation without having been exposed to sunlight. Further tests showed that the substance emits this radiation continuously even when stored in darkness for long period and that the radiation passes through glass but not through metal. The substance was described as "radioactive" because it did not need to be supplied with energy to make it emit radiations and was therefore emitting radiations actively.

Becquerel continued his research on X –rays and passed the investigation of radio activity on to his research student, Marie Curie. Within two years, Marie curie and her husband Pierre had discovered other substances which are radioactive, including two new elements, radium and polonium. Becquerel and the curies were awarded the noble prize in physics for their discoveries 1903.

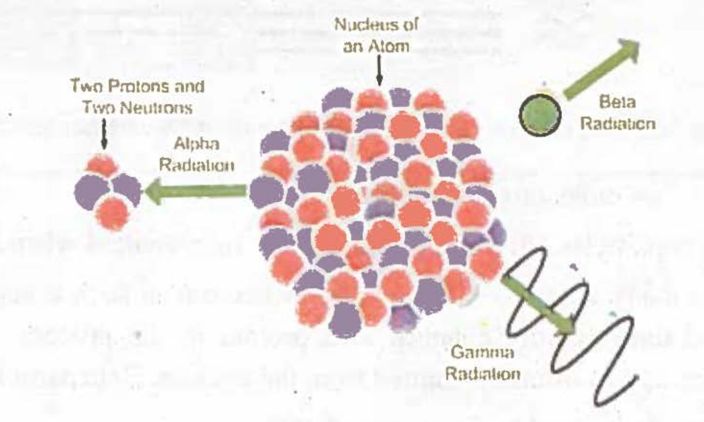


Figure 20.3 (b): nuclear radiations from radioactive element

The phenomenon of the spontaneous disintegration of heavier elements $Z > 82$ in to lighter elements along with the emission of three types of radiations is called radioactivity.

These three types of radiations are known as $\alpha-$particles, $\beta-$particles and $\gamma-$rays. The elements which possess this property, is called radioactive elements.

1. Alpha particle α consists of two protons and two neutrons i.e., these are positively charged helium nuclei. An $\alpha-$particle is emitted by a very large unstable nucleus. Alpha radiation.
   a. Is easily stopped by cardboard or thin metal.
   b. Has a range in air of no more than a few centimetres.
   c. Ionizes air molecules much more strongly than the other two types

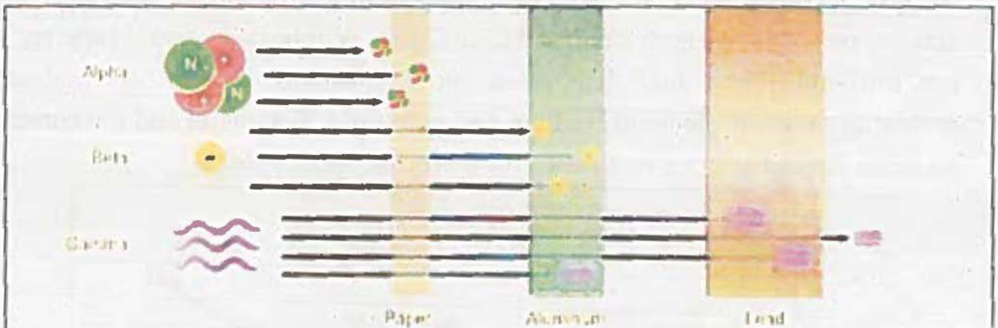


Figure 20.3 (c) : three forms of radiations with different penetration power

   of radioactive radiation.
2. Beta particles (β) consist of electrons, each emitted when a nucleus with too many neutrons disintegrates. A neutron in such a nucleus suddenly and unexpectedly changes to a proton; in the process, an electron is created and instantly emitted from the nucleus. Beta particles.
   a. Is stopped by 5 –10 mm of metal.
   b. Has a range in air of about 1m.
   c. Ionizes air molecules less strongly than $\alpha-$particles.
3. Gamma radiation (γ) consists of high energy photons. A photon is a packet of electromagnet waves. A gamma photon is emitted from a

nucleus with surplus energy after it has emitted an, α or a β-particle. Gamma radiations.

a. Is stopped only by several cm of lead.

b. Has an infinite range in air.

c. Ionizes air molecules very weakly.

Alpha Emission

Whenever an atom $_{Z}X^{A}$ disintegrates by α-emission, its atomic number reduces by 2 and the mass number reduces by 4 units. The disintegration reaction is, written as,

$$_{Z}X^{A} \longrightarrow \ _{Z-2}Y^{A-4} + _{2}He^{4} + Q$$

Q is the disintegration energy. Which is always positive, as the process is spontaneous. The decay product $_{Z-2}Y^{A-4}$ is called the daughter nucleus of the parent nucleus $_{Z}X^{A}$. The α particle is often written as $_{2}He^{4}$. The daughter nucleus may also remain unstable and undergo further disintegration till it attains stability. Following are examples of α decay.

$$_{92}U^{238} \longrightarrow {_{90}Th^{234}} + _{2}He^{4} + Q$$

$$_{88}Ra^{226} \longrightarrow {_{86}Rn^{222}} + _{2}He^{4} + Q$$

Beta Emission

The process of β emission involve no change in mass number A. It does, however, change the atomic number Z by -1 or +1 depending upon whether the particle emitted is negative β particle (electron) or positive β-particle (positron). Thus the β disintegration may lead to either of the following disintegration.

$$_{Z}X^{A} \longrightarrow {_{Z+1}X^{A}} + _{-1}\beta^{*} + \text{antineutrino} + Q_{1}$$

$$_{Z}X^{A} \longrightarrow {_{Z-1}X^{A}} + _{+1}\beta^{*} + \text{neutrino} + Q_{1}$$

As an example of negative β emission is the decay of thorium into protactinium:

$$_{90}Th^{234} \longrightarrow {_{91}Pa^{234}} + _{-1}\beta^{*} + \text{antineutrino} + Q_{1}$$

The prototype of β decay is the decay of neutron itself. The neutron, in free space, is unstable, decaying to proton and electron with a half life of 12 minutes.

$${}_{0}n^{1} \longrightarrow {}_{1}H^{1} + {}_{-1}\beta^{\cdot} + \text{antineutrino}$$

The best known example of β decay is from the naturally occurring isotope of C^{14}.

$${}_{6}C^{14} \longrightarrow {}_{7}N^{14} + {}_{-1}\beta^{\cdot} + \text{antineutrino}$$

As an example of a positron emitter is carbon 11, which decay by the reaction;

$${}_{6}C^{11} \longrightarrow {}_{5}B^{11} + {}_{1}\beta^{\circ} + \text{neutrino}$$

Gamma Emission

Most frequently the alpha or beta emission leaves the daughter nuclide in an exited state. Such a nuclide may go back to a more stable configuration and eventually to its ground state by emitting one or more γ-rays. Since γ-rays are massless photons, their emission will cause no change either in A or Z of the parent nuclide. The γ-decay process is written as follows.

$${}_{Z}X^{A} \longrightarrow \left({}_{Z}X^{A}\right)^{*} \longrightarrow {}_{Z}X^{A} + \gamma$$

Where $\left({}_{Z}X^{A}\right)^{*}$ represents an excited state of the nucleus.

20. 7 Spontaneous and Random Nuclear Decay

We know that radioactive elements disintegrates and emit α,β and γ radiations. This process is called transmutation by spontaneous disintegration. In this process each of the nuclei of a radioactive sample has a probability of decay into a daughter nucleus. The per unit time probability of decay of all nuclei is the same and has a fixed value, characteristics of material. However, the decay probability of one nucleus is quite independent of that of another nucleus. So in the natural spontaneous disintegration of a radioactive material not all the atoms disintegrates at the same time. Contrary,

different atoms decay at different times. The process of disintegration takes place randomly. When a nucleus disintegrates, nobody knows. However, it is observed that, on the average, the actual number of atoms, which decay at any instant, is proportional to the number of atoms present. As times goes on, some nuclei disintegrates and other survive. So the activity continues but with ever decreasing intensity.

20. 8 Half-life and rate of decay

The half –life of a radioactive isotope is the time taken for half the number of atoms of the isotope to disintegrate. Suppose 10000 atoms of a certain radioactive isotope "X" are present initially. The number of atoms decreases.

- From 10000 to 5000 after first half life, then
- From 5000 to 2500 second a further half life, then
- From 2500 to 1250 third a further half life, etc.

The amount of the radioactive isotope therefore decreases with time as shown in fig: 20.4 which is a half –life curve. Half –life values range from a fraction of a second to billions of years. For example, the half-life of polonium 212 is 3×10^{-7} s and that of lead 204 is 1.4×10^{7} years.

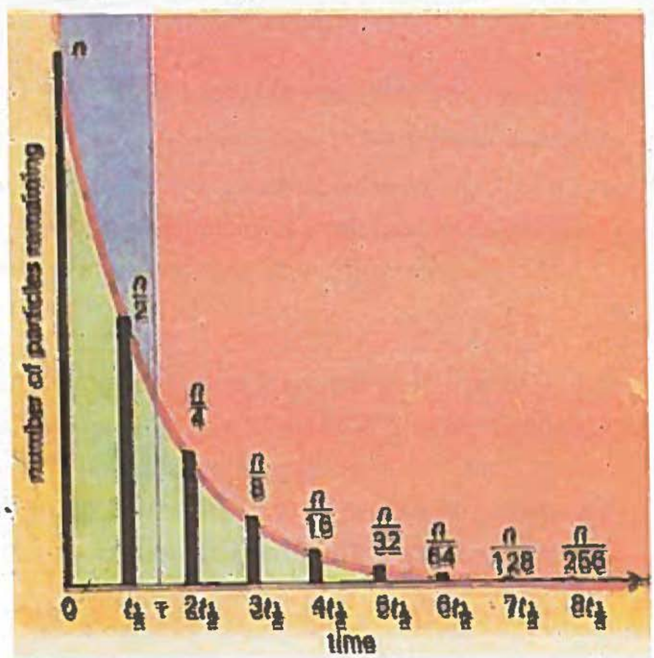


Figure 20.4 (a) :half –life curve

Radioactive disintegration is a random process.

For a large number of atoms of a given radioactive

isotope, the proportion that disintegrate per second is constant. This follows because of the random nature of radioactive disintegration.

If a radioactive sample contains "N" radioactive nuclei at some instant, one finds that the number of nuclei ΔN, that decay in a time Δt is proportional to N.

$$\frac{\Delta N}{\Delta t} \propto -N$$

$$\frac{\Delta N}{\Delta t} = -\lambda N \qquad \cdots(20.7)$$

Where λ is a constant of proportionality which depends on the nature of the element and is called decay constant and the negative sign signifies that N decreases with time, that is, ΔN is negative. The value of λ for any isotope determines the rate at which that isotope will decay.

The decay rate, or activity R, of a sample is defined as the number of decay per second. From equation 20.7 the decay rate is

$$R = -\frac{\Delta N}{\Delta t} = \lambda N \qquad \cdots(20.8)$$

Thus we see that isotopes with a large value of λ decay at a rapid rate in those with a small λ value decay slowly.

A general decay curve for a radioactive sample shown in figure (20.4a). One can show from equation (20.8) (using calculus) that the number of nuclei present varies with time according to the expression,

$$N = N_o e^{-\lambda t} \qquad \cdots(20.9)$$

Where N is the number of radioactive nuclei present at time t, N_o is the number present at time. t = 0, and e = 2.718….. is the base of the natural logarithm. Processes that obey equation (20.9) are sometimes said to undergo exponential decay. This is known as decay law of radioactive element. The unit of activity is the curie (Ci), defined as

$$1\text{Ci} = 3.70 \times 10^{10}\ \text{decay/s}.$$

This number of decay events per second was selected as the original activity unit because it is the approximate activity of 1g of radium. The S.I. unit of the activity is the Becquerel (Bq):

$$1\ \text{Bq} = 1 \text{ decay per second}$$

$$1\ \text{Ci} = 3.70\times10^{10}\,\text{Bq}$$

By substituting $N = \frac{1}{2} N_o$ and $T = T_{1/2}$ in the equation (20.9), we find that

$$\frac{1}{2}N_o = N_o e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$2 = e^{\lambda T_{1/2}}$$

Take natural logarithm of both sides and note that lne =1. We find that

$$\ln 2 = \lambda T_{1/2}$$

$$\Rightarrow \quad T_{1/2} = \frac{\ln 2}{\lambda}$$

$$T_{1/2} = \frac{0.693}{\lambda} \qquad \cdots (20.10)$$

This is the relation between the decay constant λ and the half–life $T_{1/2}$.

The half-life for radioactive isotope C^{14} is 5730 years, it means in 5730 years the 10 g of carbon disintegrated to 5 g and 5 g remain in given sample. As the time passes, the amount of remaining substance decreases but never reached to zero.

The value of half- life is constant for each radioactive element and it is possible to characterize the element by using its half -life value.

The rate of radioactive decay is directly proportional to the stability of the isotope.

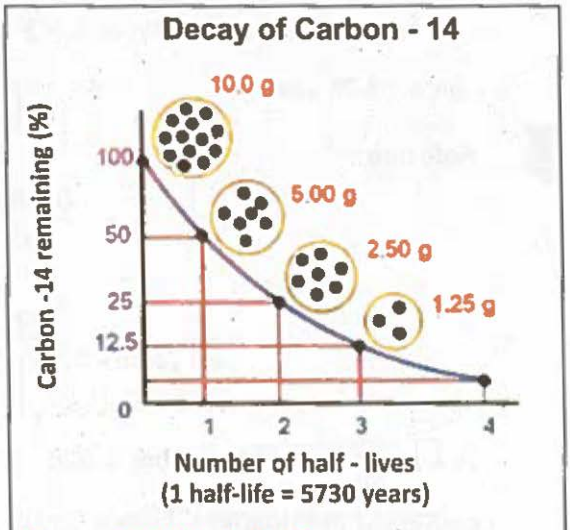


Figure 20.4 (b) : half –life curve

The half -life is a measurement of stability of radioactive elements.
The half-life of U^{238} is 4.5 x 10^9 years. So C^{14} is far less stable than U^{238}.

Example: 20.2:

The half life of radioactive nucleus ${}_{86}Ra^{226}$ is 1.6×10^3 years. Determine the decay constant.

Solution:

$$T_{1/2}=\frac{0.693}{\lambda}$$

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$T_{1/2}=1.6\times10^3 \text{ years}=\left(1.6\times10^3 \text{ years}\right)\left(3.15\times10^7 \text{ s/year}\right)$$

$$=5.0\times10^{10} \text{ s}$$

Therefore,

$$\lambda=\frac{0.693}{5.0\times10^{10} \text{ s}}=1.4\times10^{-11} \text{ s}^{-1}$$

Example (20.3):

Determine the activity of a 1 g sample of ${}_{38}^{90}Sr$ whose half –life against $\beta-$decay is 28 years.

Solution:

$$\lambda=\frac{0.693}{T_{1/2}}$$

$$=\frac{0.693}{28 \text{ years}\times3.15\times10^7 \text{ s/year}}$$

$$=7.83\times10^{-10} \text{ s}^{-1}$$

A k mole of an isotope has a mass equal to the atomic weight of that isotope expressed in kilograms. Hence 1 g of ${}_{38}^{90}Sr$ contains.

$$\frac{10^{-3}\text{kg}}{90\ \text{kg/kmole}} = 1.11\times10^{-5}\,\text{kmoles}$$

One k mole of any isotope contains Avogadro's number of atoms, and so 1g of

$^{90}_{38}Sr$ contains $1.11\times10^{-5}\,\text{kmole}\times6.025\times10^{26}$ atoms/ k mole = 6.69×10^{21} atoms

Thus the activity of the sample is,

$$R = \lambda N$$
$$=7.83\times10^{-10}\times6.69\times10^{21}\ s^{-1}$$
$$=5.23\times10^{12}\ s^{-1}$$
$$=141\ \text{curies}$$

20.9 Interaction of Radiation with Matter

1. Interaction of α – particles with matter

An α – particle travels a small distance in a medium before coming to rest. This distance is called the range of the particle. As the particle passes through a solid, liquid or gas, it loses energy due to excitation and ionization of atoms and molecules in the matter. The ionization may due to direct elastic collisions or through electrostatic attraction. Ionization is the main interaction with matter to detect the particle or to measure its energy. The range depends on the

i. Charge, mass and energy of the particle and

ii. The density of the medium and ionization potentials of the atoms of the medium.

Since α – particle is about 7000 times more massive than an electron, so it does not suffer any appreciable deflection from its straight path, provided it does not approach too closely to the nucleus of the atom. Thus α – particle continues producing intense ionization along its straight path till it loses all its energy and comes almost to rest. It, then,

captures two electrons from the medium and become a neutral helium atom.

2 Interaction of beta -particles with matter

$\beta-$particles also lose energy by producing ionization. However, its ionizing ability is about 100 times less than that of $\alpha-$particles. As a result its range is about 100 times more than $\alpha-$particles. $\beta-$particles are more easily deflected by collisions than heavy $\alpha-$particles. Thus the path of $\beta-$particles in matter is not straight but shows much straggling or scattering. The range of $\beta-$particles is measured by the effective depth of penetration into the medium not by the length of erratic path. If the density of the material is more through which the particle moves, the shorter will be its range. α and $\beta-$particles both radiate energy as $X-$rays photons when they are slow down by the electric field of the charged particles in a solid material.

3 Interaction of gamma rays with matter

Photons of $\gamma-$rays, being uncharged, cause very little ionization. Photons are removed from a beam by either scattering or absorption in the medium. They interact with matter in three distinct ways, depending mainly on their energy.

i. At low energies (less than about 0.5 MeV), the dominant process removes photons from a beam is the photoelectric effect.

ii. At intermediate energies, the dominant process is Compton scattering.

iii. At higher energies (more than 1.02 MeV), the dominant process is pair production.

In air $\gamma-$rays intensity falls off as the inverse square of the distance from the source, in the same manner as light from a lamp. In solids, the intensity decreases exponentially with increasing depth of penetration into the material. The intensity I_o of a beam after passing through a distance X in the medium is

reduced to intensity I given by the relation $I = I_o e^{-\mu x}$, where, μ is the linear absorption coefficient of the medium. This co –efficient depends on the energy of the photon as well as on the properties of matter.

Charged particles α or β and γ – radiation produce fluorescence or glow on striking some substance like zinc sulphide, sodium iodide or barium platinocyanide coated screens.

"Fluorescence is the property of absorbing radiant energy of high frequency and reemitting energy of low frequency in the visible region of electromagnetic spectrum".

4 Interaction of neutrons with matter

Neutrons, being neutral particles, are extremely penetrating particles. To be stopped or slowed, a neutron must undergo a direct collision with a nucleus or some other particles that has mass comparable to that of neutron Materials such as water or plastic, which contain more low mass nuclei per unit volume are used to stop neutrons. Neutrons produce a little indirect ionization when they interact with materials containing H – atoms and knock out protons.

20.10 Radiation Detectors

Various devices have been developed for detecting radiations. They are used for a variety of purposes including medical diagnosis, radioactive dating measurement and the measurement of background radiations.

Geiger –Muller Counter:

The Geiger –Muller counter (Fig 20.5) is perhaps the most common device used to detect radiations. It can be considered the prototype of all counters that make use of the ionization of a medium as the basic detection process. It consists of a cylindrical metal tube filled with gas at low pressure and a long wire along the axis of the tube. The wire is maintained at a high positive potential (about 1000V) with respect to the tube.

When a high energy particle or photon enters the tube through a thin window at one end, some of the atoms of the gas become ionized. The electrons removed from the atoms are attracted towards the wire, and in the process they ionize other atoms in their path. This results in an avalanche of electrons, which produces a current pulse at the output of the tube. After the pulse is amplified, it can be either used to trigger an electronic counter or delivered to a loudspeaker, which clicks each time a particle enters the detector.

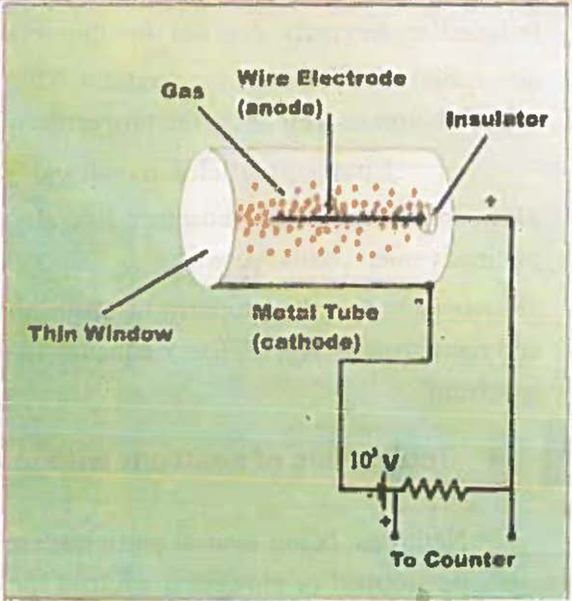


Figure 20.5 :G.M tube

Solid State Detector:

A solid state detector or semi-conductor diode detector is essentially a reversed –biased P–N junction (Fig 20.6). A P–N junction diode is a device which passes current readily when forward –biased and impedes the flow of current when reversed –biased.

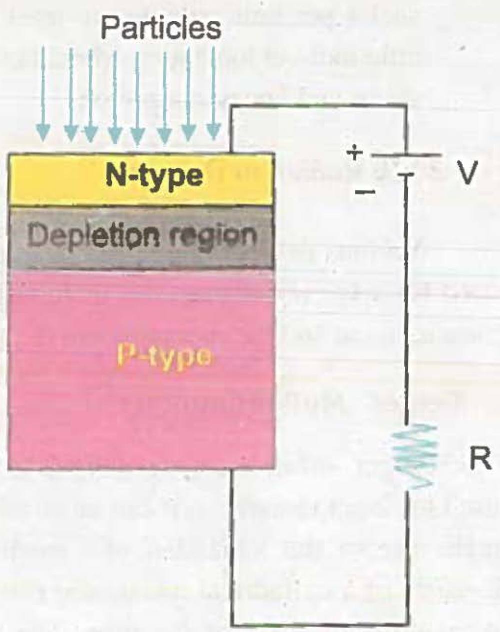


Figure 20.6: solid state detector

As an energetic particle passes through the junction, and electrons holes are simultaneously created. The internal electric field sweeps the electrons towards the side of the junction connected to positive side of the battery and the holes are swept toward the negative side. This creates a pulse of current that can be measured with any electronic counter. In a typical device, the duration of the pulse is about 10^{-7} s.

20.11 Nuclear Reactions

It is possible to change the structure of nuclei by bombarding them with energetic particles. Such collisions, which change the identity or properties of the target nuclei, are called nuclear reactions.

When a nucleus "*X*" is bombarded with some light particle "*a*", nuclear reaction take place, the product nucleus "*Y*" and a light particle "*b*" will be obtained. This will be represented by the equation.

$$X + a \longrightarrow Y + b$$

Rutherford was the first to observe nuclear reaction in 1919, using naturally occurring radioactive sources for the bombarding particles. He bombarded α-particles on nitrogen. He observed that as result of this reaction, oxygen is obtained and a proton is emitted. That is

$$_7N^{14} +_2 He^4 \longrightarrow _8O^{17} +_1 H^1 + Q \qquad (20.11)$$

The energy equivalent of the difference between the rest masses of elements on the L.H.S and those on the R.H.S is called the nuclear reaction energy and is denoted by "*Q*". Basically, "*Q*" represents the energy absorbed or evolved in any reaction. If "*Q* " is negative, energy is absorbed in the reaction (endothermic reaction) and if "*Q*" is positive, energy is evolved in the reaction (exothermic reaction). If "*Q*" is negative, the energy required to complete the reaction is usually provided by the K.E of the incoming particle unlike the case of chemical reaction, where the energy is usually provided by heating.

Conservation Laws in a Nuclear Reaction

In any nuclear reaction the following conservation laws must be obeyed. These laws form the guiding principles in determining which isotopes are formed during a nuclear reaction.

Conservation of atomic and mass number

Before and after any nuclear reaction the number of protons and neutrons must remain the same because protons and neutrons can neither be created nor destroyed using equation (20 .11) , we have

The number of nucleons on the L.H.S. = The number of nucleons on the R.H.S.

$$\text{Number of protons} = 7+2=8+1$$

$$\text{Number of neutrons} = 7+2=9+0$$

$$\text{Number of nucleons} = 18=18$$

Conservation of mass -energy

The conservation of number of nucleon does not imply the conservation of mass because the mass numbers differ from the atomic masses and the difference provides the binding energy to nucleons in the nucleus. Further, from Einstein's mass -energy relation it is known that the conservation of mass in no more a separate and independent principle but is a part of a more general principle of conservation of energy. Therefore, the principle of conservation of energy in mechanics is extended to the conservation of mass -energy in nuclear reactions. This will also include the energy difference due to changes of mass.

Based on the above conservation laws one can determined the (i) energy absorbed or liberated in any nuclear reaction and (ii) the product nucleus formed etc.

Let us calculate the reaction energy for the reaction given by equation (20.11). The rest mass of various particles on addition is

$$_2He^4 = 4.00263u \qquad _8O^{17} = 16.999133u$$

$$_7N^{14} = \frac{14.003074u\ +}{18.005677} \qquad _1H^1 = \frac{1.007825u\ +}{18.006958u}$$

Difference in rest masses before and after the reaction.

$$= 18.005677 - 18.006958$$
$$= -0.001281\ \text{u}$$
$$Q = -0.001281 \times 931$$
$$Q = -1.192\ \text{MeV}$$

Since "Q" is negative, the α-particle must have K.E 1.192 MeV for this reaction to occur. If the particle has less energy, this transformation will not take place. Usually the α-particles, i.e. more than 1.192 MeV, appears, as the K.E of product particles or nuclei.

20.12 Nuclear Fission

Knowing the fact that the emission of a β^- particle increases the atomic number by one, Fermi and his co-workers (1934) attempted to produce the elements beyond uranium (Z=92) which at that time was the last element in the periodic table. They bombarded uranium with neutrons and found that β-particles with different half -lives were emitted. Therefore, they concluded that the elements with $Z > 92$, i.e. the elements heavier than uranium, had been formed.

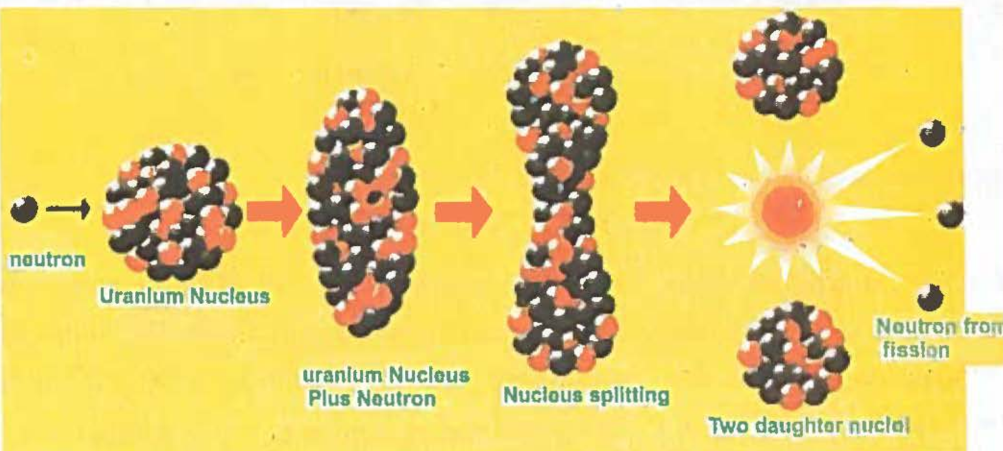


Figure 20.7 (a) : nuclear fission

Hahn and Strassmann made similar experiments in 1939. After the chemical analysis of the products they concluded that one of the product nuclei is barium and not a heavier element as predicted earlier. They concluded that the neutron bombardment can cause a uranium nucleus to break apart, producing two or more fragments of moderate and comparable size. This process was called nuclear fission. Further they found that reaction is much more pronounced with thermal neutron. Only U^{235} undergoes this process of fission – though naturally occurring uranium has 99.3% of U^{238} and 0.7% of U^{235}. We shall see that in this process there is a decrease in the mass of the system and hence energy is released. Since this process can be started automatically, it can be controlled and the energy liberated provides a good source of energy.

It was observed that when one thermal neutron strikes a uranium nuclei, three neutrons are emitted. In the reaction observed by Hahn that the product nuclei were ${}_{56}Ba^{141}$ and ${}_{36}Kr^{92}$. Therefore, the reaction can be written as

$${}_{0}n^{1} + {}_{92}U^{235} \rightarrow {}_{92}\overset{*236}{U} \rightarrow {}_{36}Kr^{92} + {}_{56}Ba^{141} + 3\,{}_{0}n^{1} + Q \qquad \cdots(20.12)$$

Where Q, is the energy of reaction which can be calculated from the value of rest masses of different nuclei. The calculation is given below

Initial masses	Final masses
$U^{235} = 235.0439\ u$	$Ba^{141} = 140.9139\ u$
${}_{0}n^{1} = 1.0087u$ +	$Kr^{92} = 91.8973u$
	$3\,{}_{0}n^{1} = 3.0261\ u$ +
$236.0526\ u$	$235.8373\ u$

The decrease in mass $= 236.0526 - 235.8373 = 0.215\ u$

$$Q = 0{\cdot}215 \times 931\ \text{MeV} = 200\ \text{MeV}$$

Therefore, when one atom of U^{235} undergoes fission 200 MeV of energy is released. If 1g of naturally occurring uranium, which has about 10^{19} atoms of U^{235} undergoes fission the total energy released would be 200×10^{19} MeV $= 3.2 \times 10^{8}\ j$. It is found that 1.0 kg of uranium deliver as much energy as the combustion of about 3000 tons of coal.

Fission Chain Reaction

As mentioned before when one uranium atom undergoes fission it releases 3 neutrons. If more than one of these neutrons is able to cause fission in the other U^{235} nuclei, the number of neutrons will increase rapidly. Thus, a chain reaction can be set up (Fig 20.7(b). The fission would produce at an ever –increasing rate and in a very short time the whole of U^{235} would be transformed with the release of a large amount of energy. If such a chain reaction is not controlled, the large energy can cause a violent explosion and destroy every thing that comes in its way.

This is the principle of the atom bomb. Further, if the amount of uranium is too small, the chain reaction can stop before it release the amount of energy required for explosion therefore, if the chain reaction is to start, it is necessary that the mass of uranium must be greater than some minimum mass called the critical mass or critical size.

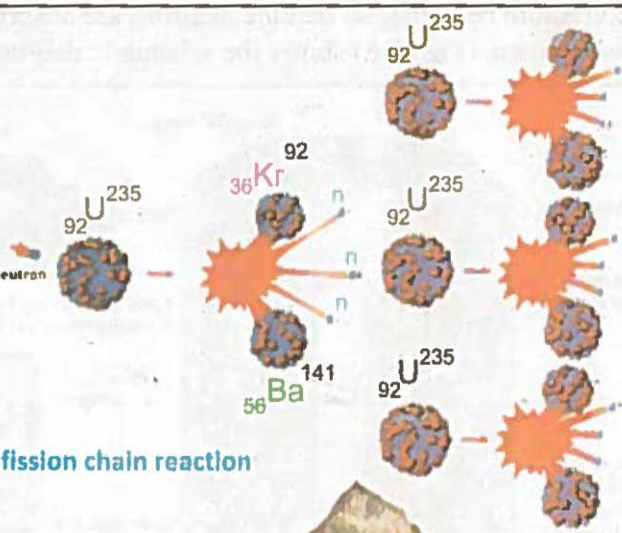


Figure 20.7 (b) : nuclear fission chain reaction

figure20.7(c) uranium ore rock

20. 13 Nuclear Reactors

The large amount of energy released in nuclear fission can be used for many useful purposes if the reaction is carried out under controlled conditions. A nuclear reactor is a device in which the fission chain reaction is a controlled one and the energy released can be used for any of the several purposes to produce power, to supply neutrons, to prepare radioiso topes, etc. The first reactor was installed and operated by Fermi and his co –workers in 1942 in the USA. In reactors, small pieces of uranium are spread throughout a material, called moderator, capable of slowing down the neutrons to thermal energies, so that thcy can cause fission in other nuclei. When a thermal neutron strikes a uranium atom, it starts the fission process which results in thc splitting of the uranium atom and the production of morc fast neutrons. These fast neutrons strike the materials and lose their K.E in repeated collision with the nuclei of the material and get thermalized. These thermalized neutrons strike another piece of uranium and again cause fission. Thus, a chain reaction is set up. Whenever this chain reaction is to be stoppe d, some material which is a strong absorber of neutrons is inserted in the uranium container so that the neutrons are absorbed and the rate of reaction is slowed down. (Fig 20.8) shows the schematic diagram of a nuclear reactor.

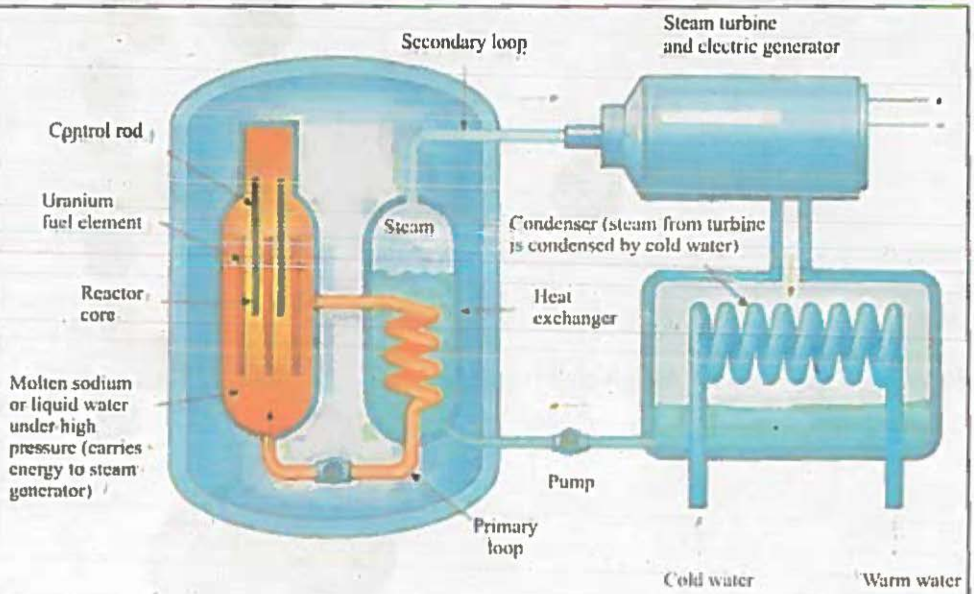


Figure 20.8 : Nuclear reactor

Basically, it consists of five parts (i) a core of nuclear fuel, (ii) a moderator for slowing down neutrons, (iii) control rods, (iv) coolant or heat exchanger for removing heat in the core, and (v) radiation shielding.

Nuclear fuel is a material that can be fissioned by thermal neutrons. It can be either one or all of the following isotopes. U^{233}, U^{235} and Pu^{239}. We shall see that when natural uranium is used, plutonium is produced in the nuclear reactor. Usually the fuel is put in different aluminium cans in cylindrical rods placed some distance apart. The fuel cans are separated by the moderator. As mentioned earlier, the moderator is used to slow down the fast neutrons produced in the fission process when thermal neutrons strike the nuclear fuel. The fast neutrons have many collisions with the materials and come out with thermal energies to strike another fuel can. The material of moderator (i) should be light, and (ii) should not absorb neutrons. Usually, graphite and heavy water (water containing deuterium instead of hydrogen) are used as moderators.

Sometimes the chain reaction, once started, can liberate an enormous amount of energy and can go out of hand and this can even blow up the reactor. To avoid such an accident, and to regulate the power level of the reactor, control rods are used. These control rods can be inserted into or drawn out of the reactor fuel core and consists of a material that absorbs neutrons, e. g. cadmium, boron or hafnium usually, cadmium control rods are used. If these rods are drawn out, the activity of neutrons increases and if they are inserted into the fuel core, the activity of neutrons decreases because the neutrons are absorbed by the rods.

The coolant, or heat exchanger, is used to cool the fuel rods and the moderator, and is capable of carrying away large amount of heat generated in the fission process. If the moderator, fuel rods, etc. are not cooled, the heat generated can melt them. The heat carried by the coolant produces steam that can run a turbine, which in turn can run an electric generator as shown in (fig 20.8).

The last part of the nuclear reactor is the shielding. Since the neutrons and the fragments in a reactor undergo radioactive decay and produce radiations which are harmful to life, there must be some shielding device to absorb those radiations. For this purpose a concrete wall which in a few feet thick is used.

20.13.1 Types of Reactors

There are two main types of nuclear reactors. These are:

(i) Thermal reactor (ii) Fast reactors.

I. **Thermal reactors** :The thermal reactors are called "thermal" because the neutron must be slowed down to "thermal energies" to produce further fission. They use natural uranium or slightly enriched uranium as fuel. Enriched uranium contains a greater percentage of U^{235} than natural uranium does. There are several designs of thermal reactors. Pressurized water reactor (PWR), are most widely used reactors in the world. In this type of reactor, the water is prevented from boiling, being kept under high pressure. This hot water is used to boil another circuit of water which produces steam for turbine rotation of electricity generators.

ii. **Fast reactor** :Fast reactor are designed to make use of $_{92}U^{238}$ which is about 99% content of natural uranium. Each $_{92}U^{238}$ nucleus absorbed a fast neutron and change into $_{94}P_u^{239}$.

$$_{92}U^{238} + {}_0n^1 \rightarrow {}_{93}N_p^{239} + {}_{-1}\beta^0$$

$$_{93}N_p^{234} \rightarrow {}_{94}P_u^{239} + {}_{-1}\beta^0$$

Plutonium can be fissioned by fast neutrons, hence, moderator is not needed in fast reactors. The core of fast reactors consists of a mixture of plutonium and uranium dioxide. Surrounded by a blanket of U^{238} Neutrons that escape from the core interact with U^{238} in the blanket, producing there by $_{94}P_u^{239}$. Thus more plutonium fuel is bred in this way and natural uranium is used more effectively.

20. 14 Nuclear Fusion

Figure 20.2 shows that the binding energy for light nuclei (those having a mass number of less than 20) is much smaller than the binding energy for heavier nuclei. This suggests a possible process that is the reverse of fission.

For your Information

Experimental research reactor called tokamaks used for fusion.

When two light nuclei combine to form a heavier nucleus, the process is called nuclear fusion.

Because the mass of the final nucleus is less than the rest masses of the original nuclei, there is a loss of mass accompanied by a release of energy. It is important to recognize that although fusion power plant have not get been developed, a great world –wide effort is under way to harness the energy from fusion reactions in the laboratory. The basic exothermic reaction in stars, including our own sun – and hence the source of nearly all of the energy in the universe – is the fusion of hydrogen nuclei into helium nucleus. This can take place under stellar conditions in two different series of processes. In one of them, the proton –proton cycle, direct collisions of protons result in the formation of heavier nuclei whose collision in turn yield helium nuclei. The other, the carbon cycle, in a series of steps in which carbon nuclei absorbed a succession of protons until they ultimately disgorge alpha particles to become carbon nuclei once more.

The initial reaction in the proton – proton cycle is

$$_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + \overset{+}{e} + \nu$$

Where, e^+ is called positron and "ν" is neutrino. A deuteron may then combine with a proton to from a $_2He^3$ nucleus:

$$_1H^1 + {}_1H^2 \longrightarrow {}_2He^3 + \gamma$$

Finally two $_2He^3$ nuclei react to produce a $_2He^4$ nucleus plus two protons:

$$_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1$$

The total energy evolved is $(\Delta m)c^2$, where Δm is the difference

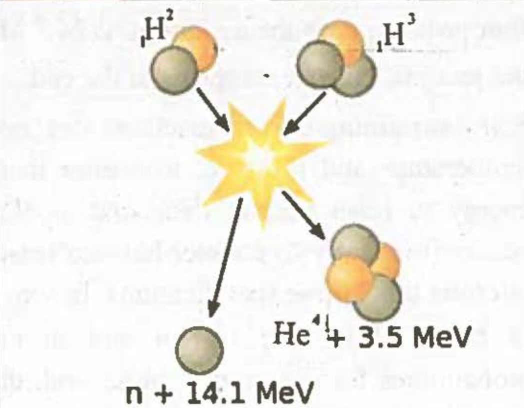


Figure 20.9 (a) : fusion reaction

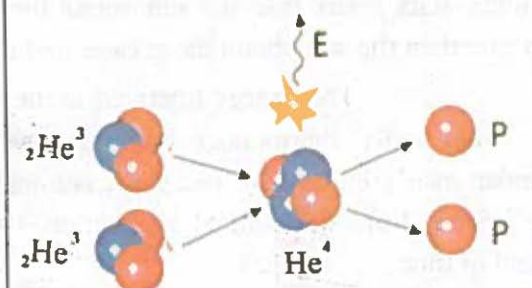


Figure 20.9 (b) : fusion reaction

between the mass of four protons and the mass of an alpha particles plus two positrons; it turn out to be 24.7 MeV.

The carbon cycle proceeds in the following way.

$$ {}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} $$

$$ {}_7N^{13} \rightarrow {}_6C^{13} + e^+ + \nu $$

$$ {}_1H^1 + {}_6C^{13} \longrightarrow {}_7N^{14} + \gamma $$

$$ {}_1H^1 + {}_7N^{14} \longrightarrow {}_8O^{15} + \gamma $$

$$ {}_8O^{15} \longrightarrow {}_7N^{15} + e^+ + \nu $$

$$ {}_1H^1 + {}_7N^{15} \longrightarrow {}_6C^{12} + {}_2He^4 $$

The net result again is the formation of an alpha particle and two positrons from four protons, with the evolution of 24.7 MeV; the initial ${}_6C^{12}$ acts as a catalyst for the process, since it reappears at the end.

Self –sustaining fusion reactions can occur only under conditions of extreme temperature and pressure, to ensure that the participating nuclei have enough energy to react despite their mutual electrostatic repulsion and that reactions occurs frequently to counter balance losses of energy to the surrounding. Stellar interiors meet these specifications. In sun, whose interior temperature is estimated to be $2\times10^6 K$, the carbon and proton –proton cycles have about equal probabilities for occurrence. In general, the carbon cycle is more efficient at high temperature, while the proton –proton cycle is more efficient at low temperature. Hence stars hotter than the sun obtain their energy the former cycle, while those cooler than the sun obtain the greater part of their energy from the latter cycle.

The energy liberated in the fusion of light nuclei into heavier ones is often called thermonuclear energy, particularly when the fusion take place under man's control on the earth neither the oroton –proton nor carbon cycle offers any hope of practical application, since their several steps required a great deal of time.

20 .15 Radiation Exposure

When a Geiger tube is used in any experiment, it records radiation even when a radioactive source is no-where near it. This is caused by radiation called background radiation. It is partly due to cosmic radiation which comes to us from outer space and partly from naturally occurring radioactive substance in the Earth's crust. The cosmic radiation consists of high energy charged particles and electromagnetic radiation. The atmosphere acts as a shield to absorb some of these radiation as well as ultraviolet rays. In recent past, the depletion of ozone layer in upper atmosphere has been detected which particularly filter ultraviolet rays reaching us. This may result in increased eye and skin diseases. The depletion of ozone layer is suspected to be caused due to excessive release of some chemicals in the atmosphere such as chlorofluorocarbons (CFC) used in refrigeration, aerosol spray and plastic foam industry. Its use is now being replaced by environmentally friendly chemicals. Many building materials contain small amounts of radioactive isotopes, (radon) radioactive carbon gas enters buildings from the ground. It gets trapped inside the building which makes radiation levels much higher from radon inside than outside. A good ventilation can reduce radon level inside the building. All types of food also contain a little radioactive substance. The most common are k^{40} and C^{14} isotopes.

Some radiation in environment is added by human activities. Medical practices, mostly diagnostic x –rays probably contribute the major portion to it. It is an unfortunate fact that many x –rays exposures such as routine chest x –rays and dental x –ray are made for no strong reason and may do more harm than good. Even x –rays exposure should have a definite justification that outweighs the risk. The other source include radioactive waste from nuclear facilities, hospital, research and industrial establishments, colour television, luminous watches and to tobacco leaves. A smoker not only inhales toxic smoke but also hazardous radiation. Low level background radiation from natural sources is normally considered to be harmless. However, higher levels of exposure are certainly damaging. We cannot avoid unnecessary exposure to any kind of ionizing radiation.

20. 16 Biological Effects of Radiation

Excessive exposure to radiation can cause damage to living tissues, cells, or organism. The degree and kind of damage caused to a particular part of the body depend upon the type, energy and dose of radiation received. There is no lower limit below which radiation damage does not occur. A number of small doses received over long period of time may lead to fatal consequences,

Radiation damage to living organism is primarily due to ionization effects in the cells. The cells is the basic unit of life. Its normal metabolic function may be disrupted as a result of interaction with the ionizing radiation. Excessive radiation does may cause death of individual cells, or produce chromosome abnormalities or genetic mutation.

The biological effects are generally of two types. Somatic and genetic. Somatic effects affect an individual directly. Skin burns, loss of hair, drop in the white blood cells and induction of cancer are example of somatic effects. The genetic effects may become apparent after a long time. The reason is that radiation can alter chemistry of the genes and may cause mutations. Even very low radiation doses reaching the reproductive organ of the body are potentially dangerous. Genetic effects may be passed on the future generations.

20. 17 Biological and Medical uses of Radiation

Although, all the isotopes of an element chemically behaves identically, but every isotopes emits radiation due to which it is easy to identify an isotope. It is this characteristic due to which the isotopes are being used in different fields of our life.

Biological Use

The chemical changes going on in an animal or a plant are very complex. The tracer method has been applied to study these changes. For example, the process of photosynthesis and the incorporation of carbon atoms in the CO_2 into giant and complex protein or carbohydrate molecules have been investigated by tracer techniques. Similarly information concerning the complex process of metabolism is obtained by means of radioisotope tracers. The

distribution of various elements, such as hydrogen, sodium, iodine, phosphorous, strontium, irons etc; in the body can be obtained by tracer technique. Genetic mutations are engineered by intense radioactively.

Radiation Therapy

High energy radiations penetrate deep into the body and can be used for intentional selective destruction of tissues, such as cancerous tumor. Radioisotopes of Co^{60} which emit $\beta-$particle and high –energy $\gamma-$rays is employed for the treatment of various types of cancers some radioisotopes are taken internally where they are selectively absorbed by certain organs and thus concentrate the radiation where it is most needed.

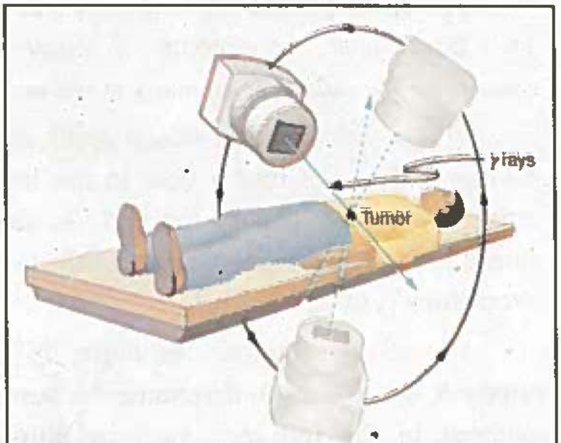


Fig 20. 10 :the Co^{60} source of γ-radiation is rotated around the patient.

For example, cancerous thyroid is treated with I^{131} radioisotope. Sometimes pellets or capsules of radioisotopes are planted close to the tumor and can be removed after treatment.

Medical Diagnostics

Hydrogen and sodium atoms are distributed uniformly throughout the body where iodine tends to concentrate in thyroids, phosphorous and strontium in bones and cobalt in liver. They can serve as tracer when injected or other wise given to patients. Radiation detectors may ascertain the passage of tracer through the body and their concentration in diseased tissues. The pattern of distribution of the radioactive tracers in a body can give a clue about normality or abnormality of the specific parts of the body.

Tracing Techniques

Radioactive particles can be used to trace chemicals participating in various reactions one of the most valuable uses of radioactive tracers is in medicine. For example, I^{131} is an artificially produced isotope of iodine. Iodine,

which is a necessary nutrient for our bodies, is obtained largely through the intake of iodized salt and seafood. The thyroid gland plays a major role in the distribution of iodine throughout the body in order to evaluate the performance of the thyroid; the patient drinks a very small amount of radioactive sodium iodide. Two hours later, the amount of iodine in the thyroid gland is determine by measuring the radiation intensity at the neck area.

A second medical application is that a salt containing radioactive sodium is injected into a vein in the leg. The time at which the radioisotope arrives at another part of the body is detected with the radiation counter. The elapsed time is a good indication of the presence or absence of constriction in the circulatory system.

The tracer technique is also useful in agricultural research. Suppose, one wishes to determine the best method of fertilizing a plant. A certain material in the fertilizer, such as nitrogen, can be tagged with one of its radioactive isotopes. The fertilizer is then sprayed on one group of plants, sprinkled on the ground for second group, and raked into soil for a third. A Geiger counter is then used to track the nitrogen through the three types of plants.

20 .18 Basic Forces of Nature

The key to understand the properties of elementary particles is to be able to describe the forces between them. All particles in nature are subjected to four fundamental forces. Strong, electromagnetic, weak, and gravitational.

The strong force is very short –ranged and is responsible for the binding of neutrons and protons into nuclei. This force represents the "glue" that hold the nucleons together and is the strongest of all the fundamental forces. The strong force is very short –ranged and is negligible for separation greater than 10^{-14} m.

The electromagnetic force, which is about 10^{-2} times the strength of the strong force, is responsible for the binding of atoms and molecules. It is a long –range force that decreases in strength as the inverse square of the separation between interacting particles.

The weak force is a short –range nuclear force that tends to produce instability in certain nuclei. It is responsible for most radioactive decay processes such as beta decay, and its strength is only about 10^{-9} time that of the strong force. Scientists now believe that the weak and electromagnetic forces are two manifestations of a single force called the electro weak force.

Finally, the gravitational force is a long –range force that has a strength of only about 10^{-38} times that of strong force. Although this familiar interaction is the force that holds the plants, stars, and galaxies together, its effect on elementary particles is negligible. Thus the gravitational force is the weakest of all the fundamental forces.

In modern physics, one often describes the interaction between particles in terms of the exchange of field particles or quanta. In the case of the familiar electromagnetic interaction, the field particles are photons. In the language of modern physics, one can say that the electromagnetic force is mediated by photons, which are the quanta of the electrometric field. Likewise, the strong force mediated by field particle called gluons, the weak force is mediated by particles called the w and z bosons, and the gravitational force is mediated by quanta of the gravitational field called gravitons.

20.19 Building Blocks of Matter

The word "atom" is from Greek word atomos, which mean indivisible. At one time atoms were thought to be the indivisible constituents of matter, that is, they were regarded to be elementary particles. Discoveries in the early part of the 20th century revealed that the atom is not elementary, but has as its constituents protons, neutrons and electrons. Up until the 1960s, physicists were bewildered by the large number and variety of elementary particles being discovered. In the last two decades, physicists have made tremendous advance in our knowledge of the structure of matter by recognizing that all particles with the exception of electrons, photons, and a few related particles are made of smaller particles called quarks. Thus, protons and neutrons, for example, are not truly elementary particles but are system of tightly bound quarks.

Classification of particles

Hadrons

Particles that interact through the strong force are called hadrons. There are two class of hadrons, known as mesons and baryons. Mesons has mass between the mass of the electron and the mass of proton. All mesons are known to be decay finally into electrons, positrons, neutrinos, and photons. The pion is the lightest of known mesons.

Baryons, which are the second class of hadrons, have mass equal to or greater than proton mass. Protons and neutrons are included in the baryon family, as are many other particles with the exception of the proton all baryons decay in such a way that the end products include a proton.

Leptons

Leptons are a group of particles that participate in the weak interaction. Include in this group are electrons, muons, and neutrinos, which are all less massive than the lightest hadron. Since lepton has no internal structure, they appear to be truly elementary particles scientists believe that there are only six leptons.

Quarks

According to quark theory initiated by M. Gell – Mann and G. Zweig, the quarks are proposed as the basic building blocks of the mesons and baryons. The quark model is based on the following assumptions.

1. There are six different types of quark, the up quark, the down quark, the strange quark, the charmed quark, the bottom quark and the top quark referred to as u, d, s, c, b and t.
2. For every type of quark, there is a corresponding antiquark.
3. Quarks combine in threes to form particles like the protons and the neutrons. Antiquarks also combine in threes to form antiparticles like the antiproton and the antineutron.
4. A meson consists of a quark and an antiquark.

In term of the charge of the electron, the u, c and t quarks each carry a charge of $+\frac{2}{3}e$ and the other three quarks carry a charge of $-\frac{1}{3}e$. An antiquark carries an equal and opposite charge to its corresponding quark. The symbol for antiquark is the same as for a quark but with a bar over the top. For example, $\bar{d}$ represents the symbol for a down antiquark.

Thus

- A proton is composed of two up quarks and a down quark.
- A neutron consists of an up quark and two down quarks, as shown in fig 20.11.

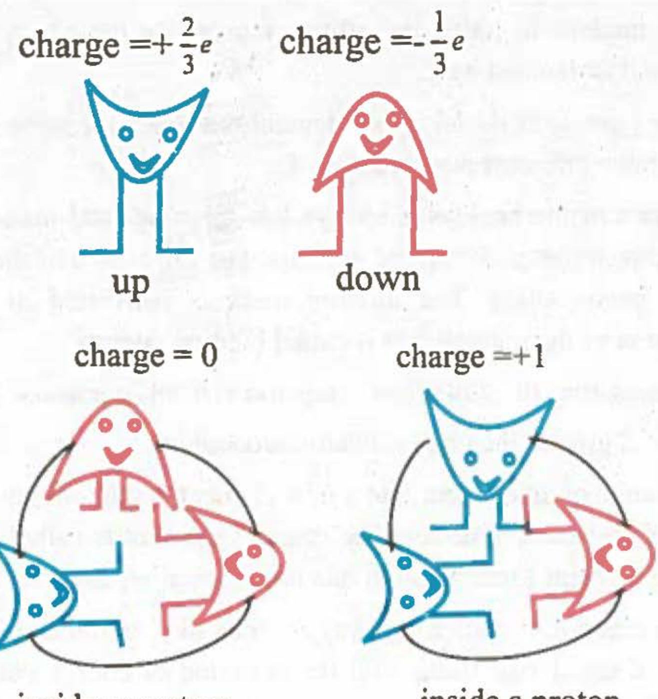


Figure 20.11: up and down quarks illustration

Key points

- The combined number of all the protons and neutrons is known as mass number and is denoted by A.
- The protons and neutrons present in the nucleus are called nucleons.
- The number of neutrons present in a nucleus is called its neutrons number and is denoted by N.
- The number of protons inside a nucleus or the number of electrons outside of the nucleus is called the atomic number or the charge number of an atom and is denoted by Z.
- Isotopes are such nuclei of an element that have the same charge number Z, but have different mass number A.
- The mass of the nucleus is always less than the total mass of the protons and neutron that make up the nucleus. The difference of the two masses is called mass defect. The missing mass is converted to energy in the formation of the nucleus and is called binding energy.
- The emission of radiations $(\alpha,\beta \text{ and } \gamma)$ from elements having charge number Z greater than 82 is called radioactivity.
- The change of an element into a new element due to emission of radiations is called radioactive decay. The original element is called parent element and the element formed due to this decay is called daughter element.
- Such a reaction in which a heavy nucleus like uranium splits up into two nuclei of equal size along with the emission of energy during reaction is called fission reaction.
- Half –life of a radioactive element is that period in which half of the atoms of the parent element decay into daughter element.
- Such a nuclear reaction in which two light nuclei merge to form a heavy nucleus along with the emission of energy is called fusion reaction.

- The strength of the radiation source is indicated by its activity measured in Becquerel. One Becquerel (Bq) is one disintegration per second. A larger unit is curie (ci) which equals 3.7×10^{10} disintegration per second.
- The effect of radiation on a body absorbing it relates to a quantity called absorbed dose D defined as the energy E absorbed from ionizing radiation per unit mass m of the absorbing body.
- **The basic forces are:**

  i. The strong force　　ii. Electromagnetic force

  iii. Weak nuclear force　　iv. Gravitational force.
- Subatomic particles are divided into following three groups:

  i. Hadrons　　ii. Leptons　　iii. Quarks

  elementary particles are the basic building blocks of matter.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. The binding energy for nucleus A is 7.7 MeV and that for nucleus B is 7.8 MeV. Which nucleus has the larger mass?

   a. Nucleus A　　b. Nucleus B

   c. Less than nucleus A　　d. None of these

2. How many neutrons are there in the nuclide Zn^{66}?

   a. 22　　b. 30

   c. 36　　d. 66

3. Mass equivalent of 931 MeV energy is

   a. 6.02×10^{-23}kg　　b. 1.766×10^{-27}kg

   c. 2.67×10^{-27}kg　　d. 6.02×10^{-27}kg

4. The energy equivalent of 1 kg of matter is about.

   a. 10^{-15} J　　b. 1 J　　c. 10^{-12} J　　d. 10^{17} J

5. The radioactive nuclide $_{86}Ra^{228}$ decays by a series of emissions of three alpha particles and one beta particle. The nuclide X finally formed is,

   a. $_{84}^{220}X$　　b. $_{86}^{222}X$

   c. $_{83}^{216}X$　　d. $_{88}^{215}X$

6. A radioactive substance has a half life of four months. 3 –fourth of the substance will decay in.

a. 6 months b. 8 months

c. 12 months d. 16 months

7. Gamma radiations are emitted due to.

a. De-excitation of atom b. De–excitation of nucleus

c. Excitation of atom d. Excitation of nucleus

8. Unit of decay constant λ is,

a. ms b. m^{-1}

c. m d. s^{-1}

9. Which of the following basic forces is able to provide an attraction between two neutrons.

a. Electrostatic and nuclear b. Electrostatic and gravitational

c. Gravitational and strong nuclear d. Only nuclear force

10. Bottom quark carries charge.

a. $\frac{2}{3}e$ b. $\frac{-2}{3}e$

c. $\frac{+1}{3}e$ d. $\frac{-1}{3}e$

Conceptual Questions

1. Why does the alpha particle not make physical contact with nucleus, when an alpha particle in headed directly toward the nucleus of an atom.
2. Why do heavier elements require more neutrons in order to maintain stability?
3. An alpha particle has twice the charge of a beta particle. Why does the former deflect less than the later when passing between electrically charged plates, assuming they both have the same speed.

4. Element X has several isotopes. What do these isotopes have in common? How do they differ?
5. How many protons are there in the nucleus $_{86}^{222}Rn$? How many neutrons? How many electrons are there in the neutral atom?
6. Ra^{226} has half –life of 1600 years.
   a. What fraction remains after 4800 years?
   b. How many half –lives does it have in 9600 years?
7. Radium has a half –life of about 1600 years. If the universe was formed five billion or more year ago, why is there any radium left now?
8. Nuclear power plants use nuclear fission reactions to generate steam to run steam –turbine generator. How does the nuclear reaction produce heat?
9. What factors make a fusion reaction difficult to achieve?
10. Discuss the similarities and differences between fission and fusion.
11. In what ways is time constant CR similar to and different from (a) radioactive decay constant, λ (b) radioactive half-life?
12. What happen to atomic number of a nucleus that emits γ-rays photons?
13. What happen to its mass?
14. Explain why neutron activated nuclides tend to decay by β^- rather than β^+.
15. Why are large nuclei unstable?
16. What happen to the atomic number and mass number of a nucleus that (a) emits an electron? (b) undergoes electron capture? (c) emits an α-particle?
17. How many α-decay occur in the decay of thorium $_{90}Th^{298}$ into $_{82}Pb^{212}$?
18. What is colour force?

Comprehensive Questions:

1. What is meant by natural radioactivity? How are the natural radioactive radiations classified into three types?
2. Explain the principle, construction, working and necessary mathematical theory of a mass spectrometer.

3. What are isotopes? Explain with examples.
4. Explain the term mass defect and binding energy related to a nucleus.
5. Define and explain the half –life of a radioactive element?
6. Define and explain nuclear reactions.
7. Write a comprehensive note on nuclear fission.
8. What is a nuclear reactor? Give the principle, construction, working and uses of a typical nuclear reactor.
9. What is meant by nuclear fusion? Discuss how can energy be released in the fusion process? Illustrate with examples.
10. What is a radiation detector? Explain the principle and working of GM counter and solid state detector.
11. Discuss the technique and use of radio isotopes in the different fields of human activities.
12. Write a comprehensive note on hadrons, leptons and quarks.
13. What are harmful effects of radiations? What measures can be adopted to safeguard us from the nuclear hazards.
14. Explain tracer technique in agricultural research.
15. Name the four fundamental interaction and the particles that mediate each interaction.
16. Discuss the differences between hadrons and leptons.

Numerical Problems:

1. Find the mass defect and binding energy for helium nucleus?

[0.03038 a·m·u, 28.3 Mev]

2. A certain radioactive isotope has half life of 8 hours. A solution containing 500 million atoms of this isotope is prepared. How many atoms of this isotope have not disintegrated after

a. 8 hours   b. 24 hours

[250 million, 62.5 million]

3. Write the nuclear equations for the beta decay of

(a) $_{82}Pb^{210}$ b. $_{83}Bi^{210}$ c. $_{90}Th^{234}$ d. $_{93}N_p^{239}$

4. Calculate the total energy released if 1 kg of U^{235} undergoes fission? Taking the disintegration energy per event to be $Q = 208\ MeV$.

$[5.32 \times 10^{26}\ MeV]$

5. Find the energy released in the following fission reaction.

$$_0n^1 + {}_{92}U^{235} \rightarrow {}_{36}Kr^{92} + {}_{56}Ba^{141} + 3{}_0n^1 + Q$$

[174 MeV]

6. Find the energy released in the fusion reaction. $_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

[17.6 MeV]

7. Complete the following nuclear reactions.

$$_7N^{14} + {}_2He^4 \rightarrow {}_1H^1 + ?$$

$$_5B^{11} + {}_1H^1 \rightarrow {}_6C^{11} + ?$$

$$_3Li^6 + ? \rightarrow {}_4Be^7 + {}_0n^1 \quad ({}_8O^{17}, {}_0n^1, {}_1H^2)$$

8. $_3Li^6$ is bombarded by deutrons. The reaction gives two α-particles along with release of energy equal to 22. 3 MeV. Knowing masses of deuteron and α-particles determine mass of lithium isotope of $_3Li^6$.

[6.017U]

9. Find the energy released when β-decay changes $_{90}Th^{234}$ into $_{91}Pa^{234}$. Mass of $_{90}Th^{234} = 234.0436u$ and $_{91}Pa^{234} = 234.042762u$. [0.279 MeV]

10. Find out the K.E to which a proton must be accelerated to induce the following nuclear reaction. $Li^7(p,n)Be^7$. [1.67 MeV]

Glossary

Atria: The **heart** is divided into four chambers that are connected by heart **valves.** The upper two heart chambers are called atria.
Alternating Current: Voltage forces electrons to flow in one direction and then quickly alternate to the opposite direction.
Ammeter A device to measure amperes (current).
Ampere Unit of current.
Alternator: A device that changes mechanical energy into an alternating current electrical energy, an AC generator.
Cerebrospinal fluid (CSF): a clear colorless bodily fluid found in the brain and spine.
Conductor A material that permits a very free exchange/movement of electrons from one atom to another.
Conventional Flow This theory states that electrons flow from positive (+) to negative (-).
Current The flow of electrons in the same direction from atom to atom.
Direct Current Voltage forces the electrons to flow continuously in one direction.
Dysfunction: abnormality in the function of an organ or part.
Dynamoelectric machine: A dynamo or generator
Disgorge: taken out from the throat what has been eaten.
Dynamo: From the Greek word dynamis, which means power
Dynamoelectric: Relating to the conversion by induction of mechanical energy into electrical energy or vice versa

Extra-terrestrial sources
Sources that are existing or originating outside the limits of the earth.
Electromagnets Do not retain their magnetism after a magnetizing force is removed.
Electromagnetic Induction: The creation of voltage in a conductor from movement of the conductor or the magnetic field.
Electron Flow: This theory states that electrons flow from negative (-) to positive (+).
Frequency The number of cycles in one second of alternating current.
Expressed in hertz (Hz). For example, 60 Hz is 60 cycles in one second.
Generator: A device that changes mechanical energy into electrical energy. Although the terms AC and DC generator are in common usage, a generator is normally considered to be a device that provides DC current.
Insulators Materials that don't readily give up electrons, thereby restricting the flow of current.
Peak inverse voltage (PIV)
The maximum voltage V_m the diode can withstand during the negative half cycle (reverse bias) is known as peak inverse voltage (PIV).
Myelin:(Life Sciences & Allied Applications / Anatomy) a white tissue forming an insulating sheath **(myelin sheath)** around certain nerve fibres. Damage to the myelin sheath causes neurological disease, as in multiple sclerosis.
Motor: From the Latin word motus, one that imparts motion, prime mover. A device that changes electrical energy into mechanical energy.

Muon
A negatively charged subatomic particle, with the same charge as an electron. It is about 200 times as massive as an electron.
Ohm Unit of resistance.
Ohm's Law Current is directly proportional to voltage and inversely proportional to resistance.
Out of step: out of step mean that the two waveforms are not synchronized.
Parallel Circuits Loads are connected across the power line to form branches.
Permanent Magnets: Retain their magnetism after a magnetizing force is removed.
Resistance The restriction to the flow of electrons.
Right-Hand Rule A current carrying conductor held in right hand will indicate the direction of lines of flux. **ELECTRICITY**
RMS Value: Root Mean Square Current is also referred to as effective current and is the square root of the average of all the instantaneous currents (current at any point on a sine wave) squared.
Series Circuit All loads in the circuit are connected one after the other.
Single-Phase: A continuous single alternating current cycle.
Spectrum : spectrum means set of freqtuencies absorbed or emitted by a substance.
Terrestrial sources
Sources that are existing or originating inside the limits of the earth.
Three-Phase A continuous series of three overlapping AC cycles offset by 120 degrees.
Transformer A device used to raise (step up) or lower (step down) a voltage level.
Ventricle - a chamber of the heart that receives blood from an atrium and pumps it to the arteries. Or the heart is divided into four chambers that are connected by heart valves. The lower two chambers of the heart are called ventricles.
Volt Unit of force applied to a conductor to free electrons, to cause electrical current flow.
Voltage The force applied to a conductor to free electrons, causing electrical current to flow.
Voltage Drop Voltage value as measured across each resistor or load.
Voltmeter A device to measure voltage.
Waveform: The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa is called its waveform or wave shape.
Watt The basic unit of power, indicating the amount of work accomplished when one volt causes one ampere to pass through a circuit.

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LIST OF PRACTICAL FOR CLASS 12

Standard experiments

1. Determine time constant by charging and discharging a capacitor through a resistor.
2. Determine resistance of wire by slide Wire Bridge.
3. Determine resistance of voltmeter by drawing graph between R and I/V.
4. Determine resistance of voltmeter by discharging a capacitor through it.
5. Analyse the variation of resistance of thermistor with temperature.
6. Determine internal resistance of a cell using potentiometer.
7. Determine emf of a cell using potentiometer.
8. Determine the emf and internal resistance of a cell by plotting V against I graph.
9. Investigate the relationship between current passing through a tungsten filament lamp and the potential applied across it.
10. Convert a galvanometer into voltmeter of range 0 – 3 V.
11. Determine the relation between current and capacitance when different capacitors are used in AC circuit using different series and parallel combinations of capacitors.

12. Determine the impedance of a RL circuit at 50Hz and hence find inductance.
13. Determine the impedance of a RC circuit at 50Hz and hence find capacitance.
14. Determine Young's modulus of the material of a given wire using Searle's apparatus.
15. Draw characteristics of semiconductor diode and calculate forward and reverse current resistances.
16. Study the half and full wave rectification by semiconductor diodes by displaying on CRO
17. Study of the variation of electric current with intensity of light using a photocell.
18. Determine Planck's constant using internal potential barrier of different light emitting diodes.
19. Observe the line spectrum of mercury with diffraction grating and spectrometer to determine the wavelength of several different lines, and hence, draw a conclusion about the width of visible spectrum.
20. Using a set of at least 100 dice, simulate the radioactive decay of nuclei and measure the simulated half life of the nuclei.
21. Draw the characteristics curve of a Geiger Muller tube.
22. Determine the amount of background radiation in your surrounding and identify their possible sources.
23. Set up a G.M. point tube and show the detection of alpha particles with the help of CRO and determine the count rate using scaler unit.

Note:

1. At least 20 standard practical alongwith exercises are required to be performed during the course of studies of class 12.
2. Use of centimetre graph paper be made compulsory.

Some Important Values

Charge of electron is **1.6022 x 10^{-19} Coulomb.**
Mass of electron: Mass of electron is **0.000548597 a.m.u.** or **9.1 x 10^{-31} kg.**
Symbol of electron: Electron is represented by **"e".**

Charge of proton is **1.6022 x 10^{-19} coulomb.**
Mass of proton: Mass of proton is **1.0072766 a.m.u.** or **1.6726 x 10^{-27} kg.**
Comparative mass: Proton is **1837** times heavier than an electron.

Mass of neutron: . Mass of neutron is **1.0086654 a.m.u.** or **1.6749 x 10^{-27} kg.**
Compartive mass: Neutron is **1842** times heavier than an electron.

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